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Statistical Analysis of Poisson Bifurcated Autoregressive Modeling

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ABSTRACT

This study evaluates the Poisson Bifurcated Autoregressive (PBAR) model for analyzing time-varying count data, focusing on exchange rate datasets from Anglophone countries. As an extension of the traditional Bifurcated Autoregressive (BAR) model, the PBAR model incorporates a Poisson-distributed error term, enabling it to capture non-Gaussian features and bifurcating structures common in degenerate economic data. Using historical exchange rates from the Central Bank of Nigeria's Statistical Bulletin, the study compares the PBAR and BAR models based on Akaike Information Criterion (AIC), Mean Absolute Error (MAE), and Root Mean Square Error (RMSE). Results show the PBAR model consistently achieves better fit and accuracy. For Equatorial Guinea (XAF), PBAR recorded an AIC of 0.4077, MAE of 0.249, and RMSE of 0.875, while the BAR model had values of 0.4694, 0.847, and 1.075, respectively. Similarly, for Ghana (GHS), PBAR produced an AIC of 0.3500, MAE of 0.152, and RMSE of 1.026, outperforming BAR's values of 0.4207, 1.080, and 1.086. Across all datasets, the PBAR model demonstrated improved accuracy and robustness. These findings highlight the PBAR model's advantage in forecasting non-linear, bifurcating time series. The study concludes that PBAR offers a significant enhancement over traditional models, making it well-suited for financial econometrics applications where accurate modeling of count-based, irregular data is essential.

1. Introduction

Time series models have been used to analyses time series dataset. The linear time series models such as Autoregressive (AR), Moving Average (MA) and Autoregressive Moving Average (ARMA), Poisson Autoregressive (PAR) etc. had been erroneously used due to its failure to characterized fluctuations, regime-shifts, flat stretches, change point behaviors, transitions etc. (Terpstra & Rao, 2001; Wong *et al*., 2009). Some non-linear time series models lacks its acumen to outburst, change points like behavior, time-varying volatilities (conditional variances), time-varying mixing weights, full range of shape changing predictive distributions (multimodalities), ability to handle cycles, bifurcating Markov chain theory, nonlinear bifurcating processes etc. (Boshnakov, 2006; Penda & Olivier, 2015). The ability to measure, assess, estimate the correlation of two detached time series via autoregressive process is what Cowan (1984) and Cowan and Staudte (1986) referred to as Bifurcated Autoregressive Processes. According to Saporta *et al*, (2018), BAR model takes into account both inherited and environmental effects to explain the evolution of the quantitative characteristics under consideration such that the two autoregressive processes are of order one and two sequence of error term respectively. The statistical method makes it possible to discover distinction patterns in the disturbance of lineage trees for meaningful contributions. The laws of large numbers, Central Limit Theorems (CLT) is usually adopted to stabilize hematopoietic subpopulations arises due to noise has a functional role in multi-cellular development indications (Gerlach *et al*., 2013). Guyon (2007) and Guyon *et al*. (2004) affirmed that one of the usefulness of BAR process with asymmetric is its similar trait with bifurcating Hidden Markov chain that makes feasible for derivation of CLT and laws of large numbers for estimators of the coefficients of the autoregressive

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processes. Hicks (2017) affirmed that establishing error term from the clamorous measurements acquired from the structural-tree or disintegrated series data are fundamental, important and difficulty at time in choosing the distributional and statistical property of the noisy data. The origin of the parental or main series is not difficult and it can be ascertain via whether the data is continuous or discrete, if it is continuous, it can take distributions such as Normal, Beta, Weibull, Lognormal, Student-t, Laplace, Gamma distributions. If it countable, which is discrete, distributions with countable ranges of values, will be suitable, distributions in such category are Poisson, Binomial, Bernoulli, Hyper-geometric, and zero-inflated distributions. Thus, the noisy lineage inherited by daughters as described by Cowan and Staudte (1986) or the noisy descending series from a financial data of uniformly time event of a threshold for bimodal of bifurcated process takes the statistical and distributional form (error term) of the progenitor series (Sandler *et al*., 2015; Hormoz *et al.,* 2016).

Undoubtedly, normality assumptions do fail in some instances. For example, if Y_t represents the number of certain type of uniformly time interval financial series or gene, it will be unquestionable that discrete Probability Mass Functions (PMFs) like Poisson will be adequate.

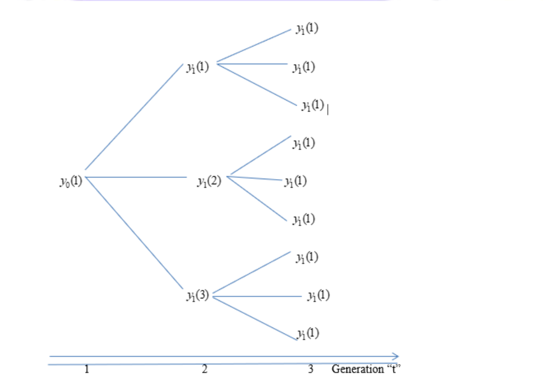

Figure 1: A tri (three)-casting tree

In generalization, if there exist t^{th}-generation such that $t \geq 1$, then a general term of t^{th} possible outcome or division is

$$H_n = \{2^t, 2^t + 1, \ldots, 2^{t+1} - 1\} \tag{1}$$

where,

$H_0 = \{1\}$ as shown in figure 2 connote the initial generation, the source, parental series or original ancestors.

$H_1 = \{2,3\}$ as shown in figure 2 is the first generation from the source.

If H_{r_t} be the generation of individual at "t" which implies that $r_t = (\log_2(t))$, since the two descendants of individual "t" are 2t and 2t+1, reversing it, the individual "t" is $\left(\frac{t}{2}\right)$, where (y) is the biggest value than y . So,

$$\Pi_t = \cup_{i=0}^{t} H_i \tag{2}$$

would be the union, sub-division, sub-detachment, sub-tree indexed of all characterization emanated from the ancestor's traits or series up to t-*th* succession. Then, in generality, the number of elements of $|H_t|$ of H_t is nothing but 2^n. The cardinality of $|\Pi_t|$ of Π_t is $2^{t+1} - 1$.

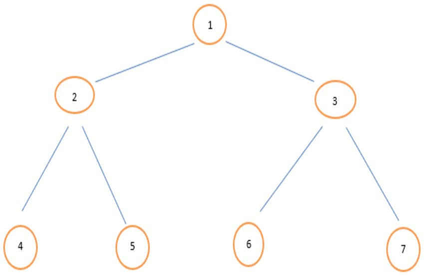

Figure 2: A Binary Integer-valued Autoregressive (BINAR)

Therefore, this paper proposed PBAR model for the degenerated exchange rate data to other currencies in Anglophone country**,** analyzed, and compare empirical study of both BAR and PBAR models with the uses of the forecast metrics to determine the better model.

2. Methods

2.1 Poisson Bifurcated Autoregressive (PBAR) model

This discusses and develops the Poisson Bifurcated Autoregressive (PBAR) model with count data (positive integer) valued error term. A Poisson distributional form was incorporated into the BAR for a degenerated count valued time- varying observations or series.

$$Y_t = \phi_0 + Y_{\left(\frac{t}{2}\right)}\phi_1 + Y_{\left(\frac{t}{4}\right)}\phi_2 + \cdots + Y_{\left(\frac{t}{p}\right)}\phi_p + \omega_t \tag{3}$$

where $\omega_{2t}, \omega_{2t+1}$ is the sequence of Independent and Identical Distribution (I.I.D)

with $E(\omega_{2t}) = E(\omega_{2t+1}) = \mu, Var(\omega_{2t}) = Var(\omega_{2t+1}) = \sigma^2, corr(\omega_{2t}, \omega_{2t+1}) = \rho$

2.2 Integer-Valued Bifurcating Autoregressive Model

As defined by Al-Osh and Alzaid (1987), the elongated first-order autoregressive to a binary integer-valued structural tree process is

$$Y_t = \phi_1 o Y_{\left(\frac{t}{2}\right)}\phi_1 + \omega_t \qquad 0 < \phi_1 < 1$$

(4)

Such that $\phi_1 o Y_{\left(\frac{t}{2}\right)}$ denotes the binomial thinning operation defined as

$$\phi_1 o Y_{\left(\frac{t}{2}\right)} = \sum_{t=1}^{Y_{\left(\frac{t}{2}\right)}} Y_t \tag{5}$$

With Y_i such that $i = 1,2, \ldots$ with a $P(Y_i = 1) = \phi_1$ and $P(Y_i = 0) = 1 - \phi_1$ for Bernoulli random variable for $(\omega_{2t}, \omega_{2t+1})$ with $t = 1,2,3, \ldots$

Considering a Poisson bifurcating autoregressive of order one. The bifurcating Poisson distribution can be written as:

$$p(y) = \frac{e^{-\lambda}\lambda^{y}}{y!} \qquad\qquad y = 0,1,2,\ldots \tag{6}$$

$$p(\omega_{2t}, \omega_{2t+1}) = e^{(\phi_1+\phi_2+\phi_3)} \sum_{i=0}^{y_1} \sum_{i=0}^{y_2} \sum_{i=0}^{i} \frac{\phi_1^{y_1-i}\phi_2^{y_2-i}\phi_3^{i}}{(y_1-1)(y_2-1)!i!} \tag{7}$$

$$p(\omega_{2t}, \omega_{2t+1}) = e^{(\phi_1+\phi_2+\phi_3)} \sum_{i=0}^{y_1 \wedge y_2} \frac{\phi_1^{y_1-i}\phi_2^{y_2-i}\phi_3^{i}}{(y_1-1)(y_2-1)!i!} \tag{8}$$

Such that $i = 1,2,3$, $\phi_i > 0$, $y_i = 0,1,2, \ldots, i = 1,2$

The marginal distribution of ω_{2t} & ω_{2t+1} are with $\mu_1 = \phi_1 + \phi_3$ and $\mu_2 = \phi_2 + \phi_3$ respectively. The covariance of ω_{2t} & ω_{2t+1} is ϕ_3.

2.3 The Central Limit Theory for the Poisson Bifurcated Autoregressive Model

Let the sum of the mean of ω_{2t} & ω_{2t+1} be λ, such that

$$\lambda = (\phi_1 + \phi_3) + (\phi_2 + \phi_3) \tag{9}$$

Let, $S_n = \sum Y_{\left(\frac{t}{2}\right)}$, $E(S_n) = n\lambda$

$\sigma(S_n) = \sqrt{n\lambda}$

The moment generating function of Z-variate as known from the normal distribution is

$$Z_n = \frac{\bar{X}-\mu}{\sqrt{\sigma}} \tag{10}$$

From the normal distribution but in Poisson model Moment Generating Function is given by Z_n

$$M_{z(t)} = Z_n = \frac{\bar{X}_{S_n(t)}-\mu}{\sqrt{n\lambda}} = \frac{M_{(t)}-\mu}{\sqrt{n\lambda}} \tag{11}$$

$$M_{z(t)} = \frac{M_{(t)}}{\sqrt{n\lambda}} - \frac{n\lambda_{(t)}}{\sqrt{n\lambda}} \tag{12}$$

$$= \left(\frac{(t)}{\sqrt{n\lambda}}\right) - t\sqrt{n\lambda} \tag{13}$$

Introducing the $M_{z(t)} = e^{tx}$

$$M_{z(t)} = e^{-t\sqrt{n\lambda}} M\left(\frac{(t)}{\sqrt{n\lambda}}\right) \tag{14}$$

Recall the $M_x(t)$ for Poisson is

$$M_x(t) = exp\lambda[e^{t} - 1] = exp - \lambda[1 - e^{t}] \tag{15}$$

$$M\left(\frac{(t)}{\sqrt{n\lambda}}\right) = exp\left[-n\lambda\left(1 - e^{-t/\sqrt{n\lambda}}\right)\right] \tag{16}$$

$$\log M_{zn}(t) = -t/\sqrt{n\lambda} - n\lambda\left[1 - e^{-t/\sqrt{n\lambda}}\right] \tag{17}$$

Recall, $e^{-x} \approx 1$

$$e^{x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\cdots$$

$$e^{-x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$$

So, $$e^{-\frac{t}{\sqrt{n\lambda}}} = 1 + \frac{t^1}{(\sqrt{n\lambda})^1 1!} + \frac{t^2}{(\sqrt{n\lambda})^2 2!} + \frac{t^3}{(\sqrt{n\lambda})^3 3!} + \cdots\cdots \tag{18}$$

So, $log\, M_{zn}\,(t) = -t\sqrt{n\lambda} - n\lambda\left(e^{-\frac{t}{\sqrt{n\lambda}}}\right)$

$$log\, M_{zn}\,(t) = -t\sqrt{n\lambda} - n\lambda\left[1 - \left(1 + \frac{t^1}{(\sqrt{n\lambda})^1 1!} + \frac{t^2}{(\sqrt{n\lambda})^2 2!} + \frac{t^3}{(\sqrt{n\lambda})^3 3!} + \cdots\cdots\right)\right] \tag{19}$$

$$= -t\sqrt{n\lambda} - n\lambda\left[1 - 1 - \frac{t^1}{(\sqrt{n\lambda})^1 1!} - \frac{t^2}{(\sqrt{n\lambda})^2 2!} - \frac{t^3}{(\sqrt{n\lambda})^3 3!} - \cdots\cdots\right] \tag{20}$$

$$= -t\sqrt{n\lambda} - n\lambda\left[-\frac{t^1}{(\sqrt{n\lambda})^1 1!} - \frac{t^2}{(\sqrt{n\lambda})^2 2!} - \frac{t^3}{(\sqrt{n\lambda})^3 3!} - \cdots\cdots\right] \tag{21}$$

$$= -t\sqrt{n\lambda} + n\lambda + \frac{n\lambda t}{(\sqrt{n\lambda})} + \frac{n\lambda t^2}{2(\sqrt{n\lambda})^2} + \frac{n\lambda t^3}{(\sqrt{n\lambda})^3 3!} + \cdots\cdots \tag{22}$$

$$= -t\sqrt{n\lambda} + t\sqrt{n\lambda} + \frac{t^2}{2} + \frac{t^3}{3!\sqrt{n\lambda}} + \cdots\cdots \tag{23}$$

$$= \frac{t^2}{2} + \frac{t^3}{3!\sqrt{n\lambda}} \tag{24}$$

But recall, $\lambda = (\phi_1 + \phi_3) + (\phi_2 + \phi_3)$

$$\Rightarrow log\, M_{zn}\,(t) = \frac{t^2}{2} + \frac{t^3}{6\sqrt{n[(\phi_1+\phi_3)+(\phi_2+\phi_3)]}} \tag{25}$$

$$= \frac{t^2}{2} + \frac{t^3}{6\left[\sqrt{n(\phi_1+\phi_3)}+\sqrt{n(\phi_2+\phi_3)}\right]} \tag{26}$$

$$= \frac{t^2}{2} + \frac{t^3}{\left[\sqrt{6n(\phi_1+\phi_3)}+\sqrt{6n(\phi_2+\phi_3)}\right]} \tag{27}$$

$$\underset{n\to\infty}{\log M_{zn}(t)} = \frac{t^2}{2} \tag{28}$$

Taking exponential of both sides

$$M_{zn}(t) = exp\left(\frac{t^2}{2}\right) \approx M.G.F(0,\sigma^2) \tag{29}$$

Z_nConverges to $(0,\sigma^2)$

This implies that has n approaches infinity the sum of $Y\left(\frac{t}{2}\right)$ turns closer to standard normal variate for $Y\left(\frac{t}{i}\right)$, such that $i = 2,4,6,8,\ldots$ (consecutive increasing even number), PBAR of any lag, say lag-p converges to $e^{\frac{t^2}{2}}$ as $n \to \infty$. This obeys and shows that PBAR holds for Strong Law of Large Numbers (SLLN) for variables.

2.4 Limit Distribution of Poisson Bifurcated Autoregressive Model

Considering

$$Y_t = \phi_0 + Y_{(\frac{t}{2})}\phi_1 + Y_{(\frac{t}{4})}\phi_2 + \cdots\cdots + Y_{(\frac{t}{p})}\phi_p + \omega_t \tag{30}$$

Representing in a general form

$$\phi(B)Y_t = \omega_t\phi_0 \tag{31}$$

where;

$$\phi(B) = 1 - \phi_1 b - \phi_2 b^2 - \phi_3 b^3 - \cdots\cdots \phi_p b^p \tag{32}$$

For $\phi(B) = 0$

$$Y_t = \sum_{i=0}^{n}\left(\omega_{[t/2]} + \phi_0\right)B \tag{33}$$

Where B are the non-negative coefficients in the coefficient expansion

$$Y_t = \sum_{i=0}^{n}\left(\omega_{[t/2]} + \phi_0\right)B \tag{34}$$

$$E(Y_t) = E\left[\sum_{i=0}^{n}\left(\left(\omega_{[t/2]} + \phi_0\right)B\right)\right] \tag{35}$$

$$= \sum_{i=0}^{n}\left[E\left(\omega_{[t/2]}\right) + E(\phi_0)E(B)\right] \tag{36}$$

Recall $E\left(\omega_{[t/2]}\right) = 0; E(\phi_0) = \phi_0$

$$= \sum_{i=0}^{n}[(0 + \phi_0)B] = \phi_0 \sum_{i=1}^{p} B_i \tag{37}$$

Since B_i is a pool of autoregressive parameters $i, \cdots\cdots, p$

$$= \phi_o\left(1 - \phi_1 b - \phi_2 b^2 - \phi_3 b^3 - \cdots\cdots \phi_p b^p\right) = \phi_0 \sum_{i=1}^{p} b_i \tag{38}$$

$$= \phi_0\left(1 - \sum_{i=1}^{p} \phi_i\right)^{-1} \tag{39}$$

$$Var(Y_t) = Var\left(\sum_{i=1}^{p} \omega_{[t/2]} B_i + \sum_{i=1}^{p} \phi_0 B_i\right) \tag{40}$$

$$= Var\left(\sum_{i=1}^{P}\omega_{[t/2]}B_i\right) + Var\left(\sum_{i=1}^{P}\phi_0 B_i\right) \tag{41}$$

Recall $Var\left(\omega_{[t/2]}\right) = \sigma^2$; also recall$Var(ax) = a^2V(x)$

$$= B_i^2 Var\left(\sum_{i=1}^{P}\omega_{[t/2]}\right) + Var\left(\sum_{i=1}^{P}\phi_0 B_i\right) \tag{42}$$

$$= \sigma^2 \sum_{i=0}^{P} B + Var\left(\sum_{i=1}^{P}\phi_0 B\right) \tag{43}$$

Recall $Var(\phi_0) = 0$

$$Var(Y_t) = \sigma^2 \sum_{i=0}^{P} B + Var(B)(0) \tag{44}$$

$$= \sigma^2 \sum_{i=0}^{P} B = \sigma^2 \sum_{i=0}^{P} b_i \tag{45}$$

$$cov(Y_t, Y_{[t/2]}) = \rho(k) = \sigma^2 \sum_{i=1}^{P} b_i b_{i+k} \tag{46}$$

Since $\phi(B) = 0$ and b_i lies outside the unit circle (that is b_i greater than zero but $|1| < 0$. $B_t = \left(1, Y_t, Y_{[t/2]}, \cdots\cdots, Y_{[t/2^{p-1}]}\right)$ such that $n^* = \frac{(n-1)}{2}$ the number of $\left(Y_t, Y_{[t/2]}, \cdots\cdots, Y_{[t/2^{p-1}]}\right)$ observed

$$\frac{1}{n^*}\sum_{t=2^{p-1}}^{n^*} B_t B_t' \to Casn < \infty$$

$$\sqrt{n}\left(\hat{\phi} - \phi\right) \to N(0, \sigma^2(1+\rho)C^{-1}) \tag{47}$$

2.5 Criteria for Selection of Optimal Models

Many criteria have been proposed for the purpose order determination by the past researchers. These include the Final prediction error (FPE) criterion, Schwarz-Rissanen criterion (SRC), Bayesian estimation criterion (BEC), Hannan-Qiunn criterion, Akaike's information criterion (AIC) and so on. The latest model selection criterion is the Akaike's information corrected criterion AICC, developed by Hurvich and Tsai (1989) as cited by (Olatayo & Adesanya, 2015). The criterion to be minimized is

$$FPE = \hat{\sigma}^2 \frac{n+p+q}{n-p-q} \tag{48}$$

where $\hat{\sigma}^2$ = variance of white noise,

n = number of observations,

p = order of the autoregressive component,

and q = order of the moving average component.

In 1970, Akaike found that FPE is asymptotically inconsistent and in 1973 he employed information-theoretic considerations to develop the Akaike's information criterion, AIC. This was designed to be an asymptotically unbiased estimate of the Kullback-Leibler index of the fitted model relative to the *true* model (Akaike, 1973). The AIC statistics is defined as

$$AIC = -2\ln Likelihood\left(\hat{\varphi}, \hat{\theta}, \hat{\sigma}^2\right) + 2(p+q+1) \tag{49}$$

where $\hat{\varphi}$ = a class of autoregressive parameters,

$\hat{\theta}$ = a class of moving average parameters,

and $\hat{\sigma}^2$, n, p and q are as defined in equation above.

In view of this, Akaike applied a Bayesian modification to AIC and finally in 1978, he came up with a consistent order selection criterion, known as Bayesian information criterion or BIC (Akaike, 1979). If the data $\{X_1, \ldots, X_n\}$ are in fact observations of an ARMA(p,q) process, then Bayesian information criterion is defined to be

$$BIC = (n-p-q)\ln\frac{n\hat{\sigma}^2}{n-p-q} + n\left(1 + \ln\sqrt{2\pi}\right) + (p+q)\ln\left[\frac{\sum_{t=1}^{n} X_t^2 - n\hat{\sigma}^2}{p+q}\right] \tag{50}$$

There is evidence to suggest that the BIC is more satisfactory than the AIC as an ARMA model selection criterion since the AIC has a tendency to pick models, which are over-parameterized, Hannan (1980).

Schwarz (1978) used a Bayesian analysis and Rissanen (1978), applied an optimal data-coding scheme to independently arrive at the same criterion, later known as Schwarz-Rissanen criterion, SIC. The criterion to be minimized is given by

$$SIC = \ln\hat{\sigma}^2 + \left(\frac{p+q}{n}\right)\ln n \tag{51}$$

Geweke and Mease (1981) suggested approximating SIC by Bayesian estimation criterion, BEC.

$$BEC = \hat{\sigma}^2 + (p_x + q_x)\hat{\sigma}_x^2 \ln\frac{n}{n-p_x-q_x} \tag{52}$$

where x denotes a quantity from pre-assigned high order ARMA model that includes all potential models.

Hannan and Quinn, (1979) and Hannan (1980) constructed Hannan-Quinn criterion from the law of the iterated logarithm. It provides a penalty function, which decreases as fast as possible for a strongly consistent estimator, as sample size increases.

Hannan-Quinn criterion is given by

$$HQ = \ln\hat{\sigma}^2 + 2(p+q)\frac{\ln(\ln n)}{n} \tag{53}$$

Hannan and Rissanen (1982) replaced the term $ln(lnn)$ by lnn to speed up the convergence of HQ. This revised version of HQ, however, was found to overestimate the model orders (Kavalieris, 1991).

2.6 Forecasting Accuracy Measures

Once forecasts are made, they can be evaluated and validated, if the actual values of the series to be forecasted were observed. There are some measurements of the forecasting accuracy metrics. These are root mean square error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) (Olatayo & Taiwo, 2016).

The mean absolute error (MAE) is defined by:

$$MAE = \frac{1}{h+1}\sum_{t=s}^{h+s}\left(\hat{X}_t - X_t\right)^2 \tag{54}$$

The root mean square forecast error (RMSE) is defined as:

$$RMSE = \sqrt{\frac{1}{h+1}\sum_{t=s}^{h+s}\left(\hat{X}_t - X_t\right)^2} \tag{55}$$

and the mean absolute percentage forecast error MAPE is given as,

$$MAPE = \frac{100}{h+s}\sum_{t=s}^{h+s}\left|\frac{\hat{X}_t - X_t}{\hat{X}_t}\right| \tag{56}$$

$where\ t = s, 1+s, \dots, h+s.$ The actual and predicted values for corresponding t values are denoted by $\hat{X}_t\ and\ X_t$ respectively. The smaller the values of *RMSE* and *MAPE* the better the forecasting performance of the model (Olatayo & Taiwo, 2015).

3. Results and Discussion

The descriptive statistics of the monthly time series exchange rates used in the study are presented in Table 1 below. Descriptive statistics describes the basic features of the data and form the basis of virtually every quantitative analysis of data in a study. This is of essence as it profiles the characteristics of the variables engaged and suggests possible pliable models for the study. It is observed that all the mean and median values are positive and similar, which suggests that the distribution of the individual variables are normal (i.e., bell-shaped).

Table 1: Descriptive Statistics of Variables

Variable	Mean	SE Mean	StDev	Variance	Skewness	Kurtosis
N/ Equ Guinea (XAF)	2.8446	0.0469	0.5383	0.2898	-0.99	-0.49
N/ Ghana (GHS)	0.012296	0.000313	0.003591	0.000013	0.29	-0.89
N/ Kenya (KES)	0.48004	0.00718	0.08255	0.00681	-1.02	-0.55
N/ S Africa (ZAR)	0.052422	0.000722	0.008296	0.000069	0.14	-1.20
N/Tunisia (TND)	0.009253	0.000083	0.000958	0.000001	-0.47	-0.69

Source: Author's Computations

3.1 Time Plot

The descriptive analysis was used to summarized the characteristics of the variables consider in this research work with a view of showing the important features of each of the variables through the use of time plot.

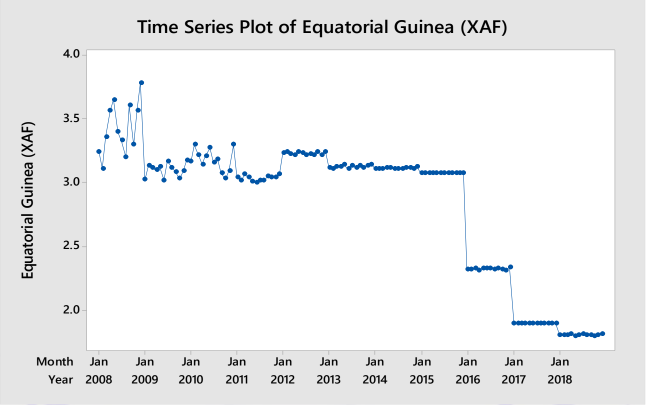

Figure 2 Time plot of N-Equatorial Guinea (XAF), 2008-2018

The time plot for the N-Equatorial Guinea (XAF) shows a short-term movement of the value in the series in different direction over the period considered. This movement is characterized by a decrease in the values of the Equatorial Guinea (XAF) over the period of time. This movement is referred to as secular variation or secular movement. By fitting a straight line freely by hand on the plotted points on the time plot for Equatorial Guinea (XAF) stretching over the period, this plotted point forms a line and this line is the trend of the time plot for Equatorial Guinea (XAF).

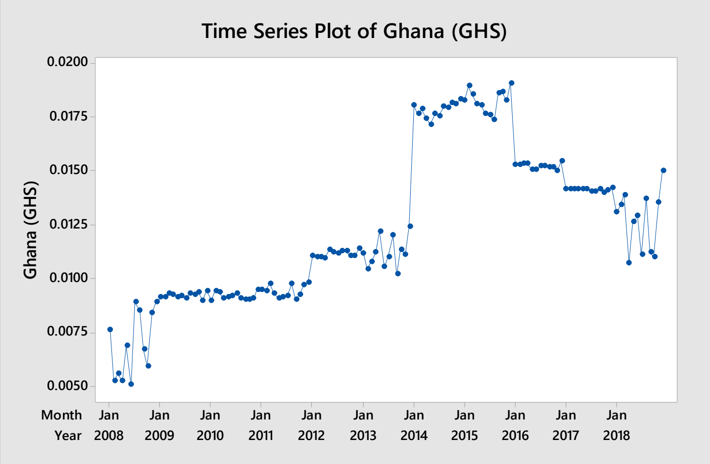

Figure 3: Time plot of N-Ghana (GHS), 2008-2018

The time plot for Ghana (GHS) shows a short-term movement in the series in different direction over the period considered. By fitting a straight line freely by hand on the plotted points on the time plot for Ghana (GHS) stretching over the period, this plotted point forms a line and this line is the trend of the time plot for Ghana (GHS).

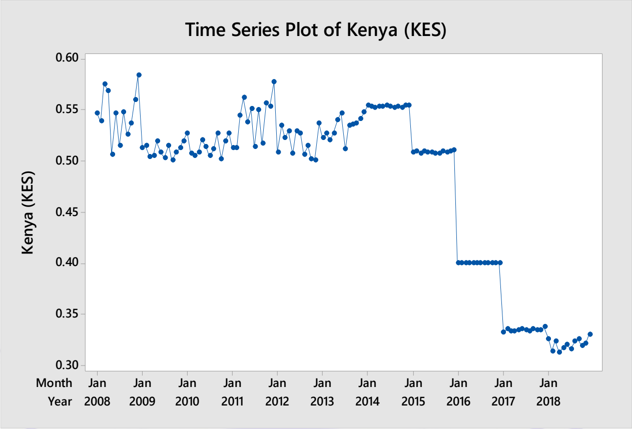

Figure 4: Time plot of N-Kenya (KES), 2008-2018

The time plot of Kenya (KES) has been on the decline overtime, which is a sign that the exchange rate between Naira to Kenya Shillings on the decrease.

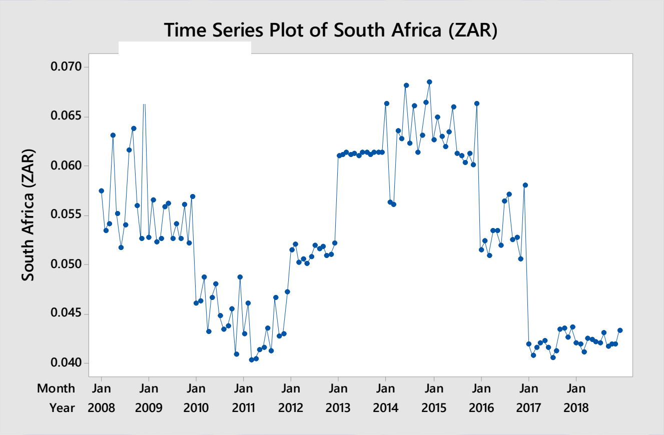

Figure 5: Time plot of N-South Africa (ZAR), 2008-2018

The time plot of the South Africa (ZAR) has also been on the increase overtime. The time plot shows a short-term movement of the value in the series in different direction over the period considered. This movement is characterized by a sinusoidal decrease in the values over the period of time. This movement is referred to as cyclical movement.

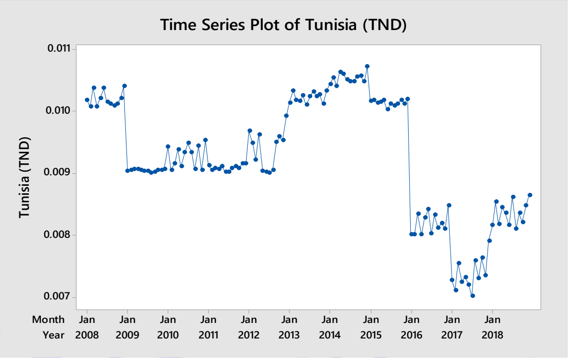

Figure 6: Time plot of N-Tunisia (TND), 2008-2018

The time plot of the Tunisia (TND) has also been on the increase overtime. The time plot shows an upward and downward movement in different direction over the period considered. This movement is characterized by a sinusoidal decrease in the values over the period of time. This movement is referred to as cyclical movement.

3.2 Unit Root Test

The Augmented Dickey-Fuller (ADF) formulae was employed to test for stationarity or the existence of unit roots in the data.

Table 2: Augmented Dickey-Fuller Unit Root Test

Series	ADFR Test Statistic	5% Critical Values	10% Critical values	Order	Remarks
Equatorial Guinea (XAF)	-0.8155	-2.8835	-2.5786	I(1)	Stationary
Ghana (GHS)	-1.8582	-2.8835	-2.5786	I(1)	Stationary
Kenya (KES)	-0.2954	-2.8837	-2.5786	I(1)	Stationary
South Africa (ZAR)	-1.8131	-2.8837	-2.5786	I(1)	Stationary
Tunisia (TND)	-1.5977	-2.8837	-2.5786	I(1)	Stationary

The empirical tests on unit root test above shows that Equatorial Guinea (XAF), Ghana (GHS), Kenya (KES), South Africa (ZAR) and Tunisia (TND) are integrated of order one. They are integrated of the same order; 1(1). From the above tables 4.3, it was found that ADF Test with trend and intercept indicated that time series are integrated of the same order. The linear combination of series integrated of the same order are said to be cointegrated. The level of their integrations indicates the number of time series have to be differenced before their stationarity is induced. Considering the ADF test statistics at 5% and 10% critical values, it is observed that test statistics are greater (in absolute term) than the critical values. Thus, the series are said to be stationary at that level.

3.3 Bifurcated Autoregressive Model for data Series

The datasets were analysed with the use of statistical software called **OxMetrics** designed by Doornik and Ooms (2004) and R-program. A one-dimensional grid values of (p) was set up with maximum values (p) = (2) and a search over all the constituent models was undertaken using the AIC to select the best fitting model.

Ojo et al. (2008), the preferred model is the one with least or lowest AIC value. The AIC methodology attempts to find the model that best explains the data with minimum of free parameters.

The model used for this study is given as: $X_t = \varphi_1 X_{\left(\frac{t}{2}\right)} + \varphi_2 X_{\left(\frac{t}{4}\right)} + \cdots + \varphi_p X_{\left(\frac{t}{2^p}\right)} + \varepsilon_t$ (57)

Tables 3 - 7 show the AIC, SC and HQC of the autoregressive models for Equatorial Guinea (XAF), Ghana (GHS), Kenya (KES), South Africa (ZAR) and Tunisia (TND).

In Table 3, two different BAR models were obtained for the dataset and the results are given below. Fitting the BAR model on the Equatorial Guinea (XAF) data and using the AIC as the model selector, it obtained the model below as: From the Table 3, $BAR(2)$ is the best model for Equatorial Guinea (XAF) considering the fact that it has the least AIC value from the selection. The model obtained is given below:

$$X_t = 0.22788 X_{\left(\frac{t}{2}\right)} + 0.086333 X_{\left(\frac{t}{4}\right)} + \varepsilon_t$$

Table 3: The AIC, SC and HQC of Equatorial Guinea (XAF)

	AIC	**SC**	**HQC**
BAR(1)	0.836028	0.852577	0.842726
BAR(2)	0.46548	0.618729	0.527553

Table 4: The AIC, SC and HQC of Ghana (GHS)

	AIC	**SC**	**HQC**
BAR(1)	0.67162	0.68817	0.67832
BAR(2)	0.42067	0.50216	0.4724

Table 5: The AIC, SC and HQC of Kenya (KES)

	AIC	**SC**	**HQC**
BAR(1)	0.51987	0.64093	0.60112
BAR(1)	0.5589	0.64283	0.59288

Table 6: The AIC, SC and HQC of South Africa (ZAR)

	AIC	**SC**	**HQC**
BAR(1)	0.85345	0.87	0.86015
BAR(2)	0.37028	0.77303	0.75326

Table 7: The AIC, SC and HQC of Tunisia (TND)

BAR(1)	**0.66271**	**0.71271**	**0.68295**
BAR(2)	0.54057	0.60747	0.56765

From Table 4 above, upon fitting the BAR model on the Ghana (GHS) data and using the AIC as the model selector, the obtained model is stated below as:

From the table, $BAR(2)$ had the least AIC value and so it is the best model for Ghana (GHS). The model obtained is given below:

$$X_t = 0.409289 X_{\left(\frac{t}{2}\right)} + 0.226148 X_{\left(\frac{t}{4}\right)} + + \varepsilon_t \qquad (58)$$

In Table 5**,** upon fitting the BAR model on the Kenya (KES) data and using the AIC as the model selector, the model below was obtained as:

From the table, $BAR(1)$ had the least AIC value, this implies that $AR(4)$ is the best model for total flow. The model obtained is given below:

$$X_t = 0.237551X_{(\frac{t}{2})}+\varepsilon_t \tag{59}$$

In Table 6 upon fitting the BAR model on the South Africa (ZAR) data and using the AIC as the model selector, the model below was obtained as:

From the table, $BAR(2)$ had the least AIC value, and so models South Africa (ZAR). The model obtained is given below:

$$X_t = 0.22788X_{(\frac{t}{2})} + 0.086333X_{(\frac{t}{4})}+\varepsilon_t \tag{60}$$

In Table 7, upon fitting the BAR model on the Tunisia (TND) data and using the AIC as the model selector, the model below was obtained as:

From the table, $BAR(1)$ had the least AIC value, and so models Tunisia (TND). The model obtained is given below:

$$X_t = 0.2537X_{(\frac{t}{2})}+\varepsilon_t$$

(61)

3.4 Poisson BAR Model

Equatorial Guinea (XAF): From a pool of models, the model with least AIC obtained is Poisson $BAR(1)$, this invariably means that $PBAR(1)$ is the best model for the dataset. This model can be written as:

$$X_t = 0.0399889X_{(\frac{t}{2})} + \varepsilon_t \tag{62}$$

Ghana (GHS): From a pool of models, the model with least AIC obtained is Poisson $BAR(2)$, this means that $PBAR(2)$ is the best model for the dataset. This model can be written as:

$$X_t = 0.69868X_{(\frac{t}{2})} + 0.932845X_{(\frac{t}{4})} + \varepsilon_t \tag{63}$$

Kenya (KES): From a pool of models, the model with least AIC obtained is Poisson $BAR(2)$, this invariably means that $PBAR(2)$ is the best model for the dataset. This model can be written as:

$$X_t = -0.0169903X_{(\frac{t}{2})} + 0.924414X_{(\frac{t}{4})} + \varepsilon_t \tag{64}$$

South Africa (ZAR): From a pool of models, the model with least AIC obtained is Poisson $BAR(1)$, this invariably means that $PBAR(1)$ is the best model for the dataset. This model can be written as:

$$X_t = 0.0894534X_{(\frac{t}{2})} + \varepsilon_t \tag{65}$$

Tunisia (TND): From a pool of models, the model with least AIC obtained is Poisson $BAR(1)$, this invariably means that $PBAR(1)$ is the best model for the dataset. This model can be written as:

$$X_t = 0.9105466X_{(\frac{t}{2})} + \varepsilon_t \tag{66}$$

3.5 Comparison of Results of the Times Series Data

Table 8: Summary of Result for the Data

Data	Model	AIC
Equatorial Guinea (XAF)	BAR	0.469368
	Poisson BAR	0.407663
Ghana (GHS)	BAR	0.42067
	Poisson BAR	0.34998

Kenya (KES)	BAR	0.51987
	Poisson BAR	0.21006
South Africa (ZAR)	BAR	0.37028
	Poisson BAR	0.34550
Tunisia (TND)	BAR	0.66271
	Poisson BAR	0.47056

From Table 8, it is quite evident that the AIC value for the bifurcated autoregressive model is higher compared to that of Poisson Bifurcated Autoregressive models. Comparatively, the AIC value for the BAR model for Equatorial Guinea (XAF) is 0.469368 and the corresponding AIC for the Poisson BAR is 0.407663. Thus, this indicates that the Poisson BAR model for Equatorial Guinea (XAF) behaves better than the BAR model. In same vein, the AIC value for the BAR model for South Africa (ZAR) is 0.37028 while the corresponding AIC value for the Poisson BAR is 0.34550. Thus, indicates that the Poisson BAR model for South Africa (ZAR) will be a better fit when compared to the BAR model. This is also equally applicable to the Kenya (KES), Ghana (GHS) and Tunisia (TND) data because the AIC values of the Poisson BAR of the data are higher than that of the Poisson BAR model with the values of 0.42067, 0.51987, 0.34998, 0.21006 and 0.66271, 0.47056 respectively.

3.6 Forecasting Performance

Finally, we evaluate the forecasting performance. Two methods were used, based on the Poisson BAR and BAR models, to produce point and probabilistic forecasts with the estimated parameters of data generated using the Poisson BAR models. The Table 9 summarizes mean absolute errors (MAE) and root mean square errors (RMSE) of the point forecasts from each method. The MAE is calculated from the conditional median and the RMSE, from the conditional mean. It also shows the empirical coverage and average length of the 95% minimum-length CI from each method. The MAE of the conditional median from the Poisson BAR model is 4.05; the BAR forecast has an MAE of 4.28. The RMSEs for the conditional means of the Poisson BAR model and the BAR model are 1.47 and 1.50, respectively. The empirical coverage of the 95% minimum-length CI for the non-Gaussian models is relatively stronger when compared to the Gaussian model. In conclusion, the point forecasts and CIs based on the Poisson BAR model are better than those based on the BAR model for these dataset.

Table 9 Summary of Point Forecast Performance

Modeled by	**Poisson BAR**			**BAR**		
	MAE	**RMSE**	**95% CI coverage**	**MAE**	**RMSE**	**95% CI coverage**
Equatorial Guinea (XAF)	0.249	0.875	95.6%	0.847	1.075	94.3%
Ghana (GHS)	0.152	1.026	95.2%	1.080	1.086	92.5%
Kenya (KES)	0.288	1.110	95.0%	1.345	1.350	91.8%
South Africa (ZAR)	0.012	0.446	95.2%	1.744	1.748	95.0%
Tunisia (TND)	0.233	0.326	95.3%	1.267	1.509	94.3%

4. Conclusion

This study demonstrates the effectiveness of the Poisson Bifurcated Autoregressive (PBAR) model in analyzing and forecasting time-varying count data, particularly in exchange rate series for Anglophone countries. As an extension of the conventional Bifurcated Autoregressive (BAR) model, the PBAR model integrates a Poisson-distributed error term, making it exceptionally suited for datasets exhibiting non-Gaussian characteristics, irregular patterns, and bifurcating structures. The comparative analysis indicates that the PBAR model consistently outperforms the BAR model across all evaluation metrics. Notably, the PBAR model achieved a lower Akaike Information Criterion (AIC), with values such as 0.4077 for Equatorial Guinea (XAF) and 0.3500 for Ghana (GHS), in contrast to higher AIC values for the BAR model, demonstrating a superior model fit. Additionally, the PBAR model recorded lower Mean Absolute

Error (MAE) and Root Mean Square Error (RMSE) scores, underscoring its enhanced forecasting accuracy. For instance, the PBAR model's MAE and RMSE for Equatorial Guinea were 0.249 and 0.875, significantly outperforming the BAR model's 0.847 and 1.075. These results validate the PBAR model as a powerful tool for financial econometrics, where accurate modeling of count-based, complex data is critical. The PBAR model's robust performance across multiple metrics underscores its potential to provide reliable, precise forecasts in contexts where traditional linear models often fall short. Thus, the PBAR model offers a valuable enhancement for applications requiring nuanced handling of non-linear and bifurcating time series data.

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