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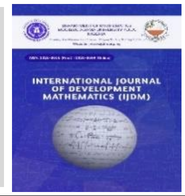

Weighted Soft Set and its Application in Parameterized Decision Making Processes

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ABSTRACT

In this research, we reviewed the notion of soft set and some of its application in decision making cases and observed that, there is need to introduce weighted parameterized soft set and its applications in real-life decision-making processes. Since in real-life situations not all decision parameters (attributes) are of the same significance. Hence the decision maker(s) tends to impose more weights to parameters of interest. The algebra of weighted soft set is presented and some fundamental results were established with relevant examples and illustrations. Various De Morgan's kinds of results with respect to the following operations, union, extended intersection, and restricted union, restricted intersection were also proved in the background of weighted soft set context.

1. Introduction

Soft set invented by (Molodtsov, 1999) as novel mathematical tool designed to handle the problems of uncertainty and imprecision that characterized the real-life situations. Therefore, (Maji *et al*., 2002, 2003) introduced the definitions of basic concepts (terms), such as equality of two soft sets, Super Set of a soft set, complement of a soft set, absolute soft set and null soft set with some working examples and illustrations. Pei and Miao (2005) advanced the work of (Maji *et al*., 2002). Ali *et al*. (2009) defined some notions such as restricted union, restricted intersection restricted difference and extended intersection of soft set. Applications of soft set in decision making were studied by various researchers, (Cagman and Enginoglu, 2010) presented soft set theory in decision making using unit interval form. Feng (2011) introduced soft rough set to multicriteria group decision making. With the growing interest in decision making (Balami, 2019) presented the application of neutrosophic soft set and its application in multicriteria decision making problems. Balami *et al*., (2021) introduced weighted intuitionistic fuzzy soft set based decision making using level soft set approach. Soft set having developed basic mathematical tools; various researchers delve into its applications in various areas of concern, such as data mining, forecast and decision making problems. Having studied its applications in decision sciences, we observed that not all parameters are of equal importance to the decision maker(s). Hence, in this research work, we introduced weighted soft set theory. In weighted soft set, each parameter or attribute is assigned a weight in the closed unit interval $[0,1]$ based on the interest or priority of the decision maker(s). Infact, introducing weighted soft set provides an avenue where an individual or group or an organization arrives at best decision without any form of regret at a long run. The algebra of weighted soft set and the established results demonstrates that soft set has indeed developed enough supporting tools where by all kind of mathematics can build on it.

Decision making is categorized into subjective decision making and objective decision making.

Subjective (personal) decision making is a decision solemnly taken by an individual base on his parameters of choice or interest.

Objective (corporate) decision making is a collective decision making taken by an organization or group of people by sampling opinion of members of the target group.

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The overall aim of decision making whether subjective decision making or objective decision making is to arrive at the best choice from alternative choice or multiple options available without regretting the choice or decision arrived at.

2. Preliminaries

Let U be a universal set and E be the set of all possible parameters under consideration with respect to U. Let the power set of U (i.e., the set of all subsets of U) be denoted by $P(U)$ and A is a subset of the parameters, E ($A \subseteq E$). The parameters are attributes, characteristics or properties associated with the objects in U.

Definition 2.1 (Molodtsov, 1999). A pair (μ, A) is called a soft set over U if and only if μ is a mapping of E into the set of all subsets of the set U. That is, a soft set is a parametrized family of subsets of the set U. For all $e \in E$, $\mu(e)$ is considered as the set of e −approximate elements of the soft set (μ, E).

Definition 2.2 (Maji *et al*., 2002). A soft set (μ, E) over a universe U is said to be *null* or *empty* soft set denoted by $\widetilde{\emptyset}$, if $\forall e \in A, \mu(e) = \emptyset$.

Definition 2.3 (Maji *et al*., 2002). A soft set (μ, A) over a universe U is called *absolute* or *universal soft set* denoted by $\widetilde{(\mu, A)}$ or $\widetilde{U}$, if $\forall e \in A, \mu(e) = U$.

Definition 2.4 (Maji *et al*., 2002). Let $E = \{e_1, e_2, e_3, \ldots, e_n\}$ be a set of parameters. The *not-set* of E denoted by $\neg E$ is defined as $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \ldots, \neg e_n\}$.

Definition 2.5 (Molodtsov, 1999). The *complement* of a soft set (μ, E), denoted by $(\mu, E)^c$, is defined as $(\mu, E)^c = (\mu^c, \neg E)$.

Where $\mu^c: \neg E \to P(U)$ is a mapping given by $\mu^c(\alpha) = U - \mu(\neg\alpha), \forall \alpha \in \neg E$. μ^c is called the soft complement function of μ. Consequently, $(\mu^c)^c = \mu$ and $((\mu, E)^c)^c = (\mu, E)$

Definition 2.6 (Molodtsov, 1999). Let (μ, A) and (Ω, B) be any two soft sets over a common universe U, (μ, A) is called a soft subset of (Ω, B), denoted by $(\mu, A) \widetilde{\subset} (\Omega, B)$ if ;

(i) $A \subset B$, and
(ii) $\forall e \in A, \mu(e) = \Omega(e)$
(μ, A) is said to be a soft super set of (Ω, B), if (Ω, B) is a subset of (μ, A) and it is denoted by $(\mu, A) \widetilde{\supset} (\Omega, B)$.

Definition 2.7 (Molodtsov, 1999). Two soft sets (μ, A) and (Ω, B) over a common universe U are said to be soft equal, denoted by $(\mu, A) = (\Omega, B)$, if (μ, A) is a soft subset of (Ω, B) and (Ω, B) is a soft subset of (μ, A).

Definition 2.8 (Ali *et al*. 2009). If (μ, A) and (Ω, B) are two soft sets then "$(\mu, A)\, AND\, (\Omega, B)$" denoted by $(\mu, A) \widetilde{\wedge} (\Omega, B)$ is defined as $(\mu, A) \widetilde{\wedge} (\Omega, B) = (H, A \times B)$, where $H(\alpha, \beta) = \mu(\alpha) \cap \Omega(\beta), \forall (\alpha, \beta) \in A \times B$.

Definition 2.9 (Ali *et al*. 2009). If (μ, A) and (Ω, B) are two soft sets then "$(\mu, A)\, OR\, (\Omega, B)$" denoted by $(\mu, A) \widetilde{\vee} (\Omega, B)$ is defined as $(\mu, A) \widetilde{\vee} (\Omega, B) = (P, A \times B)$, where, $P(\alpha, \beta) = \mu(\alpha) \cup \Omega(\beta), \forall (\alpha, \beta) \in A \times B$.

Definition 2.10 (Ali *et al*. 2009). Let (μ, A) and (Ω, B) be two soft sets over a common universe U. The union or extended union of (μ, A) and (Ω, B), denoted by $(\mu, A) \widetilde{\cup} (\Omega, B)$ or $(\mu, A) \cup_E (\Omega, B)$, is the soft set (H, C) satisfying the following conditions:

$$\text{(i) } C = A \cup B, \text{ (ii) } \forall\, e \in C,\; H(e) = \begin{cases} \mu(e) & \text{if } e \in A \backslash B \\ \Omega(e) & \text{if } e \in B \backslash A \\ \mu(e) \cup \Omega(e) & \text{if } e \in A \cap B \end{cases}$$

Definition 2.11 (Ali *et al*. 2009). The intersection of two soft sets (μ, A) and (Ω, B) over a common universe set U is the soft set (H, C), where $C = A \cap B$, and $\forall e \in C, H(e) = \mu(e)$ or $\Omega(e)$, we write $(\mu, A) \cap (\Omega, B) = (H, C)$

Definition 2.12 (Ali *et al*. 2009). The extended intersection of soft sets $(\mu, A)$and (Ω, B) over a common universe U, denoted by $(\mu, A)\ \tilde{\cup}_E\ (\Omega, B)$, is the soft set (H, C), where $C = A \cup B\ \forall e \in C$ and

$$H(e) = \begin{cases} \mu(e), & \text{if } e \in A \backslash B \\ \Omega(e), & \text{if } e \in B \backslash A \\ \mu(e) \cap \Omega(e), & \text{if } e \in A \cap B \end{cases}$$

Definition 2.13 (Ali *et al*. 2009). The restricted intersection of soft sets $(\mu, A)$and (Ω, B) over a common universe U, denoted by $(\mu, A) \Cap (\Omega, B)$, is the soft set (H, C), where $C = A \cap B \neq \emptyset$ such that $H(e) = \mu(e) \cap \Omega(e),\ \forall e \in C$.

Definition 2.14 (Ali *et al*. 2009). Let (μ, A) and (Ω, B) be two soft sets over a common universe U such that $A \cap B \neq \emptyset$. The restricted union of (μ, A) and (Ω, B), denoted by $(\mu, A)\ \tilde{\cup}_R\ (\Omega, B)$, is defined as $(\mu, A) \cup_R (\Omega, B) = (H, C)$, where $C = A \cup B$, and $\forall e \in C, H(e) = \mu(e) \cup \Omega(e)$.

Definition 2.15 (Ali *et al*. 2009). Let (μ, A) and (Ω, B) be two soft sets over a common universe U such that $A \cap B \neq \emptyset$ .The restricted difference of (μ, A) and (Ω, B) denoted by $(\mu, A) \sim_R (\Omega, B)$, is defined as $(\mu, A) \sim_R (\Omega, B) = (H, C)$, where $C = A \cap B$, and $\forall e \in C, H(e) = \mu(e) \setminus \Omega(e)$.

Definition 2.16 (Ali *et al*. 2009). The restricted symmetric difference of two soft sets (μ, A) and (Ω, B) over a common universe U is defined as $(\mu, A)\, \Delta (\Omega, B) = (\mu, A) \cup_R (\Omega, B) \sim_R \big((\Omega, B) \cup_R (\mu, A)\big)$.

3. Results and Discussions

Let U be an initial universe set, and E be a set of parameters or attributes (characteristics or qualities related to $U, P(U)$ be the power set of U and $A \subseteq E$

Definition 3.1: A triple (μ, A, Q) is a **weighted soft set**, where $\boldsymbol{\mu}$ is a mapping given by $\mu: A \longrightarrow P(U)$ and Q is a weight function, given by $Q: A \longrightarrow [0,1]$ specifying the weight $Q_j = Q(a_j)$ for all parameters $a_j \in A$ in other words, a weighted paramterized family of subsets of the universe set U. For every $a \in A, \mu(a, q)$ may be considered as the set of (a, q) elements or as the set of (a, q) approximate elements of the weighted soft set (μ, A, Q).

The weighted soft set (μ, A, Q) can be represented as a set of ordered triples as follows: $(\mu, A, Q) = \{(a, \mu(a), Q): a \in A, \mu(a) \in P(U) and\ Q \in [0,1]\}$**.**

Definition 3.2: let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U .

Then,

(i) (μ, A, Q_1) is said to be weighted soft **subset** of (Ω, B, Q_2) represented by $(\mu, A, Q_1) \subseteq (\Omega, B, Q_2)$ if $A \subseteq B, \mu(a) \subseteq \Omega(a)$ and $Q_1 \leq Q_2, \forall a \in A)$.

(ii) (μ, A, Q_1) and (Ω, B, Q_2) are said to be **equal**, denoted by $(\mu, A, Q_1) = (\Omega, B, Q_2)$, if $(\mu, A, Q_1) \subseteq (\Omega, B, Q_2)$ and $(\Omega, B, Q_2) \subseteq (\mu, A, Q_1)$ i.e $A = B, \boldsymbol{\mu(a)} = \Omega(a), \forall a \in A$ and $Q_1 = Q_2$ where $Q_1, Q_2 \in [0, 1]$.

Definition 3.3: Let (μ, A, Q) be a weighted soft set over U. Then the support of (μ, A, Q) is denoted by supp(μ, A, Q) is defined as supp $(\mu, A, Q) = \{\boldsymbol{a} \in A, \mu(a) \neq \emptyset \; and \; Q \in [0,1]\}$.

(i) (μ, A, Q) is called a **non-empty weighted soft set** if supp$(\mu, A, Q) \neq \emptyset$.

(ii) (μ, A, Q) is called **relative empty weighted soft set** denoted by $\emptyset_A$if $\mu(a) \neq \emptyset, \forall a \in A,\; Q \in [0,1]$..

(iii) (μ, A, Q) is called a **relative whole weighted soft set**, written as U_A if $\mu(a) = U, \forall a \in A,\; Q \in [0,1]$.

Definition 3.4: Let (μ, A, Q) be a weighted soft set over U. If $\mu(a) \neq \emptyset, \forall a \in A,\; Q \in [0,1]$ and, then (μ, A, Q) is called a **non-empty** weighted soft set.

Definition 3.5: Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft set over U then the **union** of (μ, A, Q_1) and (Ω, B, Q_2), written as $(\mu, A, Q_1) \,\tilde{\cup}\, (\Omega, B, Q_2)$ is a weighted soft set defined as $(\mu, A, Q_1) \,\tilde{\cup}\, (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ where $C = A \cup B$ and $\forall x \in C$,

$$\mathbb{Z}(x) = \begin{cases} \mu(x), & \text{if } x \in A \backslash B \\ \Omega(x), & \text{if } x \in B \backslash A \\ \mu(x) \cup \Omega(x), & \text{if } x \in A \cap B \end{cases}$$

Where $Q^* \in \max\,(\mathrm{Q}_1, \mathrm{Q}_2)$ and $\mathrm{Q}_1, \mathrm{Q}_2 \in [0,1]$,

Definition 3.6: Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U. The **restricted intersection** of (μ, A, Q_1) and (Ω, B, Q_2), represented by $(\mu, A, Q_1) \Cap (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ where $C = A \cap B \neq \emptyset$ and $\forall x \in C, \mathbb{Z}(x) = \mu(x) \cap \Omega(x)$, where $Q^* \in \min\,(\mathrm{Q}_1, \mathrm{Q}_2)$ and $\mathrm{Q}_1, \mathrm{Q}_2 \in [0,1]$.

Definition 3.7: Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U. The **restricted union** of (μ, A, Q_1) and (Ω, B, Q_2), denoted by $(\mu, A, Q_1) \,\tilde{\cup}_R\, (\Omega, B, Q_2)$ is a weighted soft set defined as $(\mu, A, Q_1) \,\tilde{\cup}_R\, (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ where $C = A \cap B \neq \emptyset$ and for all $x \in C$,

$\mathbb{Z}(x) = \mu(x) \cup \Omega(x)$ where $Q^* \in min(Q_1, Q_2)$ and $Q_1, Q_2 \in [0,1]$.

Definition 3.8: Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U. **The extended intersection** of (μ, A, Q_1) and (Ω, B, Q_2), denoted by $(\mu, A, Q_1) \,\tilde{\cap}_E\, (\Omega, B, Q_2)$ is defined as $(\mu, A, Q_1) \,\tilde{\cap}_E\, (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ where $C = A \cup B$ and for all $x \in C$

$$\mathbb{Z}(x) = \begin{cases} \mu(x), & \text{if } x \in A \backslash B \\ \Omega(x), & \text{if } x \in B \backslash A \\ \mu(x) \cap \Omega(x), & \text{if } x \in A \cap B \end{cases}.$$ Where $Q^* \in max(Q_1, Q_2)$ and $Q_1, Q_2 \in [0,1]$.

Definition 3.9: Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U. The **AND – product** or **AND – intersection** of (μ, A, Q_1) and (Ω, B, Q_2) denoted by $(\mu, A, Q_1) \tilde{\wedge} (\Omega, B, Q_2)$ is a weighted soft set defined by $(\mu, A, Q_1) \tilde{\wedge} (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ where $C = A \times B$ and for all $(x, y) \in A \times B, \mathbb{Z}(x, y) = \boldsymbol{\mu(x)} \cap \Omega(x)$ where $Q^* \in (Q_1 \times Q_2) \in [0,1]$ and $Q_1, Q_2 \in [0,1]$.

Definition 3.10: Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U. The **OR-product** or **OR-Union** of (μ, A, Q_1) and (Ω, B, Q_2) denoted by $(\mu, A, Q_1) \tilde{\wedge} (\Omega, B, Q_2)$ is a weighted soft set defined by $(\mu, A, Q_1) \tilde{\vee} (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ where $C = A \times B$ and for all $(x, y) \in A \times B, \mathbb{Z}(x, y) = \mu(x) \cup \Omega(x)$ where $Q^* \in (Q_1 \times Q_2) \in [0,1]$ and $Q_1, Q_2 \in [0,1]$.

Definition 3.11: A weighted soft set (μ, A, Q) over a universe U is said to be a **null weighted soft set** or an **empty weighted soft set** denoted by $\tilde{\emptyset}_A$ if for all $a \in A$, there exists $Q \in [0,1]$, such that $\mu(a) = \emptyset$.

Definition 3.12: A weighted soft set (μ, A, Q) over a universe U is said to be a **absolute weighted soft set** or an **universal weighted soft set** denoted by $\widetilde{U}_A$ if for all $a \in A$, there exists $Q \in [0,1]$, such that $\mu(a) = U$.

Example 3.1: Let U be a set of University final year accounting students, a new generation Bank X is considering to employ after their graduation and A be a set of decision parameters or attributes. Each weighted parameter is a word or a sentence. Consider,

$$A = \{First\ Class, \mathrm{Sec}ond\ Class Upper, \mathrm{Sec}ond\ Class\, Lower, Tall, beautiful, Eloquent, Healthy\}$$

Suppose, there are six graduating students in the universe set U, given by $U = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ and set of parameters $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ $A_w = \{a_1 w_1, a_2 w_2, a_3 w_3, a_4 w_4, a_5 w_5, a_6 w_6, a_7 w_7\}$ where $W = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$ $w_i \in [0,1], i = 1, 2, ...7$

$$W_1 = 0.7, W_2 = 0.7, W_3 = 0.5, W_4 = 0.4, W_5 = 0.6, W_6 = 0.9, W_7 = 0.95$$

Suppose

$$F(a_1, 0.7) = \{S_1, S_2, S_5\}$$

$$F(a_2, 0.7) = \{S_1, S_3, S_4, S_6\}$$

$$F(a_3, 0.5) = \{S_3, S_5\}$$

$$F(a_4, 0.4) = \{S_1, S_4, S_6\}$$

$$F(a_5, 0.6) = \{S_3, S_4, S_6\}$$

$$F(a_6, 0.9) = \{S_1, S_5, S_6\}$$

$$F(a_7, 0.95) = \{S_1, S_2, S_5\}.$$

$CV - Choice\, Value\, (C_i)$.

Algorithm

Step I: Input the weighted soft set (μ, A, Q)

Step II: Compute $C_i = w_i \times d_{ij}$

Step III: Optimal decision is to select C_k, if $C_k = max C_i$ from each U.

Step IV: If there are more than one C_k, then any one of C_k may be chosen.

Note: If unique choice is needed, then the decision maker(s) will go back to **Step I** and impose more weights to parameters of interest, so as to enable the decision maker(s) obtain a unique choice from the multiple options.

Table 3.1: Tabular Representation of Weighted Soft set with choice value

A,W U	$a_1,0.7$	$a_2,0.7$	$a_3,0.5$	$a_4,0.4$	$a_5,0.6$	$a_6,0.9$	$a_7,0.95$	$Choice Value(CV)$
S_1	1	1	0	1	0	1	1	3.65
S_2	1	0	0	0	0	0	1	1.55
S_3	0	1	1	0	1	0	0	1.80
S_4	0	1	0	1	1	0	0	1.70
S_5	1	0	1	0	0	1	1	3.05
S_6	0	1	0	1	1	1	0	2.60

From Table 3.1, the maximum choice value $C_1 = 3.65$ and this next is $C_5 = 3.05$ suppose only two candidates are to be chosen, candidate S_1 and S_5 are the most qualified for the job.

Theorem 3.1: Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U, $q_1 : A \to [0,1]$ and $q_2 : A \to [0,1]$ are specifying weight functions, i.e $q_1 = Q(a_i)$ and $q_2 = Q(b_j)$ for every parameter $a_i \in A$ and $b_i \in B$.

Then

i. $(\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2)$ is a weighted soft set over U, if it is non-empty.

ii. $(\mu, A, Q_1) \Cap (\Omega, B, Q_2)$ is a weighted soft set over U, if it is non-empty.

iii. $(\mu, A, Q_1) \mp (\Omega, B, Q_2)$ Is a weighted soft set over U, if it is non-empty

Proof:

(i) Let $(\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$, where $C = A \cup B \neq \emptyset$ and for all $x \in C$ there exists $Q^* \in min(Q_1, Q_2)$ and $Q_1, Q_2 \in [0,1]$ such that

$$\mathbb{Z}(x) = \begin{cases} \mu(x), & \text{if } x \in A \backslash B \\ \Omega(x), & \text{if } x \in B \backslash A \\ \mu(x) \cup \Omega(x), & \text{if } x \in A \cap B \end{cases}.$$

By hypothesis $(\mathbb{Z}, C, Q^*)$ is a non-empty weighted soft set over U. If $x \in C$ and $Q^* \in [0,1]$, then $\mathbb{Z}(x) = \mu(x) \cup \Omega(x) \neq \emptyset$ and $Q^*: C \to [0,1]$. It implies that $\mu(x) \neq \emptyset$ and $\Omega(x) \neq \emptyset$ and are both subsets of U. It follows that, $\mu(x)$ is a subsets of U for all $x \in supp(\mathbb{Z}, C, Q^*)$ and $Q^* = min(Q_1, Q_2) \in [0,1]$. Hence, $(\mathbb{Z}, C, Q^*)$ is a weighted soft set over U.

(ii) Let $(\mu, A, Q_1) \Cap (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ where $\mathbb{Z}(x) = \mu(x) \cap \Omega(x)$ and for all $x \in C = A \cap B \neq \emptyset$, there exists $Q^* = \min(Q_1, Q_2) \in [0,1]$. By the hypothesis, $(\mathbb{Z}, C, Q^*)$ is a non-empty weighted soft set. If $x \in supp(\mathbb{Z}, C, Q^*)$ and $Q^* \in [0,1]$ then $\mathbb{Z}(x) = \mu(x) \cap \Omega(x) \neq \emptyset$ and $Q^*: C \longrightarrow [0,1]$. It follows that $\mu(x) \neq \emptyset$ and $\Omega(x) \neq \emptyset$ are both subsets of U, for all $x \in supp(\mathbb{Z}, C, Q^*)$ there exists $Q^* = min(Q_1, Q_2) \in [0,1]$. Thus, $(\mathbb{Z}, C, Q^*)$ is a weighted soft set over U.

(iii) Let $(\mu, A, Q_1) \mp (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ and $Q^* = \min(Q_1, Q_2) \in [0,1]$ such that $\mathbb{Z}(x) = \mu(x) \cap \Omega(x)$. By the hypothesis, $\mu(x) \cup \Omega(x) \neq \emptyset$. It implies that $\mu(x) \neq \emptyset$ and $\Omega(x) \neq \emptyset$ are both weighted soft sets over U for all $x \in supp(\mathbb{Z}, C, Q^*)$ and $Q^* \in [0,1]$. Therefore, $(\mu, A, Q_1) \mp (\Omega, B, Q_2)$ is a weighted soft sets over U.

Theorem 3.2: Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U, $Q_1: A \to [0,1]$ and $Q_2: B \to [0,1]$ are specifying weight functions, i.e $Q_1 = Q(a_i)$ and $Q_2 = Q(b_j)$ for every parameter $a_i \in A$ and $b_i \in B$.

Then

i. $(\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2)$ is a weighted soft set over U if it is non-empty and $\mu(x)$ and $\Omega(x)$ are ordered by inclusion relation for all.
ii. $(\mu, A, Q_1) \tilde{\cup} (\Omega, B, Q_2)$ is a weighted soft set over U if it is non-empty and if A and B are disjoint.
iii. $(\mu, A, Q_1) \tilde{\wedge} (\Omega, B, Q_2)$ is a weighted soft set over U if it is non-empty.
iv. $(\mu, A, Q_1) \tilde{\vee} (\Omega, B, Q_2)$ is non-empty and if $\mu(x)$ and $\Omega(x)$ are ordered by inclusion relation for all $(x_1, x_2) \in supp(\mu, A, Q_1) \tilde{\vee} (\Omega, B, Q_2)$.

Proof:

(i) Let $(\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ for every $x \in C = A \cap B$ and $A \cap B \neq \emptyset$ there exists $Q^* = \min(Q_1, Q_2) \in [0,1]$ such that $\mathbb{Z}(x) = \mu(x) \cup \Omega(x)$. By the hypothesis $(\mathbb{Z}, C, Q^*)$ is a non-empty weighted soft set over U and $Q^*: C \to [0,1]$. If $x \in supp(\mathbb{Z}, C, Q^*)$ then there exists $Q^* \in [0,1]$ such That $\mathbb{Z}(x) = \mu(x) \cup \Omega(x) \neq \emptyset$. Since $\mu(x)$ and $\Omega(x)$ are ordered by inclusion relation for every $x \in supp(\mathbb{Z}, C, Q^*)$ there exists $Q^* = \max(Q_1, Q_2) \in [0,1]$ such that $\mu(x) \cup \Omega(x) = \mu(x)$ or $\mu(x) \cup \Omega(x) = \Omega(x)$. Since $\mu(x) \neq \emptyset$ and $\Omega(x) \neq \emptyset$ are weighted soft sets over U. Then $\mathbb{Z}(x)$ is a weighted soft set over U, for every $x \in supp(\mathbb{Z}, C, Q^*)$ and $Q^* \in [0,1]$. Therefore, $(\mathbb{Z}, C, Q^*)$ is a weighted soft set.

(ii) Let $(\mu, A, Q_1) \cup (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$, where $C = A \cup B$ and for all $x \in C$ there exists $Q^* \in max(Q_1, Q_2)$ and $Q_1, Q_2 \in [0,1]$ and $A \cap B \neq \emptyset$ such that

$$\mathbb{Z}(x) = \begin{cases} \mu(x), & \text{if } x \in A \backslash B \\ \Omega(x), & \text{if } x \in B \backslash A \\ \mu(x) \cup \Omega(x), & \text{if } x \in A \cap B \end{cases}.$$

It follows that either $x \in A/B$ or $x \in B/A$ for all $x \in C$ and $Q^* \in max(Q_1, Q_2) \in [0,1]$. If $x \in A/B$, there exists $Q_1 \in [0,1]$ such that $\mathbb{Z}(x) = \mu(x)$ is a weighted soft set over U and if $x \in B/A$, there exists $Q_2 \in [0,1]$ such that $\mathbb{Z}(x) = \Omega(x)$ is a weighted soft set over U. Therefore, $(\mu, A, Q_1) \cup (\Omega, B, Q_2)$ is a weighted soft set over U.

(iii) Let $(\mu, A, Q_1) \tilde{\wedge} (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ and for all $(x_1, x_2) \in C$, where $C = A \times B$, then there exists $Q^* = (Q_1 \times Q_2) \in [0,1]$ such that $\mathbb{Z}(x_1, x_2) = \mu(x_1) \cap \Omega(x_2)$. Then by the hypothesis, $(\mathbb{Z}, C, Q^*)$ is a non-empty weighted soft set over U. And $Q^* \in [0,1]$, Where $Q^*: C \longrightarrow [0,1]$. If $(x_1, x_2) \in supp(\mathbb{Z}, C, Q^*)$ then there exists $Q^* \in [0,1]$ such that $\mathbb{Z}(x_1, x_2) = \mu(x_1) \cap \Omega(x_2) \neq \emptyset$ are both weighted soft sets over U for all $(x_1, x_2) \in supp(\mathbb{Z}, C, Q^*)$. Hence, $(\mu, A, Q_1) \tilde{\wedge} (\Omega, B, Q_2)$ is weighted soft set over U.

(iv) Let $(\mu, A, Q_1) \tilde{\vee} (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$, for all $(x_1, x_2) \in C$, where $C = A \times B$, there exists $Q^* = (Q_1 \times Q_2) \in [0,1]$ such that $\mathbb{Z}(x_1, x_2) = \mu(x_1) \cup \Omega(x_2)$. By the hypothesis $(\mathbb{Z}, C, Q^*)$ is a non-empty weighted soft set over U and $Q^* \in [0,1]$, where $Q^*: C \longrightarrow [0,1]$. If $(x_1, x_2) \in supp(\mathbb{Z}, C, Q^*)$, then there exists $Q^* = (Q_1 \times Q_2) \in [0,1]$ such that $\mathbb{Z}(x_1, x_2) = \mu(x_1) \cup \Omega(x_2) \neq \emptyset$. Since $\mu(x_1)$ and $\Omega(x_2)$ are ordered by inclusion relation for all $(x_1, x_2) \in supp(\mathbb{Z}, C, Q^*)$ and $Q^* \in [0,1]$, $\mu(x_1) \cup \Omega(x_2) = \mu(x_1)$ or $\mu(x_1) \cup \Omega(x_2) = \Omega(x_2)$, since $\mu(x_1) \neq \emptyset$ and $\Omega(x_2) \neq \emptyset$ are both weighted soft sets over U for all $(x_1, x_2) \in supp(\mathbb{Z}, C, Q^*)$ and $Q^* \in [0,1]$. Thus, $(\mu, A, Q_1) \tilde{\vee} (\Omega, B, Q_2)$ is a weighted soft set over U.

Theorem 3.4 demonstrates that, the commutative and associative laws hold in weighted soft sets.

Theorem 3.4:

Let (μ, A, Q_1), (Ω, B, Q_2) and (Θ, D, Q_3) be three weighted soft sets over U and $Q_1, Q_3, Q_3 \in [0,1]$. Then

i. $(\mu, A, Q_1) \Cap (\Omega, B, Q_2) = (\Omega, B, Q_2) \Cap (\mu, A, Q_1)$
ii. $\big((\mu, A, Q_1) \Cap (\Omega, B, Q_2)\big) \Cap (\Theta, D, Q_3) = (\mu, A, Q_1) \Cap ((\Omega, B, Q_2) \Cap (\Theta, D, Q_3))$.

Proof:

i. L H S: Let $(\mu, A, Q_1) \Cap (\Omega, B, Q_2) = (W, C, Q^*)$, where $C = A \cap B$ and for all $x \in C$, there exists $Q^* = \min(Q_1, Q_2) \in [0,1]$ such that
$W(x) = \mu(x) \cap \Omega(x)$ **(1)**
R H S: Let $(\Omega, B, Q_2) \Cap (\mu, A, Q_1) = (K, C, Q^*)$, where $C = B \cap A = A \cap B$, similarly, for every $x \in C$, there exists $Q^* = \min(Q_2, Q_1) = \min(Q_1, Q_2) \in [0,1]$ such that
$K(x) = \mu(x) \cap \Omega(x)$ **(2)**.
Since $(\mathbf{1}) = (\mathbf{2})$, therefore the mappings W and K are the same, hence

$(\mu, A, Q_1) \Cap (\Omega, B, Q_2) = (\Omega, B, Q_2) \Cap (\mu, A, Q_1)$.

ii. L H S: Let $\big((\mu, A, Q_1) \Cap (\Omega, B, Q_2)\big) \Cap (\Theta, D, Q_3) = (R, C, Q^*)$, where $C = (A \cap B) \cap D$ and for each $x \in C$, there exists $Q^* = \min(Q_1, Q_2, Q_3) \in [0,1]$,
such that $R(x) = \mu(x) \cap \Omega(x) \cap \Theta(x)$ **(3)**.
R H S: Let $(\mu, A, Q_1) \Cap ((\Omega, B, Q_2) \Cap (\Theta, D, Q_3)) = (g, C, Q^*)$, where $C = A \cap (B \cap D)$ and for every $x \in C$, there exists $Q^* = \min(Q_1, Q_2, Q_3) \in [0,1]$,
such that $R(x) = \mu(x) \cap \Omega(x) \cap \Theta(x)$ **(4)**.

Since (**3**) = (**4**), therefore the result has been established.

Theorem 3.5:

Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U and $Q_1, Q_2 \in [0,1]$. Then,

i. $\big((\mu, A, Q_1) \mathbin{\tilde{\cup}} (\Omega, B, Q_2)\big) \Cap (\mu, A, Q_1) = (\mu, A, Q_1)$
ii. $\big((\mu, A, Q_1) \Cap (\Omega, B, Q_2)\big) \mathbin{\tilde{\cup}} (\mu, A, Q_1) = (\mu, A, Q_1)$

Proof:

i. Let $\big((\mu, A, Q_1) \mathbin{\tilde{\cup}} (\Omega, B, Q_2)\big) \Cap (\mu, A, Q_1) = (\text{Þ}, (A \cup B) \cap A)$. For every $x \in A$, there exists $Q^* \in [0,1]$ with the following cases being considered.
First case: $x \in B$, then $\text{Þ}(x) = (\mu(x) \cup \Omega(x)) \cap \mu(x)$.
Second Case: $x \notin B$, then $\text{Þ}(x) = (\mu(x) \cap \mu(x) = \mu(x)$ Therefore, μ and Þ are the same mapping and have $\big((\mu, A, Q_1) \mathbin{\tilde{\cup}} (\Omega, B, Q_2)\big) \Cap (\mu, A, Q_1) = (\mu, A, Q_1)$.

ii. Let $\big((\mu, A, Q_1) \Cap (\Omega, B, Q_2)\big) \mathbin{\tilde{\cup}} (\mu, A, Q_1) = (\text{Þ}, (A \cap B) \cup A, Q^*)$. For every $x \in A$, there exists $Q^* \in [0,1]$ with the following cases being considered.
Case 1: If $x \in B$, then $V(x) = (\mu(x) \cap \Omega(x)) \cup \mu(x)$.

Case 2: If $x \notin B$, then $V(x) = (\emptyset \cup \mu(x) = \mu(x)$ Therefore, the mappings μ and V are the same and hence $\left((\mu, A, Q_1) \tilde{\cup} (\Omega, B, Q_2)\right) \Cap (\mu, A, Q_1) = (\mu, A, Q_1)$

Theorem 3.6:

Let (μ, A, Q_1) and (Ω, B, Q_2) be two weighted soft sets over U and $Q_1, Q_2 \in [0,1]$. Then,

i. $\left((\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2)\right) \tilde{\cup}_R (\mu, A, Q_1) = (\mu, A, Q_1)$
ii. $\left((\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2)\right) \tilde{\cap}_E (\mu, A, Q_1) = (\mu, A, Q_1)$

Proof:

i. Let $\left((\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2)\right) \tilde{\cup}_R (\mu, A, Q_1) = (Z, (A \cup B) \cap A, Q^*)$. Now, for every $x \in A$, there exists $Q^* = ((Q_1 \cup Q_2) \cap Q_3) \in [0,1]$ with the following two cases being considered.

Case 1: If $x \in B$, then $Z(x) = (\mu(x) \cup \Omega(x)) \cap \mu(x)$.
Case 2: If $x \notin B$, then $Z(x) = (\mu(x) \cap \mu(x) = \mu(x)$ Therefore, the mappings μ and Z are the same mappings and hence $\left((\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2)\right) \tilde{\cup}_R (\mu, A, Q_1) = (\mu, A, Q_1)$.

ii. Let $\left((\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2)\right) \tilde{\cap}_E (\mu, A, Q_1) = (J, (A \cap B) \cup A, Q^*)$. Now, for each $x \in A$, there exists $Q^* = ((Q_1 \cap Q_2) \cup Q_3) \in [0,1]$ with the following similar two cases being considered.

Case 1: If $x \in B$, then $J(x) = (\mu(x) \cap \Omega(x)) \cup \mu(x)$.
Case 2: If $x \notin B$, then $J(x) = (\emptyset \cup \mu(x) = \mu(x)$ Therefore, the mappings μ and J are the same mappings and hence $\left((\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2)\right) \tilde{\cap}_E (\mu, A, Q_1) = (\mu, A, Q_1)$.

Theorem 3.7: Let (μ, A, Q) be weighted soft sets over the universe set U and $Q \in [0,1]$. Then,

i. $(\mu, A, Q) \tilde{\cup}\ U_E = U_E$.
ii. $(\mu, A, Q) \Cap\ U_E = (\mu, A, Q)$.
iii. $(\mu, A, Q) \tilde{\cup}\ (\mu, A, Q) = (\mu, A, Q)$.
iv. $(\mu, A, Q) \Cap\ (\mu, A, Q) = (\mu, A, Q)$

Proof:

i. Let $(\mu, A, Q) = (\mu, A, Q_1)$ and $U_E = (\Omega, E, Q_2)$
$(\mu, A, Q_1) \tilde{\cup} (\Omega, E, Q_2) = (T, C, Q^*)$, where $C = A \cup E = E$ and for all $x \in C$ there exists $Q^* = Q_1 \cup Q_2 = \max(Q_1, Q_2) \in [0,1]$, such that
$T(x) = \mu(x) \cup \Omega(x) = \Omega(x)$.
Since $T(x)$ and $\Omega(x)$ are the same mappings. Therefore, $(\mu, A, Q) \tilde{\cup}\ U_E = U_E$.

ii. Let $(\mu, A, Q) = (\mu, A, Q_1)$ and $U_E = (\Omega, E, Q_2)$
$(\mu, A, Q_1) \Cap (\Omega, E, Q_2) = (T, C, Q^*)$, where $C = A \cap E = A$ and for all $x \in C$ there exists $Q^* = Q_1 \cap Q_2 = \min(Q_1, Q_2) \in [0,1]$, such that
$T(x) = \mu(x) \cap \Omega(x) = \mu(x)$.
Since the mappings $T(x)$ and $\mu(x)$ are the same,
therefore, $(\mu, A, Q) \tilde{\cup}\ U_E = (\mu, A, Q)$.

iii. Let $(\mu, A, Q) \tilde{\cup}\ (\mu, A, Q) = (W, C, Q^*)$, where $C = A \cup A = A$ and for every $x \in C$, then there exists $Q^* = Q \cup Q = Q \in [0,1]$, such that
$W(x) = \mu(x) \cup \mu(x) = \mu(x)$. This implies $W(x) = \mu(x)$.
Hence, $(\mu, A, Q) \tilde{\cup}\ (\mu, A, Q) = (\mu, A, Q)$.

iv. Let $(\mu, A, Q) \Cap (\mu, A, Q) = (V, C, Q^*)$, where $C = A \cap A = A$ and for every $x \in C$, then there exists $Q^* = Q \cap Q = Q \in [0,1]$, such that
$V(x) = \mu(x) \cap \mu(x) = \mu(x)$. This implies $V(x) = \mu(x)$.
Hence, the result has been established.

Theorem 3.8, shows that commutative and associative properties of union ($\widetilde{\cup}$) hold true in weighted soft set.

Theorem 3.8:

Let (μ, A, Q_1), (Ω, B, Q_2) and (Θ, D, Q_3) be three weighted soft sets over U and $Q_1, Q_3, Q_3 \in [0,1]$. Then

i. $(\mu, A, Q_1) \widetilde{\cup} (\Omega, B, Q_2) = (\Omega, B, Q_2) \widetilde{\cup} (\mu, A, Q_1)$.
ii. $\big((\mu, A, Q_1) \widetilde{\cup} (\Omega, B, Q_2)\big) \widetilde{\cup} (\Theta, D, Q_3) = (\mu, A, Q_1) \widetilde{\cup} ((\Omega, B, Q_2) \widetilde{\cup} (\Theta, D, Q_3))$.

Proof:

i. L H S: Let $(\mu, A, Q_1) \widetilde{\cup} (\Omega, B, Q_2) = (\delta, C, Q^*)$, where $C = A \cup B$ and for all $x \in C$, there exists $Q^* = \max(Q_1, Q_2) \in [0,1]$ such that
$\delta(x) = \mu(x) \cup \Omega(x)$ **(5)**
R H S: Let $(\Omega, B, Q_2) \widetilde{\cup} (\mu, A, Q_1) = (\varphi, C, Q^*)$, where $C = B \cup A = A \cup B$, similarly, for every $x \in C$, there exists $Q^* = \max(Q_2, Q_1) = \max(Q_1, Q_2) \in [0,1]$ such that
$\varphi(x) = \mu(x) \cup \Omega(x)$ **(6)**
Since $(\mathbf{5}) = (\mathbf{6})$, therefore the mappings δ and φ are the same, hence

$(\mu, A, Q_1) \widetilde{\cup} (\Omega, B, Q_2) = (\Omega, B, Q_2) \widetilde{\cup} (\mu, A, Q_1)$.

ii. L H S: Let $\big((\mu, A, Q_1) \widetilde{\cup} (\Omega, B, Q_2)\big) \widetilde{\cup} (\Theta, D, Q_3) = (\Delta, C, Q^*)$, where $C = (A \cup B) \cup D$ and for each $x \in C$, there exists $Q^* = \max(Q_1, Q_2, Q_3) \in [0,1]$,
such that $\Delta(x) = \mu(x) \cap \Omega(x) \cap \Theta(x)$ **(7)**
R H S: Let $(\mu, A, Q_1) \widetilde{\cup} ((\Omega, B, Q_2) \widetilde{\cup} (\Theta, D, Q_3)) = (\nabla, C, Q^*)$, where $C = A \cup (B \cup D)$ and for every $x \in C$, there exists $Q^* = \max(Q_1, Q_2, Q_3) \in [0,1]$,
such that $\nabla(x) = \mu(x) \cap \Omega(x) \cap \Theta(x)$ **(8)**

Since (**7**) = (**8**), therefore the result has been established.

Theorem 3.9: Let (μ, A, Q_1), (Ω, B, Q_2) and (Θ, D, Q_3) be three weighted soft sets over U and $Q_1, Q_3, Q_3 \in [0,1]$. Then,

i. $(\mu, A, Q_1) \widetilde{\cap}_E (\mu, A, Q_1) = (\mu, A, Q_1)$.
ii. $(\mu, A, Q_1) \widetilde{\cap}_E (\Omega, B, Q_2) = (\Omega, B, Q_2) \widetilde{\cap}_E (\mu, A, Q_1)$.
iii. $((\mu, A, Q_1) \widetilde{\cap}_E (\Omega, B, Q_2)) \widetilde{\cap}_E (\Theta, D, Q_3) = (\mu, A, Q_1) \widetilde{\cap}_E ((\Omega, B, Q_2) \widetilde{\cap}_E (\Theta, D, Q_3))$

Proof:

i. Let $(\mu, A, Q_1) \widetilde{\cap}_E (\mu, A, Q_1) = (\alpha, C, Q^*)$, where $C = A \cup A = A$ and for each $x \in C$, then there exists $Q^* = Q_1 \cup Q_1 = Q_1 \in [0,1]$, such that
$\alpha(x) = \mu(x) \cap \mu(x) = \mu(x)$. This implies $\alpha(x) = \mu(x)$.
Hence, $(\mu, A, Q_1) \widetilde{\cap}_E (\mu, A, Q_1) = (\mu, A, Q_1)$.

ii. L H S: Let $(\mu, A, Q_1) \widetilde{\cap}_E (\Omega, B, Q_2) = (\pi, C, Q^*)$, where $C = A \cup B$ and for all $x \in C$, there exists $Q^* = \max(Q_1, Q_2) \in [0,1]$ such that
$\pi(x) = \mu(x) \cap \Omega(x)$.

R H S: Let $(\Omega, B, Q_2) \tilde{\cup} (\mu, A, Q_1) = (\tau, C, Q^*)$, where $C = B \cup A = A \cup B$, similarly, for every $x \in C$, there exists $Q^* = \max(Q_2, Q_1) = \max(Q_1, Q_2) \in [0,1]$ such that
$\tau(x) = \mu(x) \cup \Omega(x)$.

Since $\tau(x) = \pi(x)$, therefore the mappings τ and π are the same, hence

$$(\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2) = (\Omega, B, Q_2) \tilde{\cap}_E (\mu, A, Q_1).$$

iii. L H S: Let $\left((\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2)\right) \tilde{\cap}_E (\Theta, D, Q_3) = (\psi, C, Q^*)$, where $C = (A \cup B) \cup D$ and for each $x \in C$, there exists $Q^* = \max(Q_1, Q_2, Q_3) \in [0,1]$,
such that $\psi(x) = \mu(x) \cap \Omega(x) \cap \Theta(x)$.
R H S: Let $(\mu, A, Q_1) \tilde{\cup} ((\Omega, B, Q_2) \tilde{\cup} (\Theta, D, Q_3)) = (Ƒ, C, Q^*)$, where $C = A \cup (B \cup D)$ and for every $x \in C$, there exists $Q^* = \max(Q_1, Q_2, Q_3) \in [0,1]$,
such that $Ƒ(x) = \mu(x) \cap \Omega(x) \cap \Theta(x)$.

Since, $\psi(x) = Ƒ(x)$, therefore the result has been established.

Theorem 3.10: Let (μ, A, Q_1), (Ω, B, Q_2) and (Θ, D, Q_3) be three weighted soft sets over U and $Q_1, Q_3, Q_3 \in [0,1]$. Then,

i. $(\mu, A, Q_1) \tilde{\cup}_R (\mu, A, Q_1) = (\mu, A, Q_1)$.
ii. $(\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2) = (\Omega, B, Q_2) \tilde{\cup}_R (\mu, A, Q_1)$.
iii. $((\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2)) \tilde{\cup}_R (\Theta, D, Q_3) = (\mu, A, Q_1) \tilde{\cup}_R ((\Omega, B, Q_2) \tilde{\cup}_R (\Theta, D, Q_3))$

Proof:

i. Let $(\mu, A, Q_1) \tilde{\cup}_R (\mu, A, Q_1) = (Ӥ, C, Q^*)$, where $C = A \cap A = A$ and for every $x \in C$, then there exists $Q^* = Q_1 \cap Q_1 = Q_1 \in [0,1]$, such that
$Ӥ(x) = \mu(x) \cap \mu(x) = \mu(x)$. This implies $Ӥ(x) = \mu(x)$.
Hence, $(\mu, A, Q_1) \tilde{\cup}_R (\mu, A, Q_1) = (\mu, A, Q_1)$.

ii. L H S: Let $(\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2) = (ɏ, C, Q^*)$, where $C = A \cap B$ and for all $x \in C$, there exists $Q^* = \min(Q_1, Q_2) \in [0,1]$, such that
$ɏ(x) = \mu(x) \cup \Omega(x)$.
R H S: Let $(\Omega, B, Q_2) \tilde{\cup}_R (\mu, A, Q_1) = (Ⱪ, C, Q^*)$, where $C = B \cap A = A \cap B$, similarly, for every $x \in C$, there exists $Q^* = \min(Q_2, Q_1) = \min(Q_1, Q_2) \in [0,1]$ such that
$Ⱪ(x) = \mu(x) \cup \Omega(x)$.

Since $ɏ(x) = Ⱪ(x)$, therefore $(\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2) = (\Omega, B, Q_2) \tilde{\cup}_R (\mu, A, Q_1)$..

iii. L H S: Let $\left((\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2)\right) \tilde{\cup}_R (\Theta, D, Q_3) = (Ꚍ, C, Q^*)$, where $C = (A \cap B) \cap D$ and for all $x \in C$, there exists $Q^* = \min(Q_1, Q_2, Q_3) \in [0,1]$,
such that $Ꚍ(x) = \mu(x) \cup \Omega(x) \cup \Theta(x)$.
R H S: Let $(\mu, A, Q_1) \tilde{\cup}_R ((\Omega, B, Q_2) \tilde{\cup}_R (\Theta, D, Q_3)) = (Ŀ, C, Q^*)$, where $C = A \cap (B \cap D)$ and for all $x \in C$, there exists $Q^* = \min(Q_1, Q_2, Q_3) \in [0,1]$,
such that $Ŀ(x) = \mu(x) \cap \Omega(x) \cap \Theta(x)$.

Since $Ꚍ(x) = Ŀ(x)$, therefore the result has been established.

4. De Morgan's Theorems on weighted soft set

In classical set theory, it is a fact that, De Morgan's laws relate union and intersection operations through complements. Similarly, De Morgan's laws relate union and extended intersection operations and restricted union and restricted intersection through complements in weighted soft set theory context.

Theorem 4.1: Let (μ, A, Q_1) and (Ω, B, Q_2) weighted soft sets over U and $Q_1, Q_2 \in [0,1]$. Then,

i. $((\mu, A, Q_1) \tilde{\cup} (\Omega, B, Q_2))^C = (\mu, A, Q_1)^C \tilde{\cap}_E (\Omega, B, Q_2)^C$.
ii. $((\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2))^C = (\mu, A, Q_1)^C \tilde{\cup} (\Omega, B, Q_2)^C$.

Proof:

i. Suppose that $(\mu, A, Q_1) \tilde{\cup} (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ and for all $x \in C$, where $C = A \cup B$, there exists $Q \in [0,1]$ such that,

$$\mathbb{Z}(x) = \begin{cases} \mu(x), & \text{if } x \in A \backslash B \\ \Omega(x), & \text{if } x \in B \backslash A \\ \mu(x) \cup \Omega(x), & \text{if } x \in A \cap B \end{cases}.$$

Now, $((\mu, A, Q_1) \tilde{\cup} (\Omega, B, Q_2))^C = (\mathbb{Z}, C, Q^*)^C = (\mathbb{Z}^C, \neg C, Q^*)$. For each $x \in \neg C = \neg A \cup \neg B$, there exists $Q \in [0,1]$ such that,

$$\mathbb{Z}^C(x) = \begin{cases} \mu^C(x), & \text{if } x \in \neg A \backslash \neg B \\ \Omega^C(x), & \text{if } x \in \neg B \backslash \neg A \\ \mu^C(x) \cap \Omega^C(x), & \text{if } x \in \neg A \cap \neg B \end{cases} \quad \textbf{(9)}$$

Also, let $(\mu, A, Q_1)^C \tilde{\cap}_E (\Omega, B, Q_2)^C = (\mu^C, \neg A, Q_1) \tilde{\cap}_E (\Omega^C, \neg B, Q_2) = (\mathfrak{f}, \neg C, Q^*)$ and for all $x \in \neg C$ where $\neg C = \neg A \cup \neg B$, there exists $Q^* \in [0,1]$ such that,

$$\mathfrak{f}(x) = \begin{cases} \mu^C(x), & \text{if } x \in \neg A \backslash \neg B \\ \Omega^C(x), & \text{if } x \in \neg B \backslash \neg A \\ \mu^C(x) \cap \Omega^C(x), & \text{if } x \in \neg A \cap \neg B \end{cases} \quad \textbf{(10)}$$

From (**9**) and (**10**), it is obvious that $\mathbb{Z}^C(x) = \mathfrak{f}(x)$. Hence, the result has been proved.

ii. Let $(\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ for all $x \in C = A \cup B$, there exists $Q \in [0,1]$ such that,

$$\mathbb{Z}(x) = \begin{cases} \mu(x), & \text{if } x \in A \backslash B \\ \Omega(x), & \text{if } x \in B \backslash A \\ \mu(x) \cap \Omega(x), & \text{if } x \in A \cap B \end{cases}.$$

Now, $((\mu, A, Q_1) \tilde{\cap}_E (\Omega, B, Q_2))^C = (\mathbb{Z}, C, Q^*)^C = (\mathbb{Z}^C, \neg C, Q^*)$ and for all $x \in \neg C = \neg \mathrm{A} \cup \neg \mathrm{B}$, there exists $Q^* \in [0,1]$, such that

$$\mathbb{Z}^C(x) = \begin{cases} \mu^C(x), & \text{if } x \in \neg A \backslash \neg B \\ \Omega^C(x), & \text{if } x \in \neg B \backslash \neg A \\ \mu^C(x) \cup \Omega^C(x), & \text{if } x \in \neg A \cap \neg B \end{cases} \quad \textbf{(11)}$$

Also, $(\mu, A, Q_1)^C \tilde{\cup} (\Omega, B, Q_2)^C = (\mu^C, \neg A, Q_1) \tilde{\cup} (\Omega^C, \neg B, Q_2) = (T, \neg C, Q^*)$, say and for all $x \in \neg C$, where $\neg C = \neg A \cup \neg B$, there exists $Q^* \in [0,1]$, such that

$$\mathrm{T}(x) = \begin{cases} \mu^C(x), & \text{if } x \in \neg A \backslash \neg B \\ \Omega^C(x), & \text{if } x \in \neg B \backslash \neg A \\ \mu^C(x) \cup \Omega^C(x), & \text{if } x \in \neg A \cap \neg B \end{cases} \quad \textbf{(12)}$$

Since **(11)** =**(12)**, clearly, $\mathbb{Z}^C(x) = \mathrm{T}(x)$. Therefore, the result has been proved.

Theorem 4.2: Let (μ, A, Q_1) and (Ω, B, Q_2) weighted soft sets over U and $Q_1, Q_2 \in [0,1]$. Then,

i. $((\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2))^C = (\mu, A, Q_1)^C \Cap (\Omega, B, Q_2)^C$, if $A \cap B \neq \emptyset$.

ii. $((\mu, A, Q_1) \Cap (\Omega, B, Q_2))^C = (\mu, A, Q_1)^C \tilde{\cup}_R (\Omega, B, Q_2)^C$, if $A \cap B \neq \emptyset$.

Proof:

i. Let $(\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$ for all $x \in C = A \cap B$, there exists $Q \in [0,1]$ such that, $\mathbb{Z}(x) = \mu(x) \cup \Omega(x)$.

Now, let $((\mu, A, Q_1) \tilde{\cup}_R (\Omega, B, Q_2))^C = (\mathbb{Z}, C, Q^*)^C = (\mathbb{Z}^C, \neg C, Q^*)$, then for all $x \in \neg C = \neg A \cap \neg B$, there exists $Q \in [0,1]$, such that

$$(\mathbb{Z}(x))^C = (\mu(x) \cup \Omega(x))^C = \mu^C(x) \cap \Omega^C(x) \quad \textbf{(13)}$$

Also, Let $(\mu, A, Q_1)^C \Cap (\Omega, B, Q_2)^C = (\mu^C, \neg A, Q_1) \Cap (\Omega^C, \neg B, Q_2) = (J, \neg C, Q^*)$, and for every $x \in \neg C = \neg A \cap \neg B \neq \emptyset$, there exists $Q \in [0,1]$ such that,

$$J(x) = \mu^C(x) \cap \Omega^C(x) \quad \textbf{(14)}$$

Since (**13**) = (**14**), that is, $\mathbb{Z}^C(x) = J(x)$. Thus, the result has been established.

ii. Let $(\mu, A, Q_1) \Cap (\Omega, B, Q_2) = (\mathbb{Z}, C, Q^*)$, for all $x \in C = A \cap B \neq \emptyset$, there exists $Q \in [0,1]$ such that, $\mathbb{Z}(x) = \mu(x) \cap \Omega(x)$.

Now, $((\mu, A, Q_1) \Cap (\Omega, B, Q_2))^C = (\mathbb{Z}, C, Q^*)^C = (\mathbb{Z}^C, \neg C, Q^*)$ and for all $x \in \neg C = \neg A \cap \neg B \neq \emptyset$, there exists $Q \in [0,1]$ such that,

$$\mathbb{Z}^C(x) = \mu^C(x) \cup \Omega^C(x) \quad \textbf{(15)}$$

Also, $(\mu, A, Q_1)^C \tilde{\cup}_R (\Omega, B, Q_2)^C = (\mu^C, \neg A, Q_1) \tilde{\cup}_R (\Omega^C, \neg B, Q_2) = (k, \neg C, Q^*)$, say, for all $x \in \neg C$, such that $k(x) = \mu^C(x) \cup \Omega^C(x)$ **(16)**

Since **(15)** = **(16)**, therefore, $\mathbb{Z}^C(x) = k(x)$. Therefore, the result has been established.

5. Conclusion

In this research paper, we explicitly introduced weighted soft set and its application in decision making, showing clearly significance of weights imposition based on the preference and priority of the decision maker(s). Weighted soft set and its decision making processes can be applied in all aspects of human endeavors, whether personal or subjective decision making or objective or organizational decision making. The richness of the parameterization tools of soft set and flexibility of the choice of the weight imposition makes it desirable in all facets of decision making problems. The algebra of the weighted soft set were presented with basic algebraic properties. De Mogan's kind of results were established in weighted soft set theory context.

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