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Modeling the Impacts of Diagnosis and Treatment of Snakebite Envenoming Victims with Anti-Snake Venom in a Community

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ABSTRACT

Snakebite envenoming (SBE) is one of the major causes of deaths and disabilities in tropical and subtropical countries especially in the continents of Asia and Africa. Report revealed that on average, 5-8 and about 3,000 cases of SBE are recorded daily and yearly respectively in northeast Nigeria which population are largely agriculturists. Thus, snakebite (SB) led to economy setback and slows down productivity among the workforce as a result of disabilities induced by SBE. Consequently, the revenue generation even including income facing declined. This paper proposes population-based mathematical model for studying the dynamic impacts of diagnosis and treatment of snakebite envenoming victims with anti-snake venom in a community. The model is rigorously analyzed using relevant techniques and theorems in dynamical systems. The model was validated using monthly reported data on snakebite envenoming from 2020–2022, collected from snakebite treatment and research hospital (STRH) Kaltungo in northeast Nigeria. The qualitative study revealed that the model has snakebite-envenoming-free and snakebite-envenoming-endemic equilibrium points. It is shown via the Lyapunov function in conjunction with the Lassale invariance principle that these two equilibrium points are globally asymptotically stable. Furthermore, numerical simulations of the model were carried out to investigate the impact of the diagnosis of snakebite envenoming victims and the treatment of the same on the burden of snakebite envenoming. The results of the simulations revealed that appropriate diagnosis and treatment of snakebite envenoming victims with anti-snake venom could significantly reduce the number of snakebite-induced deaths and disabilities. This work can serve as a framework for making policy and decision in addressing the issues of SBE in the public healthcares in the study region and beyond.

1. Introduction

Snakebite (SB) is an injury caused by a bite from a snake, resulting in puncture wounds inflicted by the fangs, while snakebite envenoming (SBE) is a potentially life-threatening disease which occurs as a result of an injection of a complex mixture of different toxins following the bite of a venomous snake (Warrell, 2010 and Williams, *et al*,. 2019). The severity of snakebite depends on some factors, such as the kind of snake, the part of the body bitten, the quantity of venom injected, and the general health of the person bitten (Abdullahi, Habib & Hussaini, 2021). Generally, there are more than 3000 species of snakes in the world, and they live in terrestrial and aquatic ecosystems. They are predatory carnivores with a wide range of prey species. Approximately 15% of the 3000 species of snakes worldwide are considered dangerous to humans and responsible for significant morbidity and mortality, (WHO, 2017; Gold, Dart & Barish, 2002). Snake venom is the most complex of all natural venoms and poisons and it varies in composition from species to species, (Menez, 2003). There is a considerable variation also in the venom constituents within a single species within a geographical distribution at different season (Warrel, 1996). The four key toxins of snake venom includes serine protein (SVSP), three finger toxins (3FTx), phospholipase A_2 (PLA_2) and snake venom metalloproteinease (SVMP), (Casewell & Clevers, 2020).

SBE is a significant public health concern in many parts of the world, particularly in rural areas of tropical and subtropical regions. It is estimated that 1.7 billion lives globally are threatened annually by neglected tropical diseases, of which SBE is among them. An estimated 5.4 million cases of SB occur every year, and it is responsible for causing around 2.7 million cases of envenoming, with about 138,000 cases of deaths and more than 400,000 people suffering from disabilities around the world, (Warrell, 2010; Williams, *et al*,. 2019 and WHO, 2019). SBE is most prevalent in Africa, Asia, and Latin America. According to research, individuals living in rural areas of these regions are at a higher

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risk of SBE due to their occupational exposure to snakes. SBE causes morbidity and mortality among farmers, pastoralists, hunters, and children, (WHO, 2019; Longbottom, *et al.,* and Gampin, Nassouri, Chippaux & Semde, 2016). Those who suffered the most from SB are the young agriculturists, the most economically productive group, followed by children, (Warrell, 2010 & WHO, 2019).

SBE can lead to a wide range of clinical manifestations, including local effects (such as pain, swelling, and tissue damage), systemic effects (such as bleeding disorders, cardiovascular collapse, and renal failure), and neurotoxic effects (such as paralysis and respiratory failure). Limited access to healthcare facilities, especially in rural areas, is a major challenge in the management of SBE. This often results in delayed treatment and increased morbidity and mortality, (Gutierrez *et al.,* 2017). SBE can be treated when adequately diagnosed. Usually, the clinical outcomes of the diagnosis of SBE cases for management can be classified as mild, moderate, or severe. The toxicity effects induced by SBE are currently treated with intravenous administration of anti-snake venom (ASV) in combination with analgesic fluid therapy, hemodialysis, and/or antibiotics, (Harrison & Williams, 2019; Gutierrez, *et al.,* 2007 and Gutierrez *et al.,* 2017). The availability and accessibility of effective ASV are crucial for treating SBE. However, some studies have shown significant challenges related to the production, distribution, and affordability of ASV in many affected regions (Habib & Brown 2018 and Gutiérrez, 2018).

Despite the dangers posed by snakes, they benefit humans by killing unwanted insects and rodents in food stores and farms, (Katie & Kennymac, 2013 and Abdullahi, Musa, He & Bello 2021). Snake-skins are used to make shoes, handbags, and other articles. And snake park serve as tourism attractions. The venom is used for producing life-saving ASV, biomedical research and other medicinal products, and to control infections such as protozoan infections, cancers, and so on, (Abubakar *et al.,* 2010; Agom, Moses & Priti,.2022; Alams & Gomes, 2003; Zainal Abidin, Lee, Lekhsan & Rakesh, 2019 and Hussaini, Okuneye & Gumel, 2017).

It is noteworthy to mention that several mathematical models have been developed to understand the transmission dynamics and control of some neglected tropical diseases, such as Leishmaniasis, Rabies, Chikungunya, Dengue, Chagas disease, Onchocerciasis, Lymphatic Filariasis, and Schistosomiasis See, for instance, Adamu & Hussaini, 2019; Shigui, 2017; Zhang, Jin, Sun, Zhou & Ruan, 2011; Yakob & Clements 2013; Sabiu & Hussaini, 2017; Bhunu & Mushayabasa, 2012; Dumont & Chiroleu 2010; Omondi, Nyabadza, Bonyah, & Badu, 2017; Sabiu, Hussaini, Zhao & Daihai, 2020 and Woolhouse, 1992) and references therein. Unlike some of those mentioned neglected tropical diseases, it has been recognized to the best of the authors' knowledge that not many mathematical models have been constructed to analyze the dynamics and control of SBE. However, some few researches include prediction of snake distributions in relation to its transmission using ecological niche modeling, estimation of incidence of snakebite by application of the law of mass action, Goldstein, Erinjery, Martin, Kasturiratne, Ediriweera & de Silva (2021) predicting snakebite risk using species distribution model Casewell & Clevers (2020), mathematical model based on social demography, economic and health infrastructure to examine the impact of behavior on the overall mortality outcomes of snakebite victims Goldstein *et al.,* (2021) and evaluation of intervention strategies with effective cost in management of snakebite using mechanistic model (Yañez-Arenas, Peterson, Mokondoko, Rojas-Soto & Martínez-Meyer, 2014). Some of other few ones can be found in the following studies (Abdullahi, Habib & Hussaini, 2021; Bravo-Vega, Santos-Vega, Juan & Cordovez, 2021; Martin, Erinjery, Ediriweera, de Silva, Lalloo, Iwamura & Murray, 2022; Kumar, Vedat Suat Erturk, Govindaraj, & Baleanu, 2023 and Abdullahi, Habib. & Hussaini, 2024) and references therein.

In management of snakebite, effective treatment is associated to proper diagnosis (mild, moderate and severe) of victim of snakebite envenoming which were not captured in previous SBE models. Therefore, this study aims to develop a population-based mathematical model to investigate the impacts of the diagnosis of SBE victims and their treatment with ASV on the dynamics of SBE. A notable feature of the model is the incorporation of the rate of diagnosis of SBE victims with three different conditions, mild, moderate, and severe. The model also includes the death rate of snakes due to human activities. The paper is organized as follows: The model is formulated in Section 2, and the analysis of the mathematical properties of the model and its validation via model fitting are presented in Section 3. The numerical simulations are explored in Section 4, and the conclusion is presented in Section 5.

2. Methods

2.1 Model formulation

In this section, the procedures for formulating the model are presented.

2.1.1 Assumptions of the model

The following are some of the major assumptions made in the formulation of the model:

i. all species of venomous snakes are grouped into a single class;
ii. effective treatment is associated with a proper diagnosis;
iii. recovered individuals are not immune to snakebite thus they transit to the susceptible class after recovery;
iv. snakebite-induced death is negligible in all treatment classes.

2.1.2 *Description of the model*

The human population at time t, denoted by $\left(N_h\left(t\right)\right)$, is divided into nine (9) mutually exclusive compartments, namely: population of susceptible persons $\left(S\left(t\right)\right)$, population of undiagnosed SBE victims $\left(I\left(t\right)\right)$, population of SBE victims diagnosed with mild conditions $\left(I_{ml}\left(t\right)\right)$, population of SBE victims diagnosed with moderate conditions $\left(I_{mo}\left(t\right)\right)$, population of SBE victims diagnosed with severe conditions $\left(I_s\left(t\right)\right)$, population of SBE victims receiving treatment with ASV $\left(T\left(t\right)\right)$, population of persons suffering from early adverse reactions during treatment with ASV $\left(E_{ar}\left(t\right)\right)$, population of persons who recovered with disability $\left(R_d\left(t\right)\right)$ and population of persons who recovered without disability $\left(R_w\left(t\right)\right)$. Thus, the total human population is given by

$$N_h\left(t\right)=S\left(t\right)+I\left(t\right)+I_{ml}\left(t\right)+I_{mo}\left(t\right)+I_s\left(t\right)+T\left(t\right)+E_{ar}\left(t\right)+R_d\left(t\right)+R_w\left(t\right). \quad (1)$$

The total population of venomous snakes is represented by $\left(V_s\left(t\right)\right)$. In this work, susceptible persons refer to those at risk of being bitten by a venomous snake. The population of susceptible persons is generated by a constant recruitment rate given by Λ_h, recovered persons with disabilities and without disabilities who become susceptible again at the rates κ_1 and κ_2 respectively. It diminishes by susceptible persons who become victims of SBE at a rate λ and natural death rate μ_h. Thus, the equation governing the dynamics of the susceptible population is given by

$$\frac{dS}{dt}=\Lambda_h+\kappa_1R_d+\kappa_2R_w-\left(\lambda+\mu_h\right)S, \quad (2)$$

where

$\lambda=\frac{\beta V_s}{N_h}$ and β is the effective contact rate between venomous snakes and humans.

The population of undiagnosed SBE victims is generated by susceptible persons who are bitten by snakes and become victims of SBE as given by λS. The population decreased due to the diagnosis of SBE victims with mild, moderate and severe conditions at the rates, θ_1,θ_2 and θ_3 respectively. It also diminishes due to SBE-induced mortality at a rate δ_1 and natural mortality at the rate μ_h.. Hence, we have the following equation describing the dynamics of undiagnosed SBE victims:

$$\frac{dI}{dt}=\lambda S-\left(\theta_1+\theta_2+\theta_3+\delta_1+\mu_h\right)I. \quad (3)$$

The population of SBE victims diagnosed with mild conditions is generated by those SBE victims diagnosed with mild conditions at the rate θ_1 and diminishes by those receiving treatment at the rate τ_1 and natural mortality at the rate μ_h .. Thus, we obtain the following equation for SBE victims diagnosed with mild conditions:

$$\frac{dI_{ml}}{dt} = \theta_1 I - \left(\tau_1 + \mu_h\right) I_{ml} . \tag{4}$$

The population of SBE victims diagnosed with moderate conditions is generated by those SBE victims diagnosed with mild conditions at the rate θ_2 and the population diminishes by those receiving treatment at a rate τ_2 , and natural mortality at the rate μ_h . Thus, we obtain the following equation:

$$\frac{dI_{mo}}{dt} = \theta_2 I - \left(\tau_2 + \mu_h\right) I_{mo}. \tag{5}$$

The population of SBE victims diagnosed with severe conditions is generated by those SBE victims diagnosed with severe conditions at the rate θ_3 , it diminishes by those receiving treatment at a rate τ_3 , SBE-induced mortality at the rate δ_2 , and natural mortality at the rate μ_h . Hence, we have the following equation:

$$\frac{dI_s}{dt} = \theta_3 I - \left(\tau_3 + \delta_2 + \mu_h\right) I_s . \tag{6}$$

The population of SBE victims receiving treatment with ASV is generated by SBE victims diagnosed with mild, moderate, and severe conditions at the rates τ_1, τ_2 and τ_3 , respectively. This population is reduced by proportions of persons who recovered with or without disability at a rate σ , those who suffer from early adverse reactions at a rate γ and natural mortality at a rate μ_h . Hence, we have the following equation:

$$\frac{dT}{dt} = \tau_1 I_{mi} + \tau_2 I_{mo} + \tau_3 I_s - \left(\sigma + \gamma + \mu_h\right) T . \tag{7}$$

The population of persons suffering from early adverse reactions due to ASV treatment is generated by SBE victims on treatment who suffer from early adverse reactions due to ASV at the rate γ . This population diminishes due to those persons who recovered with or without disability at a rate ψ and natural mortality at a rate μ_h . Thus, we have the following equation:

$$\frac{dE_{ar}}{dt} = \gamma T - \left(\psi + \mu_h\right) E_{ar} . \tag{8}$$

The population of persons recovered with disability is generated by the proportions of persons who recovered with disabilities in the treatment class at the rate $\sigma\rho$ and in the early adverse reaction class at the rate $\psi\phi$. It diminishes because of movement to a susceptible class at the rate κ_1 and natural mortality at the rate μ_h . Hence, we have the following equation:

$$\frac{dR_d}{dt} = \sigma\rho T + \psi\phi E_{ar} - \left(\kappa_1 + \mu_h\right) R_d . \tag{9}$$

The population of persons who recovered without disabilities is generated by the proportions of persons in the treatment class who recovered without disabilities at the rate $\sigma\left(1-\rho\right)$ and in the early adverse reaction classat the rate $\psi\left(1-\phi\right)$. It diminishes because of transition to susceptible class at the rate κ_2 and natural mortality at the rate μ_h . Thus, we have the following equation:

$$\frac{dR_w}{dt} = \sigma(1-\rho)T + \psi(1-\phi)E_{ar} - (\kappa_2 + \mu_h)R_w. \tag{10}$$

The venomous snake population increases at a rate Λ_s ,which is reduced by natural mortality rate μ_s and human activities induce mortality at a rate ζ . Thus, we obtain the equation governing the dynamics of the venomous snakes as

$$\frac{dV_s}{dt} = \Lambda_s - (\zeta + \mu_s)V_s. \tag{11}$$

2.1.3 *The model equation*

Therefore, the model describing the dynamics of SBE is given by the following system of first-order nonlinear ordinary differential equations (the model schematic diagram is depicted in Fig. 1, and the associated state variables and parameters are presented in Tables 1 and 2, respectively).

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda_h + \kappa_1 R_d + \kappa_2 R_w - (\lambda + P_1)S,\\
\frac{dI}{dt} &= \lambda S - P_2 I,\\
\frac{dI_{ml}}{dt} &= \theta_1 I - P_3 I_{ml},\\
\frac{dI_{mo}}{dt} &= \theta_2 I - P_4 I_{mo},\\
\frac{dI_s}{dt} &= \theta_3 I - P_5 I_s,\\
\frac{dT}{dt} &= \tau_1 I_{ml} + \tau_2 I_{mo} + \tau_3 I_s - P_6 T,\\
\frac{dE_{ar}}{dt} &= \gamma T - P_7 E_{ar},\\
\frac{dR_d}{dt} &= \sigma\rho T + \psi\phi E_{ar} - P_8 R_d,\\
\frac{dR_w}{dt} &= \sigma\Gamma_1 T + \psi\Gamma_2 E_{ar} - P_9 R_w,\\
\frac{dV_s}{dt} &= \Lambda_s - P_{10} V_s,
\end{aligned} \tag{12}$$

with the following initial conditions

$$S(0) > 0, I(0) \geq 0, I_{ml}(0) \geq 0, I_{mo}(0) \geq 0, I_s(0) \geq 0, T(0) \geq 0, E_{ar}(0) \geq 0, R_d(0) \geq 0, R_w(0) \geq 0, V_s(0) > 0,$$

where

$$P_1 = \mu_h,\ P_2 = (\theta_1 + \theta_2 + \theta_3 + \delta_1 + \mu_h),\ P_3 = (\tau_1 + \mu_h),\ P_4 = (\tau_2 + \mu_h), P_5 = (\tau_3 + \delta_2 + \mu_h), P_6 = (\sigma + \gamma + \mu_h),$$

$$P_7 = (\psi + \mu_h),\ P_8 = (\kappa_1 + \mu_h),\ P_9 = (\kappa_2 + \mu_h),\ P_{10} = (\zeta + \mu_s), \Gamma_1 = (1-\rho),\ \Gamma_2 = (1-\phi), \lambda = \frac{\beta V_s}{N_h}.$$

Table 1: Definition of State Variables of the Model

Variable(s)	Description
S(t)	Population of susceptible persons at time t

$I(t)$	Population of undiagnosed SBE victims at time t
$I_{ml}(t)$	Population of SBE victims diagnosed with mild conditions at time t
$I_{mo}(t)$	Population of SBE victims diagnosed with moderate conditions at time t
$I_s(t)$	Population of SBE victims diagnosed with severe conditions at time t
$T(t)$	Population of SBE victims receiving treatment with ASV at time t
$E_{ar}(t)$	Population of persons who suffer from the early adverse reaction at time t
$R_d(t)$	Population of persons who recovered with disabilities at time t
$R_w(t)$	Population of persons who recovered without disabilities at time t
$V_s(t)$	Population of venomous snakes at time t

Table 2: Definition of Parameters of the Model

Parameter(s)	Description
$\Lambda_h\left(\Lambda_s\right)$	Recruitment rate into susceptible class(Venomous snakes)
λ	Rate of susceptible persons who become victims of SBE
$\theta_i\left(i=1,2,3\right)$	Rates of diagnosis of SBE victims that produce mild, moderate and severe conditions respectively
$\tau_i\left(i=1,2,3\right)$	Rate of treatment of SBE victims with mild, moderate and severe conditions respectively
$\delta_i\left(i=1,2\right)$	SBE-induced death rates in I and I_s classes respectively
ρ	Proportion of persons who recovered with disabilities in T class
ϕ	Proportion of persons who recovered with disabilities in E_{ar} class
σ	Recovery rate of persons in T class
ψ	Recovery rate of persons in E_{ar} class
ζ	Death rate of venomous snakes due to human activities
$\mu_h\left(\mu_s\right)$	Humans (venomous snakes) natural mortality rates
$\kappa_1\left(\kappa_2\right)$	Transition rates to susceptible classof recovered persons with disability (without disability)
β	Effective contact rate between venomous snakes and humans
γ	Rate of persons who suffered from early adverse reactions

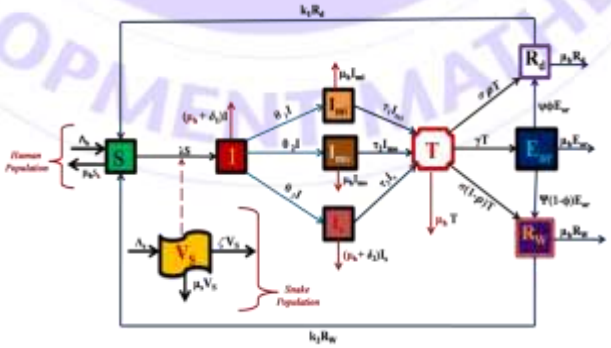

Figure 1: Schematic diagram of the model

3. Results

3.1 Model Analysis

For model analysis, we assume that all the model parameters are positive. Further, the assumption that the recovered person transit to the susceptible class is relaxed (i.e., $\kappa_1 = \kappa_2 = 0$) for simplicity and mathematical convenience.

3.1.1 Basic properties of the model

The basic properties of the model in terms of positivity and boundedness of solution are explored in the following theorems:

Theorem 1: *For any non-negative initial conditions of the model equations given by system (12), the solution* $\left(S, I, I_{ml}, I_{mo}, I_s, T, E_{ar}, R_w, R_d, V_s\right)$ *of the model equation will be positive for all* $t > 0$.

Proof. *Let*

$t_1 = \sup\left\{t > 0 : S > 0, I > 0, I_{ml} > 0, I_{mo} > 0, I_s > 0, T > 0, E_{ar} > 0, R_d > 0, R_w > 0, V_s > 0\right\}$, *therefore, t> 0. Consider the first equation of system (12) as follows:*

$$\frac{dS}{dt} = \Lambda_h - \left(\lambda + P_1\right)S, \text{ since } \Lambda_h > 0 \text{ we have } \frac{dS}{dt} \geq -\left(\lambda + P_1\right)S.$$

Separating the variables and integrating both sides from $t = 0$ to $t = t_1$ *we obtain*

$$S\left(t_1\right) \geq S\left(0\right)\exp\left(-\left[P_1 t_1 + \int_0^{t_1} \lambda\left(y\right)dy\right]\right) > 0, \text{ thus, } S\left(t\right) > 0, \text{ for every } t > 0.$$ *Using a similar approach it can be shown that the remaining state variables, i.e.,*

$I\left(t\right) > 0, I_{ml}\left(t\right) > 0, I_{mo}\left(t\right) > 0, I_s\left(t\right) > 0, T\left(t\right) > 0,$

$E_{ar}\left(t\right) > 0, R_d\left(t\right) > 0, R_w\left(t\right) > 0, V_s\left(t\right) > 0$ *for all* $t > 0$, *hence the proof is complete.*

Lemma 1. *The solution of the model equations given by system (12) is bounded in the closed set*

$$\Omega = \Omega_h \cup \Omega_s \subset \mathbb{R}_+^9 \times \mathbb{R}_+, \text{where } \Omega_h = \left\{\left(S, I, I_{ml}, I_{mo}, I_s, T, E_{ar}, R_d, R_w\right) \in \mathbb{R}_+^9 : 0 \leq N_h \leq \frac{\Lambda_h}{\mu_h}\right\} \text{ and}$$

$$\Omega_s = \left\{V_s \in \mathbb{R}_+ : 0 \leq V_s \leq \frac{\Lambda_s}{\mu_s}\right\}.$$

Furthermore, the set Ω *is positively invariant with respect to the model equations.*

Proof. *The total human population is given by*

$$N_h = S + I + I_{ml} + I_{mo} + I_s + T + E_{ar} + R_d + R_w \quad . \tag{13}$$

Thus, by differentiating equation (13), we have

$$\frac{dN_h}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dI_{ml}}{dt} + \frac{dI_{mo}}{dt} + \frac{dI_s}{dt} + \frac{dT}{dt} + \frac{dE_{ar}}{dt} + \frac{dR_d}{dt} + \frac{dR_w}{dt}, \tag{14}$$

substituting the right-hand sides of the system (12) into equation (14) and simplifying we obtain

$$\frac{dN_h}{dt} = \Lambda_h - N_h\mu_h - \delta_1 I - \delta_2 I_s.$$

Neglecting SBE-induced mortality in I and I_s *classes, we have*

$$\frac{dN_h}{dt} \leq \Lambda_h - N_h\mu_h \text{ and } \frac{dN_h}{dt} \geq -N_h\mu_h$$

Separating the variables and integrating both sides we have

$N_h(t) \geq N_h(0)e^{-\mu_h t}$ *and* $N_h(t) \leq \frac{\Lambda_h}{\mu_h} - \left[\frac{\Lambda_h - \mu_h N_h(0)}{\mu_h}\right]e^{-\mu_h t}$ *. This can be rewritten as*

$N_h(0)e^{-\mu_h t} \leq N_h(t) \leq \frac{\Lambda_h}{\mu_h} - \left[\frac{\Lambda_h - \mu_h N_h(0)}{\mu_h}\right]e^{-\mu_h t}$ *. It follows that*

$0 \leq \liminf_{t\to\infty} N_h(t) \leq \limsup_{t\to\infty} N_h(t) \leq \frac{\Lambda_h}{\mu_h}$ *.Similarly, considering the equation for the venomous snake population from system (12), we have* $\frac{dV_s}{dt} = \Lambda_s - \mu_s V_s - \zeta V_s$*. Neglecting the venomous snakes' death rate due to human activities, we have* $\frac{dV_s}{dt} \leq \Lambda_s - \mu_s V_s$ and $\frac{dV_s}{dt} \geq -\mu_s V_s$*. Separating the variables and integrating both sides, we obtain* $V_s(0)e^{-\mu_s t} \leq V_s(t) \leq \frac{\Lambda_s}{\mu_s} - \left[\frac{\Lambda_s - \mu_s V_s(0)}{\mu_s}\right]e^{-\mu_s t}$ *. It follows that*

$0 \leq \liminf_{t\to\infty} V_s(t) \leq \limsup_{t\to\infty} V_s(t) \leq \frac{\Lambda_s}{\mu_s}$ *.Now, if* $N_h(0) \leq \frac{\Lambda_h}{\mu_h}$ and $V_s(0) \leq \frac{\Lambda_s}{\mu_s}$*, then we have* $N_h(t) \leq \frac{\Lambda_h}{\mu_h}$ and $V_s(t) \leq \frac{\Lambda_s}{\mu_s}$ *respectively. Thus the region* Ω *is positively Invariant meaning that the solution starting in* Ω *stays in* Ω *for t>0. In addition, if* $N_h(0) > \frac{\Lambda_h}{\mu_h}$ and $V_s(0) > \frac{\Lambda_s}{\mu_s}$*, then either the solution enters the region* Ω *in finite time or* $N_h(t)$ *approaches* $\frac{\Lambda_h}{\mu_h}$ *asymptotically, and* $N_s(t)$ *approaches* $\frac{\Lambda_s}{\mu_s}$ *asymptotically. Thus, the region* Ω *attracts all solutions in* $\mathbb{R}^{10}_+$*, so the proof is complete.*

Since the region Ω is epidemiologically and mathematically realistic, it suffices to study the dynamics of the model equations given by system (12) in the region, where the usual existence, uniqueness and continuation of solution hold.

3.2 Model Fitting and Parameter Estimation

Monthly data on snakebite envenoming for the period 2020–2022 were collected from the snakebite treatment and research hospital (STRH) in Kaltungo, Gombe State. The data is used to estimate the unknown values of parameters as well as to validate the model. This is done by using the nonlinear least squares method implemented in Matlab. The graphs depicted in Figures 2–10 demonstrate how well the model fits the reported data with respect to the cumulative number of monthly cases of the following: undiagnosed SBE victims; SBE victims diagnosed with mild conditions; SBE victims diagnosed with moderate conditions; SBE victims diagnosed with severe conditions; SBE victims receiving treatment with ASV; persons suffering from early adverse reactions; persons recovered with disabilities; persons recovered without disabilities; and SBE-induced deaths. In addition, the estimated values of the initial conditions and the parameters in the model are presented in Table 3 and Table 4, respectively. Since the estimated outcomes of the model are in good agreement with the actual reported data, the model can be used to predict the cases of SBE and study its dynamics in a given population.

Table 3: Reported data and the estimated values of the state variables

Variable(s)	Value(s)	Reference
$S(0)$	1.9315×10^{6}	Fitted
$I(0)$	69	Data(STRH)
$I_{ml}(0)$	9	Data(STRH)
$I_{mo}(0)$	8	Data(STRH)
$I_s(0)$	29	Data(STRH)
$T(0)$	46	Data(STRH)
$E_{ar}(0)$	1	Data(STRH)
$R_d(0)$	0	Data(STRH)
$R_w(0)$	46	Data(STRH)
$V_s(0)$	1.2431×10^{4}	Fitted

Table 4: Estimated values of the state model parameters

Parameter(s)	Value(s)	Unit	Reference
Λ_h	1336	Day^{-1}	(Abdullahi, Habib & Hussaini, 2021)
Λ_s	0.2206	Day^{-1}	Fitted
$\theta_i(i = 1, 2, 3)$	$0.2775, 0.1830, 1.0000$	Day^{-1}	Fitted
$\tau_i(i = 1, 2, 3)$	$1.0000, 1.0000, 0.4888$	Day^{-1}	Fitted
γ	0.0166	Day^{-1}	Fitted
δ_1, δ_2	$0.0014, 1.9066\times10^{-8}$	Day^{-1}	Fitted
ρ	1.8805×10^{-4}	Nill	Fitted
σ	0.8751	Day^{-1}	Fitted
ψ	0.1713	Nill	Fitted
ζ	0.0537	Day^{-1}	Fitted
μ_h	5.04×10^{-6},	Day^{-1}	(Abdullahi, Habib & Hussaini, 2021)
μ_s	2.283×10^{-4}	Day^{-1}	Fitted
κ_1, κ_2	0.9973, 0.8290	Day^{-1}	Fitted
ϕ	0.9995	Day^{-1}	Fitted
β	0.08367	Nill	Fitted
K_s	4.1040×10^{8}	Day^{-1}	Fitted

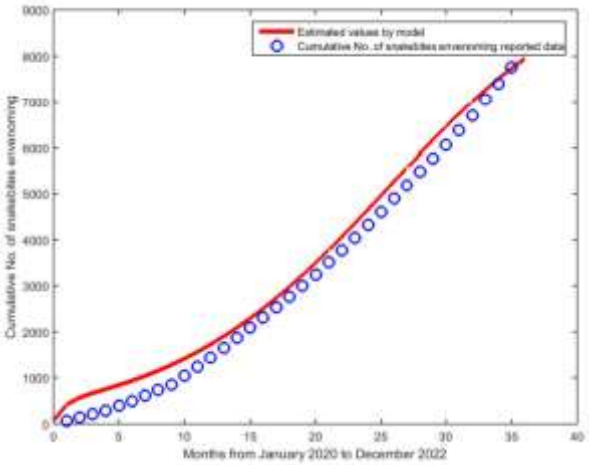

Figure 2: Graph showing the results of the model fitting with the cumulative number of reported data on undiagnosed SBE victims.

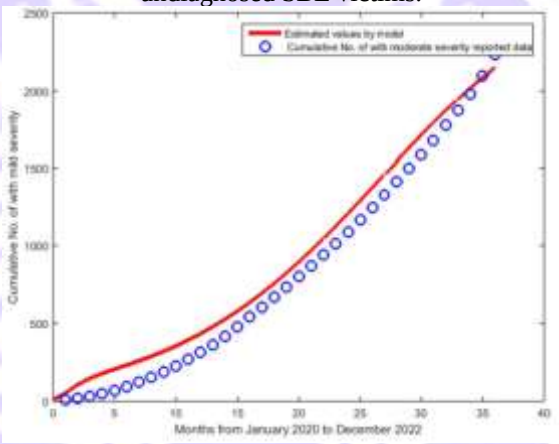

Figure 3: Graph showing the model results fitting with the cumulative number of reported data on SBE victims diagnosed with mild conditions.

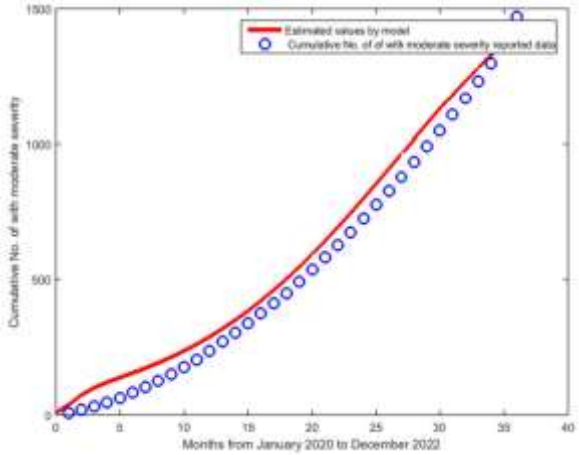

Figure 4: Graph showing the results of the model fitting with the cumulative number of reported data on SBE victims diagnosed with moderate conditions.

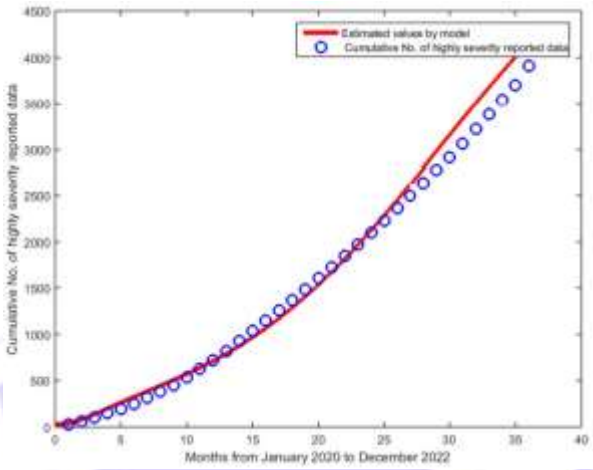

Figure 5: Graph showing the results of the model fitting with the cumulative number of reported data on SBE victims diagnosed with severe conditions.

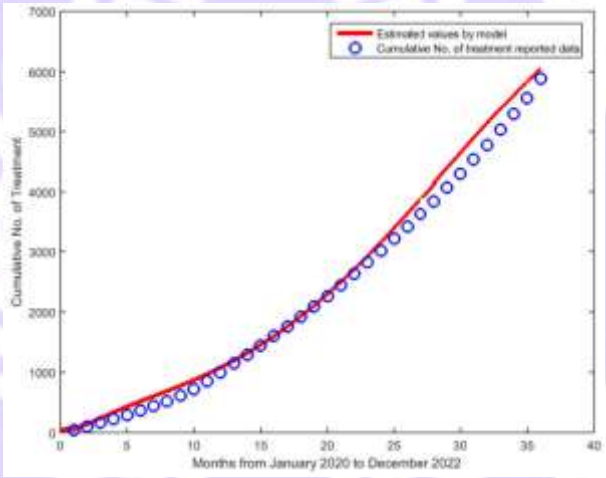

Figure 6: Graph showing the results of the model fitting with the cumulative number of reported data on SBE victims receiving treatment.

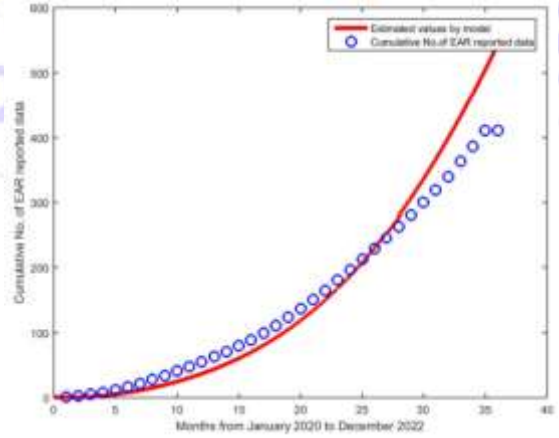

Figure 7: Graph showing the results of the model fitting with the cumulative number of reported data on individuals suffering from early adverse reactions uring treatment.

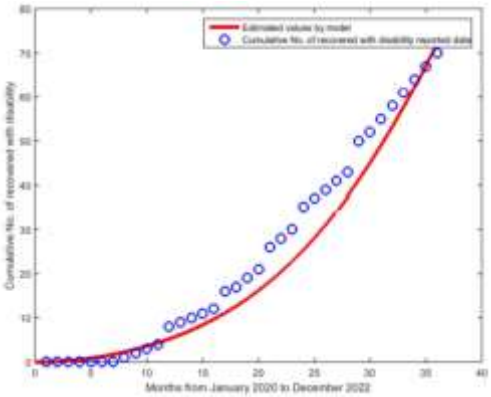

Figure 8: Graph showing the results of the model fitting with the cumulative number of reported data on persons who recovered with disabilities.

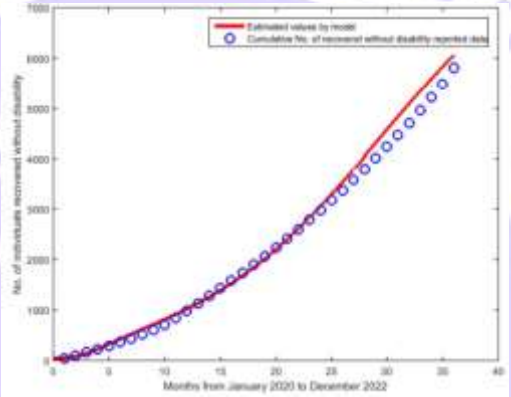

Figure 9: Graph showing the results of the model fitting with the cumulative number of reported data on persons who recovered without disabilities.

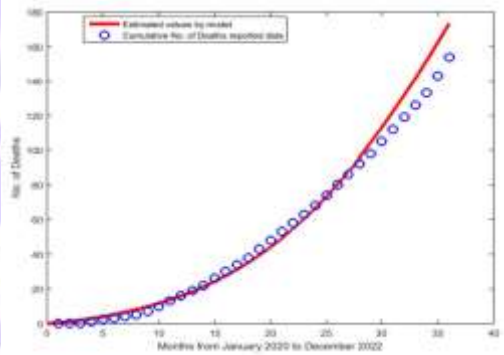

Figure 10: Graph showing the results of the model fitting with the cumulative number of reported data on SBE-induced deaths.

3.3 Equilibrium Points of the Model

In this section, we establish the conditions for the existence of equilibrium points in the model equations given by the system (12).

3.3.1 Existence of snakebite envenoming free equilibrium point

The Snakebite-envenoming free equilibrium point describes a circumstance where a community will be free from SBE. Biologically speaking, this equilibrium point can only exist if there is no contact between venomous snakes

and humans, i.e., $\beta = 0$ which also implies that the force of envenoming, $\lambda = 0$. Therefore, the existence of snakebite-envenoming free equilibrium point denoted by E^* is established and is presented as follows:

$$E^* = \left(S^*, I^*, I_{ml}^*, I_{mo}^*, I_s^*, T^*, E_{ar}^*, R_d^*, R_w^*, V_s^*\right) = \left(\frac{\Lambda_h}{P_1}, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\Lambda_s}{P_{10}}\right).$$

3.3.2 Existence of snakebite envenoming endemic equilibrium point

The Snakebite-envenoming endemic equilibrium point demonstrates a situation where SBE will persist in a given community. Biologically, this can due to interaction between venomous snakes and humans, i.e., $\beta > 0$, which also implies the force of envenoming, $\lambda > 0$. Now, to obtain the conditions for the existence of the snakebite-envenoming endemic equilibrium point,

Let $E^{**} = \left(S^{**}, I^{**}, I_{ml}^{**}, I_{mo}^{**}, I_s^{**}, T^{**}, E_{ar}^{**}, R_d^{**}, R_w^{**}, V_s^{**}\right)$ be an arbitrary endemic equilibrium point of the model equation given by system (12). In addition, let

$$\lambda^{**} = \frac{\beta V_s^{**}}{N_h^{**}}, \tag{15}$$

be the associated force of envenoming of the model equation at steady state. Now, the equations of the model given by system (12) are solved in terms of the force of envenoming at a steady state given by equation (15). Solving for the state variables in the model equation given by system (12) at steady-state gives

$$S^{**} = \frac{\Lambda_h}{\lambda^{**} + P_1}, I^{**} = \frac{\lambda^{**}\Lambda_h}{P_2\left(\lambda^{**} + P_1\right)}, I_{ml}^{**} = \frac{\theta_1\Lambda_h\lambda^{**}}{P_2P_3\left(\lambda^{**} + P_1\right)}, I_{mo}^{*} = \frac{\theta_2\Lambda_h\lambda^{**}}{P_2P_4\left(\lambda^{**} + P_1\right)}, I_s^{**} = \frac{\theta_3\Lambda_h\lambda^{**}}{P_2P_5\left(\lambda^{**} + P_1\right)},$$

$$T^{**} = \frac{\tau\Lambda_h\left(P_4P_5\theta_1 + P_3P_5\theta_2 + P_3P_4\theta_3\right)\lambda^{**}}{P_2P_3P_4P_5P_6\left(\lambda^{**} + P_1\right)}, E_{ar}^{**} = \frac{\gamma\tau\Lambda_h\left(P_4P_5\theta_1 + P_3P_5\theta_2 + P_3P_4\theta_3\right)\lambda^{**}}{P_2P_3P_4P_5P_6P_7\left(\lambda^{**} + P_1\right)},$$

$$R_d^{**} = \frac{\tau\Lambda_h\left(\gamma\phi\psi + \rho\sigma P_7\right)\left(P_3P_4\theta_3 + P_3P_5\theta_2 + P_4P_5\theta_1\right)\lambda^{**}}{P_2P_3P_4P_5P_6P_7P_8\left(\lambda^{**} + P_1\right)},$$

$$R_w^{**} = \frac{\tau\Lambda_h\left(\gamma\psi\Gamma_2 + \sigma\Gamma_1P_7\right)\left(P_4P_5\theta_1 + P_3P_5\theta_2 + P_3P_4\theta_3\right)\lambda^{**}}{P_2P_3P_4P_5P_6P_7P_9\left(\lambda^{**} + P_1\right)}, V_s^{**} = \frac{\Lambda_s}{P_{10}}.$$

Substituting the values of these state variables at steady state into the corresponding force of envenoming given by equation (15), we obtain the following quadratic equation in terms of λ^{**}, $A_2\lambda^{**2} + A_1\lambda^{**} - A_0 = 0$, (16)

where

$$A_2 = \gamma\phi\psi\tau\Lambda_hP_9P_{10}\left(P_4P_5\theta_1 + P_3P_5\theta_2 + P_3P_4\theta_3\right) + \gamma\psi\tau\Gamma_2\Lambda_hP_3P_8P_{10}\left(P_4P_5\theta_1 + P_5\theta_2 + P_4\theta_3\right)$$
$$+\rho\sigma\tau\Lambda_hP_7P_9P_{10}\left(P_4P_5\theta_1 + P_3P_5\theta_2 + P_3P_4\theta_3\right) + \sigma\tau\Gamma_1\Lambda_hP_7P_8P_{10}\left(P_4P_5\theta_1 + P_3P_5\theta_2 + P_3P_4\theta_3\right)$$
$$+\gamma\tau\Lambda_hP_8P_9P_{10}\left(P_4P_5\theta_1 + P_3P_5\theta_2 + P_3P_4\theta_3\right) + \tau\Lambda_h\left(P_4P_5\theta_1 + P_3P_5\theta_2 + P_3P_4\theta_3\right)$$
$$+P_3P_4P_5P_6P_7P_8P_9\left(\Lambda_hP_{10} + \Lambda_sP_2\right) + \Lambda_hP_6P_7P_8P_9P_{10}\left(P_4P_5\theta_1 + P_3P_5\theta_2 + P_3P_4\theta_3\right),$$
$$A_1 = P_2P_3P_4P_5P_6P_7P_8P_9\left(\Lambda_sP_1 + \Lambda_hP_{10} - \Lambda_s\beta\right),$$
$$A_0 = \Lambda_s\,\beta\,P_1P_2P_3P_4P_5P_6P_7P_8P_9.$$

Now, for $\beta > 0$ which also implies $\lambda^{**} > 0$, it is clear that $A_i > 0, \left(i = 0, 2\right)$, since all the model parameters are positive. Also, $A_1 > 0$ or $A_1 < 0$ whenever $\Lambda_sP_1 + \Lambda_hP_{10} > \Lambda_s\beta$ or $\Lambda_sP_1 + \Lambda_hP_{10} < \Lambda_s\beta$, respectively.

Applying the Descartes Rule of Signs in each case showsthat only one sign changes, establishing the existence of one positive real root, i.e., a unique snakebite-envenoming endemic equilibrium point of the model denoted by E^{**} is established. Thus, the following proposition is presented:

Proposition 1.*If* $\beta > 0$ *with either* $\Lambda_s P_1 + \Lambda_h P_{10} > \Lambda_s \beta$ *or* $\Lambda_s P_1 + \Lambda_h P_{10} < \Lambda_s \beta$ *, then the model equation given by system (12) has a unique snakebite-envenoming endemic equilibrium point denoted by* $\left(E^{**}\right)$.

3.4 Local Stability of the Equilibrium Point of the Model

The local asymptotic stability of the equilibrium points of the model is established using the method of linearization, and the results are reported in the following theorems:

Theorem 2.*The snakebite-envenoming free equilibrium point,* $\left(E^{*}\right)$ *and the snakebite-envenoming endemic equilibrium point,* $\left(E^{**}\right)$ *of the model equation given by system (12) are locally asymptotically stable (LAS) in the region* Ω.

Proof. The Jacobian corresponding to the model equation given by system (12) is given as

$$J(X)=\begin{bmatrix} -\frac{\beta V_s}{N_h^*}-P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\beta S}{N_h^*} \\ \frac{\beta V_s}{N_h^*} & -\mathrm{P}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\beta S}{N_h^*} \\ 0 & \theta_1 & -\mathrm{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_2 & 0 & -\mathrm{P}_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_3 & 0 & 0 & -\mathrm{P}_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_1 & \tau_2 & \tau_3 & -\mathrm{P}_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma & -\mathrm{P}_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma\rho & \psi\phi & -\mathrm{P}_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma\Gamma_1 & \psi\Gamma_2 & 0 & -\mathrm{P}_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{P}_{10} \end{bmatrix}, \tag{17}$$

where $X=\left(S, I, I_{ml}, I_{mo}, I_s, T, E_{ar}, R_d, R_w, V_s\right)$ and $N_h = N_h^*$ are taken as a constant for simplicity. Let $\beta = 0$, so that the snakebite-envenoming free equilibrium, E^* exists. Now, the Jacobian of the system (12) evaluated at E^* is given by

$$J\left(E^{*}\right)=\begin{bmatrix} -P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mathrm{P}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_1 & -\mathrm{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_2 & 0 & -\mathrm{P}_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_3 & 0 & 0 & -\mathrm{P}_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_1 & \tau_2 & \tau_3 & -\mathrm{P}_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma & -\mathrm{P}_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma\rho & \psi\phi & -\mathrm{P}_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma\Gamma_1 & \psi\Gamma_2 & 0 & -\mathrm{P}_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{P}_{10} \end{bmatrix}. \tag{18}$$

The characteristic equation of the Jacobian matrix evaluated at the snakebite-envenoming free equilibrium point given by equation(18)is presented as

$$\det\left(J(E^*)-\Upsilon I_{10}\right)=0. \tag{19}$$

The eigenvalues of equation (19) are presented as follows:

$$\Upsilon_i=-P_i,\quad (i=1,2...,10). \tag{20}$$

It follows from equation (20) that all the eigen values, Υ_i, for $i=1,2,...,10$,have satisfied the negativity requirement for local asymptotic stability of E^* for the reason that $P_i>0,$ for $i=1,2,...,10$, since, all the model parameters are positive. Thus, the snakebite-envenoming free equilibrium point E^* for the system (12) is locally asymptotically stable. Similarly, suppose that the result in Proposition 1 holds so that a unique snakebite-envenoming endemic equilibrium point E^{**} exists. The Jacobian matrix evaluated at the snakebite-envenoming endemic equilibrium point E^{**} is presented as

$$J\left(E^{**}\right)=\begin{bmatrix}
-\frac{\beta V_s^{**}}{N_h^*}-P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\beta S^{**}}{N_h^*}\\
\frac{\beta V_s^{**}}{N_h^*} & -\mathrm{P}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\beta S^{**}}{N_h^*}\\
0 & \theta_1 & -\mathrm{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \theta_2 & 0 & -\mathrm{P}_4 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \theta_3 & 0 & 0 & -\mathrm{P}_5 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \tau_1 & \tau_2 & \tau_3 & -\mathrm{P}_6 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \gamma & -\mathrm{P}_7 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \sigma\rho & \psi\phi & -\mathrm{P}_8 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \sigma\Gamma_1 & \psi\Gamma_2 & 0 & -\mathrm{P}_9 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{P}_{10}
\end{bmatrix}. \tag{21}$$

The characteristic equation of the Jacobian matrix evaluated at the snakebite-envenoming endemic equilibrium point is given by

$$\det\left(J(E^{**})-\Phi I_{10}\right)=0. \tag{22}$$

The eigenvalues of equation (22) are presented as follows:

$$\Phi_j=-P_i,\quad (i=2,3...,10),(j=1,2,3...,9) \text{ and } \Phi_{10}=-\frac{\beta V_s^{**}+N_h^*P_1}{N_h^*}. \tag{23}$$

From equation (23), it is obvious that $\Phi_{10}<0$. Also, observe that the first nine eigenvalues of equation (22) and equation (19) are the same. Hence, following similar arguments that established the LAS of the snakebite-envenoming free equilibrium point, we conclude that the snakebite-envenoming endemic equilibrium point of the model is also locally asymptotically stable (LAS), and hence the proof is complete.

3.5 Global Stability of Equilibrium Point of the Model

To ensure that the stabilities of snakebite-envenoming free and snakebite-envenoming endemic equilibrium points are independent of the initial conditions, we shall carry out global stability analysis using Lyapunov function theory in conjunction with LaSalle's invariance principle. These will be achieved through the following theorems:

Theorem 3. *The snakebite-envenoming free equilibrium point of the model is globally asymptotically stable in the region*Ω.

Proof. Consider the following candidate Lyapunov function given by:

$$\mathcal{F}(t)=f_1I+f_2I_{ml}+f_3I_{mo}+f_4I_s+f_5T+f_6E_{ar}+f_7\left(V_s-V_s^*-V_s^*\ln\left(\frac{V_s}{V_s^*}\right)\right), \tag{24}$$

where $f_i,(i=1,2...,7)$ are positive constants to be determined. Now, the time derivative of the Lyapunov function along the solution of system (12) is given by

$$\dot{\mathcal{F}}(t) = f_1\dot{I} + f_2\dot{I}_{ml} + f_3\dot{I}_{mo} + f_4\dot{I}_s + f_5\dot{T} + f_6\dot{E}_{ar} + f_7\left(1-\frac{V_s^*}{V_s}\right)\dot{V}_s \quad . \tag{25}$$

Using the right-hand sides of system (12) in equation (24) we have

$$\begin{aligned}\dot{\mathcal{F}}(t) &= f_1(\lambda S - P_2 I) + f_2(\theta_1 I - P_3 I_{ml}) + f_3(\theta_2 I - P_4 I_{mo}) + f_4(\theta_3 I - P_5 I_s)\\ &+ f_5(\tau_1 I_{ml} + \tau_2 I_{mo} + \tau_3 I_s - P_6 T) + f_6(\gamma T - P_7 E_{ar}) + f_7\left(1-\frac{V_s^*}{V_s}\right)(\Lambda_s - P_{10}V_s)\cdot\end{aligned} \tag{26}$$

Expanding and collecting like terms in equation (26) and keeping into consideration that $S \le N_h$ the following equation is obtained:

$$\begin{aligned}\dot{\mathcal{F}}(t) &\le \left(f_2\theta_1 + f_3\theta_2 + f_4\theta_3 - f_1(\theta_1+\theta_2+\theta_3)\right)I - f_1(\delta_1+\mu_h)I + (f_5\tau_1 - f_2\tau_1)I_{ml} - f_2\mu_h I_{ml}\\ &+ (f_5\tau_2 - f_3\tau_2)I_{mo} - f_3\mu_h I_{mo} + (f_5\tau_3 - f_4\tau_3)I_s - f_4(\delta_2+\mu_h)I_s + (f_6\gamma - f_5P_6)T - f_6P_7E_{ar}\\ &+ (f_1\beta - f_7P_{10})V_s - f_7\frac{\Lambda_s V_s^*}{V_s} + f_7\Lambda_s + f_7V_s^*P_{10}\cdot\end{aligned} \tag{27}$$

From equation (27) the following Lyaponuv coefficients were derived

$$f_1 = f_2 = f_3 = f_4 = f_5 = 1,\ f_6 = \frac{P_6}{\gamma} \text{ and } f_7 = \frac{\beta}{P_{10}} \quad . \tag{28}$$

Thus, equation (27) becomes

$$\dot{\mathcal{F}}(t) \le -(\delta_1+\mu_h)I - \mu_h(I_{ml}+I_{mo}) - (\delta_2+\mu_h)I_s - \frac{P_6P_7}{\gamma}E_{ar} - \frac{\beta}{P_{10}}\left(\frac{\Lambda_s V_s^*}{V_s} - \Lambda_s - V_s^*P_{10}\right) \tag{29}$$

Since $\beta = 0$, it follows that $\dot{\mathcal{F}}(t) \le 0$, with $\dot{\mathcal{F}}(t) = 0$, if $I = I_{ml} = I_{mo} = I_s = T = E_{ar} = 0$. Since, Ω is positively invariant and attracting, The largest positive invariant set in Ω is the singleton E^*.Thus, according to LaSalle's invariance principle [38],the snakebite-envenoming free equilibrium point E^* is globally attractive. Therefore, the snakebite-envenoming free equilibrium point is globally asymptotically stable.

Theorem 4. The snakebite-envenoming endemic equilibrium point of the model is globally asymptotically stable in the region Ω.

Proof. Suppose the result in Proposition 1 holds so that the existence of unique snakebite-envenoming endemic equilibrium point is assured. Consider the following quadratic Lyapunov function with the following candidate:

$$\begin{aligned}G(X) &= \frac{g_1}{2}\left(S-S^{**}\right)^2 + \frac{g_2}{2}\left(I-I^{**}\right)^2 + \frac{g_3}{2}\left(I_{ml}-I_{ml}^{**}\right)^2 + \frac{g_4}{2}\left(I_{mo}-I_{mo}^{**}\right) + \frac{g_5}{2}\left(I_s-I_s^{**}\right) + \frac{g_6}{2}\left(T-T^{**}\right)^2\\ &+ \frac{g_7}{2}\left(E_{ar}-E_{ar}^{**}\right)^2 + \frac{g_8}{2}\left(R_d-R_d^{**}\right)^2 + \frac{g_9}{2}\left(R_w-R_w^{**}\right)^2 + \frac{g_{10}}{2}\left(V_s-V_s^{**}\right)^2,\end{aligned} \tag{30}$$

where $X = S, I, I_{ml}, I_{mo}, I_s, T, E_{ar}, R_d, R_w, V_s$.

Now, differentiating equation (30) along the solution of the model equation given by system (12) and choosing $g_i = 1, (i = 1,2,3,...10)$ gives

$$\begin{aligned}\frac{dG}{dt} &= \left[\left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right)\right]\\ &\frac{d}{dt}\left[S+I+I_{ml}+I_{mo}+I_s+T+E_{ar}+R_d+R_w\right]+\left[V_s-V_s^{**}\right]\frac{dV_s}{dt}\cdot\end{aligned} \tag{31}$$

From system (12), we have

$$\frac{d}{dt}\left[S+I+I_{ml}+I_{mo}+I_s+T+E_{ar}+R_d+R_w\right] = \Lambda_h - \mu_h\left[S+I+I_{ml}+I_{mo}+I_s+T+E_{ar}+R_d+R_w\right] - \delta_1 I - \delta_2 I_s \tag{32}$$

and

$$\frac{dV_s}{dt} = \Lambda_s - P_{10}V_s. \tag{33}$$

Using equations (32) and (33) in equation (31), we obtain

$$\frac{dG}{dt} = \left[\left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right)\right]$$
$$\left[\Lambda_h-\mu_h\left[S+I+I_{ml}+I_{mo}+I_s+T+E_{ar}+R_d+R_w\right]-\delta_1 I-\delta_2 I_s\right]+\left[V_s-V_s^{**}\right]\left[\Lambda_s-P_{10}V_s\right]. \tag{34}$$

At steady state, we have

$$\Lambda_h = \mu_h\left[S^{**}+I^{**}+I_{ml}^{**}+I_{mo}^{**}+I_s^{**}+T^{**}+E_{ar}^{**}+R_d^{**}+R_w^{*}\right]+\delta_1 I^{**}+\delta_2 I_s^{**} \text{ and } \Lambda_s = P_{10}V_s^{**}. \tag{35}$$

Substituting equation (35) in equation (34) gives

$$\frac{dG}{dt} = \left[\left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right)\right]$$
$$\left[\mu_h\left[S^{**}+I^{**}+I_{ml}^{**}+I_{mo}^{**}+I_s^{**}+T^{**}+E_{ar}^{**}+R_d^{**}+R_w^{*}\right]+\delta_1 I^{**}+\delta_2 I_s^{**}-\mu_h\left[S+I+I_{ml}+I_{mo}+I_s+T+E_{ar}+R_d+R_w\right]\right]$$
$$\left[-\delta_1 I-\delta_2 I_s\right]+\left[V_s-V_s^{**}\right]\left[P_{10}V_s^{**}-P_{10}V_s\right]. \tag{36}$$

Collecting like terms gives

$$\frac{dG}{dt} = \left[\left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right)\right]$$
$$\left[-\mu_h\left(S-S^{**}\right)-\mu_h\left(I-I^{**}\right)-\mu_h\left(I_{ml}-I_{ml}^{**}\right)-\mu_h\left(I_{mo}-I_{mo}^{**}\right)-\mu_h\left(I_s-I_s^{**}\right)-\mu_h\left(T-T^{**}\right)-\mu_h\left(E_{ar}-E_{ar}^{**}\right)-\mu_h\left(R_d-R_d^{**}\right)-\mu_h\left(R_w-R_w^{**}\right)\right]$$
$$\left[-\delta_1\left(I-I^{**}\right)-\delta_2\left(I_s-I_s^{**}\right)\right]-P_{10}\left(V_s-V_s^{**}\right)\left(V_s-V_s^{**}\right). \tag{37}$$

After expansion and some series of simplifications, the following equation is obtained:

$$\frac{dG}{dt} = -\left(S-S^{**}\right)\left[\mu_h\left[\left(S-S^{**}\right)+\chi_1\right]+Z\right]-\left(I-I^{**}\right)\left[\mu_h\left[\left(I-I^{**}\right)+\chi_2\right]+Z\right]$$
$$-\left(I_{ml}-I_{ml}^{**}\right)\left[\mu_h\left[\left(I_{ml}-I_{ml}^{**}\right)+\chi_3\right]+Z\right]-\left(I_{mo}-I_{mo}^{**}\right)\left[\mu_h\left[\left(I_{mo}-I_{mo}^{**}\right)+\chi_4\right]+Z\right]$$
$$-\left(I_s-I_s^{**}\right)\left[\mu_h\left[\left(I_s-I_s^{**}\right)+\chi_5\right]+Z\right]-\left(T-T^{**}\right)\left[\mu_h\left[\left(T-T^{**}\right)+\chi_6\right]+Z\right]$$
$$-\left(E_{ar}-E_{ar}^{**}\right)\left[\mu_h\left[\left(E_{ar}-E_{ar}^{**}\right)+\chi_7\right]+Z\right]-\left(R_d-R_d^{**}\right)\left[\mu_h\left[\left(R_d-R_d^{**}\right)+\chi_8\right]+Z\right]$$
$$-\left(R_w-R_w^{**}\right)\left[\mu_h\left[\left(R_w-R_w^{**}\right)+\chi_9\right]+Z\right]-P_{10}\left[V_s-V_s^{**}\right]^2, \tag{38}$$

where

$$\chi_1 = \left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right),$$
$$\chi_2 = \left(S-S^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right),$$
$$\chi_3 = \left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right),$$
$$\chi_4 = \left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right),$$
$$\chi_5 = \left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right),$$
$$\chi_6 = \left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right),$$
$$\chi_7 = \left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(R_d-R_d^{**}\right)+\left(R_w-R_w^{**}\right),$$
$$\chi_8 = \left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_w-R_w^{**}\right),$$
$$\chi_9 = \left(S-S^{**}\right)+\left(I-I^{**}\right)+\left(I_{ml}-I_{ml}^{**}\right)+\left(I_{mo}-I_{mo}^{**}\right)+\left(I_s-I_s^{**}\right)+\left(T-T^{**}\right)+\left(E_{ar}-E_{ar}^{**}\right)+\left(R_d-R_d^{**}\right),$$
$$Z = \delta_1\left(I-I^{**}\right)+\delta_2\left(I_s-I_s^{**}\right).$$

From equation (38) it is clear that $\frac{dG}{dt} \leq 0$ and $\frac{dG}{dt} = 0$ if and only if $S = S^{**}$, $I = I^{**}$, $I_{ml} = I_{ml}^{**}$, $I_{mo} = I_{mo}^{**}$, $I_s = I_s^{**}, T = T^{**}, E_{ar} = E_{ar}^{**}, R_d = R_d^{**}, R_w = R_w^{**}, V_s = V_s^{**}$. Furthermore, every solution of the model equation given by system (12) approaches the snakebite-envenoming endemic equilibrium, $\left(E^{**}\right)$ as $t \to \infty$. Hence, the largest positively invariant set in $\left\{\left(S, I, I_{ml}, I_{mo}, I_s, T, E_{ar}, R_d, R_w, V_s\right) \in \Omega : \frac{dG}{dt} = 0\right\}$ is the singleton $\left(E^{**}\right)$. Hence, according to the Lasalle's invariance principle [38], the snakebite-envenoming endemic equilibrium point is globally asymptotically stable in the region Ω.

4. Numerical Simulations

The data in Tables 3 and 4 are used as baseline values for the state variable and the parameters of the model. Numerical simulations to investigate the impacts of the diagnosis of SBE victims and the treatment with ASV on the the burden of SBE were performed. The diagnosis- and treatment-related parameters were varied as low (25%), moderate (50%), and high (75%). The results of the simulations of the model showing the impacts of the diagnosis of SBE victims and the treatment with ASV on the burden of SBE in terms of SBE-induced deaths and disability are presented in Figures. 11 and 12, respectively. The simulation result in Figure 11 clearly shows that an increase in the rates of diagnosis of SBE victims combined with ASV treatment resulted in a reduction in the burden of SBE in terms of SBE-induced deaths presented for one year. Similarly, the outcome of the simulation depicted in Figure 12 reveals that when rates of diagnosis of SBE victims and treatment with ASV are increased, the number of SBE-induced disabilities prevented increases. It is worth noting that when the rate of diagnosis of SBE victims and treatment with ASV is set to zero, the number of SBE-induced deaths and disabilities increases. The results of these numerical simulations indicate that the diagnosis of SBE victims for proper treatment with ASV will help in the struggle against SBE in terms of reducing its burden. Thus, the affected community should be encouraged to always visit the hospital for proper diagnosis of SBE victims and treatment with ASV.

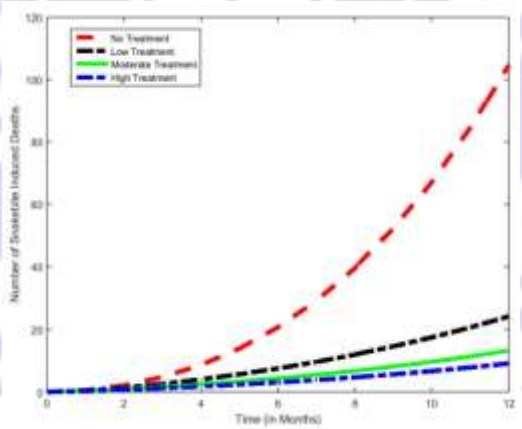

Figure 11: Simulation showing the impact of diagnosis and treatment with ASV on SBE-induced deaths

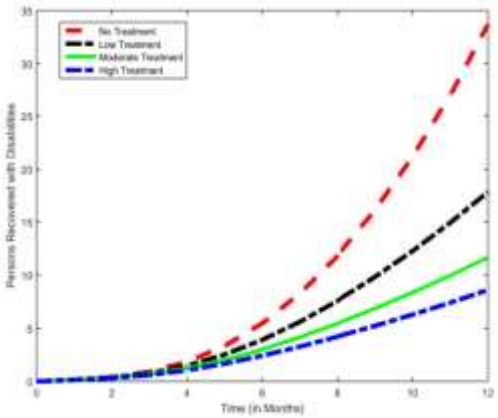

Figure 12: Simulation showing the impact of diagnosis and treatment with ASV on SBE-induced disability

5. Conclusion

In this work, a population-based mathematical model was developed to investigate the impact of the diagnosis of SBE victims and treatment with ASV. The basic properties of the model in terms of positivity and boundedness of the model solution were explored. The model is fitted using real data on SBE collected from snakebite treatment and research hospitals in Kaltungo, Gombe State, in northeast Nigeria. The dynamical behavior of the model in terms of equilibrium points and their associated stability analyses were carried out. Furthermore, numerical simulations of the model were performed. Thus, the main qualitative and epidemiological findings of this study are summarized as follows:

i. the outcome of the model fitting has shown that the model fitted well with the real data, suggesting that the model can be used to study the dynamics of SBE in a given community;
ii. the qualitative study of the model has established that the model has two equilibrium points, namely, snakebite-envenoming free and snakebite-envenoming endemic;
iii. the results of the stability analyses revealed that both the two equilibrium points, i.e., snakebite-envenoming free and snakebite-envenoming endemic, are locally and globally asymptotically stable;
iv. numerical simulations of the model have shown that diagnosis of SBE victims and treatment with ASV could reduce the burden of SBE in terms of preventing the number of SBE-induced deaths and disabilities.

Therefore, based on the epidemiological findings in this work, we recommend that people living in a high-risk community be encouraged to attend health facilities for appropriate diagnosis of snakebite victims and treatment.

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