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Forecasting of Monthly Naira-Dollar Exchange Rate Series: Seasonal or Non-seasonal Univariate Time Series Model?

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A B S T R A C T

Over the past eight decades, the United States Dollar has become the vehicle currency driving other major foreign currencies as well as minor currencies emerging in the financial markets. Consequently, the goal of developed and developing countries is to maintain good exchange rates with the USD through effective monetary policy management. To assess the monthly Naira-Dollar exchange rates (EXR) relationship, this study applied the non-seasonal Autoregressive Integrated Moving Average (ARIMA) and Seasonal ARIMA (SARIMA) models to examine the forecasting dynamics of the series. The pre-test results confirmed that EXR is a stationary difference process of order one {I(1)}, contains seasonality. SARIMA $(0, 1, 1)(0, 1, 1)_{12}$ was observed to outperform its ARIMA (3, 1, 0) counterpart. The preference of the Akaike Information Criterion (AIC) in selecting the model over Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) was highlighted. A diagnostic check was carried out on the identified model and it revealed that the residual of the model is a white noise since it is homoscedastic, stationary, and non-autocorrelated. Findings from this study established that the Naira is projected to continue to fluctuate and further lose value relative to the USD within the forecasted time frame. Based on these findings, both the Government and policymakers need to formulate policies that will further enhance investment incentives, currency stabilization, and local production support. The practical implications of the forecast results on the country's economy include rising costs of living, higher inflation, reduced foreign direct investment, job losses, worsening trade deficit, and skilled worker migration.

1. Introduction

Low-frequency monthly time series data, say crude oil, air passengers, exchange rates, etc., often exhibit some certain level of seasonality, which makes them not examinable by the usual Box-Jenkins approach. Just like monthly data, other low-frequency time series variables that are measured quarterly and bi-annually have the potential to exhibit seasonality because the regular patterns within the data can lead to detectable seasonality. As a result, it is important to always subject monthly, quarterly, and bi-annual time series data to seasonality tests before fitting a univariate time series model for such data. However, all yearly series cannot display seasonality unless a periodic pattern spans several years (Adeboye & Ogunnusi, 2020). In this case, the seasonal impacts would be more indicative of long-term cyclicality. Consequently, researchers do not usually test for seasonality in annual or multiyear time series data since they are expected to be adequately examined by non-seasonal versions of ARIMA variants (Akanni & Adeniyi, 2020; Aje et al., 2024; Garba et al., 2023; Muhammed & Abdulmuahymin, 2016).

The exchange rate of the Nigerian Naira to the United States Dollar (EXR) is measurable at high frequency (daily or intra-day) or low frequency (monthly, quarterly, bi-annually, annually, or multi-year). When the EXR is measured at high frequency, it becomes very volatile in such a way that both the seasonal and non-seasonal ARIMA variants fail to capture their forecasting dynamics due to the non-homoscedasticity of the models' residuals (Engle, 1982). As a

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result, the volatility models are usually appropriate for modelling it at high frequency. However, when the EXR is measured monthly, it is necessary to check for stationarity in the datasets before fitting an appropriate non-causal (ARIMA) model for the series. This is because modelling a seasonal EXR series using a non-seasonal ARIMA model will yield spurious or nonsense time series regression (Gujarati & Porter, 2009).

Recently, various researchers have applied either ARIMA variants (see Etuk, 2012; Nwosu et al., 2021; Muhammed & Abdulmuahymin, 2016; Gabriel, 2022; Bakawu et al., 2020; Akintunde & Ampitan, 2024) or other models in their various studies to examine the EXR series. For illustration, Adenomon et al. (2019) compared various univariate time series models for forecasting the monthly exchange rates. The result revealed that among the models considered, Holt-Winters' model was seen to perform best for forecasting monthly Nigerian exchange rates. Okon & Ikpang (2020) forecasted the exchange rate of Nigerian Naira (NGN) to USD during COVID-19 using ARIMA (2, 1, 3), the result showed that the Naira's downward trend against the US dollar is likely to persist throughout the forecast period. Obite et al. (2021) used ARIMA, random forest, and artificial neural networks (ANN) to forecast the Naira-Dollar exchange rates. The study findings revealed that the ANN (5, 5, 1) outperformed both the ARIMA and the random forest in predicting future data points of the series. Adedotun et al. (2022) applied volatility model variants GARCH, EGARCH, and TGARCH models to forecast the annual naira/dollar exchange rate spanning 1981 to 2020. Their findings revealed that of all these models, GARCH (1, 2) best predicted the series while EGARCH (1, 4) explained the asymmetric effect. Etuk et al. (2014) employed the SARIMA model to examine the Nigerian exchange rates. The study examined the rate of change of the NGN to West African CFA franc (XOF) and found out that the SARIMA (0, 1, 1) $(0, 1, 1)_{12}$ model best captured seasonality within series. The study by Olowe (2009) was on the fluctuation of monthly exchange rates of NGN to USD using GARCH-Type models. The findings revealed that volatility persisted across all considered models, even though with monthly data. Gabriel (2022) models the monthly Naira/Rupee exchange rate (NREXR) using the ARIMA methodology. The study findings also revealed the choice of ARIMA (1, 1, 2) as the best model among competing models. Research by Garba et al. (2021) revealed that Pounds Sterling–Naira exchange rates and Euro–Naira exchange rates can be predicted by US Dollar–Naira exchange rates in the foreign exchange markets. Olaniran et al. (2022) developed the Bayesian Regularized Neural Network (BRNN) to show the appropriateness of BRNN in modeling the short-term monthly exchange rates dynamics of NGN to USD. Results from the study of Musa and Abubakar (2014) established that the TGARCH (1, 1) and TS-GARCH (1, 1) models are the best fit for the NGN-USD data. Other studies on Nigeria exchange rates include that of Musa et al. (2014), Ajao et al. (2017), Olakorede et al. (2018), Oyenuga et al. (2019), Garba et al. (2023), Adenomon & Emmanuel (2024), just to mention a few.

This study intends to substantially contribute to the existing body of research in two key areas. Number one is that a comparison shall be made between the identified non-causal and causal models of EXR data for forecast performance. The second is to compare different model selection criteria in selecting an appropriate model. Liew (2004) has expatiated the appropriateness of lag selection criteria to be employed in time series econometrics.

2. Materials and Methods

In this study, low-frequency monthly time series data of Naira-Dollar exchange rates (EXR) collected from the Statistical Bulletin of the Central Bank of Nigeria (CBN) via their repository http://data.worldbank.org were used. The EXR time series data spans January 2006 to December 2023.

The Seasonal ARIMA (SARIMA) is an extended version of the ARIMA (p, d, q) model, which is best known for capturing seasonality. SARIMA will always be preferred for modelling monthly EXR whenever there is sufficient evidence of a seasonal component in the series, otherwise, ARIMA will be preferred. This is because SARIMA will adequately capture the recurring patterns over time if present in the series. Unlike the classical ARIMA which incorporates only non-seasonal differencing (d) plus non-seasonal orders p and q of AR and MA components of the model, SARIMA incorporates both seasonal differencing (D) plus seasonal orders P and Q of AR and MA components of the model, which makes it effective in capturing periodic fluctuations which may result from tourism trends, trade cycles and fiscal policies.

2.1 Non-Seasonal ARIMA Model Specification

Let $EXR_t =$ EXR at time t, mathematically, the non-causal model or ARIMA is given as:

$$EXR_t = \mu + \phi_1 EXR_{t-1} + ... + \phi_p EXR_{t-p} + \theta_1 \varepsilon_{t-1} + ...\theta_q \varepsilon_{t-q} + \varepsilon_t \quad (1)$$

Where:

$EXR_t =$ EXR at current time t, $EXR_{t-1} =$ EXR at time t-1, $EXR_{t-p} =$ EXR at time t-p, $\varepsilon_t =$ disturbance or stochastic fluctuation at time t, while μ, ϕ_p, and θ_q are the model parameters.

Using the backshift operator, equation (1) can be re-specified as follows:

$$\phi(B)(1-B)^d EXR_t = \theta(B)\varepsilon_t \quad (2)$$

where $\phi(B)$ and $\theta(B)$ are the AR(p) and MA(q) characteristic functions respectively and ε_t is a white noise process (Hipel & McLeod, 1994; Flaherty & Lombardo, 2000).

Equating $\phi(B)$ and $\theta(B)$ to zero, gives the characteristic equations for the AR(p) and MA(q) components of the model. For the model to be stationary, $|\phi(B)| < 1$ for the AR component while for invertibility condition $|\theta(B)| > 1$. Once these two conditions are met, then the process EXR_t is said to be stationary and invertible.

From (2), the following equation can be specified as follows:

$$(1-\sum_{i=1}^{p}\phi_i B^i)(1-B)^d EXR_t = (1+\sum_{j=1}^{q}\theta_q B^j)\varepsilon_t \quad (3)$$

2.2 Seasonal ARIMA Model Specification

Adams et al (2019) noted that the SARIMA model integrates both seasonal and non-seasonal components into a single, multiplicative framework given by:

$$SARIMA(p,d,q)\times(P,D,Q)_S \quad (4)$$

Where the seasonal part (P,D,Q) comprises the autoregressive, AR(P), D differencing and moving averages, MA(Q), with the non-seasonal part given as AR(p), d differencing and MA(q) orders. S represents the length of seasonal cycle. For monthly data, S=12, for quarterly, and bi-annual time series data S assumes (4) and (2) respectively. Mathematically, the non-difference version of equation (4) is stated as equation (5) below

$$\varphi(B^s)\varphi(B)(EXR_t - \mu) = \theta(B^s)\theta(B)W_t \quad (5)$$

From (5), the non-seasonal components are re-written and stated as equations (6) and (7) below:

Where

AR: $\phi(B) = 1 - \phi_1 B - ... - \phi_p B^p$ (6)

MA: $\theta(B) = 1 + \theta_1 B + ... + \theta_q B^q$ (7)

Also, from (5), the seasonal components are re-written and stated as equations (8) and (9)

Seasonal AR: $\varphi(B^s) = 1 - \varphi_1 B^s - ... - \varphi_p B^{ps}$ (8)

Seasonal MA: $\theta(B^s) = 1 + \theta_1 B^s + ... + \theta_q B^{qs}$ (9)

SARIMA modeling aims to find a parsimonious model that best describes the underlying stochastic process. Stepwise procedures for achieving the best ARIMA/SARIMA models are as follows:

Step 1: Visualization

Every analysis in time series modelling usually begins with visualizing the data using a time series plot. This work uses both the time series plots and box plots for visualizing the Nara-Dollar exchange rates (EXR). The essence of visualization is to pre-determine the stationarity status of the EXR series.

Step 2: Decomposing the data

Here, the Naira-Dollar Exchange Rates (EXR) series are decomposed to determine whether seasonality is present in the data or not. If there is no seasonality, we proceed to examine the series with an ARIMA (p, d, q) model, otherwise a SARIMA model will be considered.

Step 3: Data Transformation for Stationarity

In this section, the EXR series must attain stationarity before further analysis. Stationarity refers to the statistical properties of the process do not change with time. In other words, the series is said to be time-invariant. Sometimes the raw series might achieve stationarity and sometimes, a transformation might be required. Transformation, such as differencing, taking logarithms, etc., are commonly employed.

Step 4: Model Selection

For ARIMA (p, d, q) model, correlograms are usually employed for pre-determined or tentative suggestions of likely models that fit the series. Different selection criteria can then be used to select the best model out of all the competing models earlier identified. In most cases, the Akaike Information Criterion (AIC) given in (10) is used.

$$AIC(a) = n\ln(\frac{\sigma\varepsilon_t}{n}) + 2a \tag{10}$$

Where: a is the number of parameters, and n is the number of observations.

Other evaluation metrics Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) used here are

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(EXR_i - EXR_i)^2 \tag{11}$$

$$RMSE = \sqrt{MSE} \tag{12}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|EXR_i - EXR_i\right| \tag{13}$$

Where n is the sample size,

EXR_i is the actual series, EXR_i is the predicted value, $(EXR_i - EXR_i)^2$ is the square of the difference (error) between the actual and predicted values for the *ith* observation, and $\left|EXR_i - EXR_i\right|$ is the absolute error for each prediction.

Step 5: Model Estimation

Once the non-seasonal orders p and q and seasonal orders P and Q are determined, the next thing is to estimate model parameters.

Step 6: Model Diagnostics

This study follows Aje et al. (2024) by examining the residuals for homoscedasticity, stationarity, and autocorrelation using 'tsdiag' plots of R statistical software. Forecasting comes next, if the fitted model satisfies these conditions.

Step 7: Forecasting

This study adopted an out-sample forecasting to be able to predict the anticipated behavior of EXR dynamics.

3. Results and Discussion

This section presents the results of analysis carried out on the Naira-Dollar exchange rates (EXR) series using the seasonal and non-seasonal univariate time series techniques earlier described in the research methodology section. The summary statistics in Table 1 revealed a significant variation in the Naira-Dollar exchange rates (EXR) series around the mean or average value of $S242.

Table 1: Descriptive or summary statistics for the Exchange Rates (EXR) series

Variable	N	Mean	Sd	Min	Max	Skew	Kurtosis
EXR	216	242	130.8	113.7	795.04	1.53	3.09

Moreover, the distribution has a skewness of 1.53 and kurtosis of 3.09. This implies that the distribution of the system is positively skewed, slightly more peaks and heavy tailed than a normal distribution. Figure 1 represents a time series plot of monthly EXR (2006 to 2023).

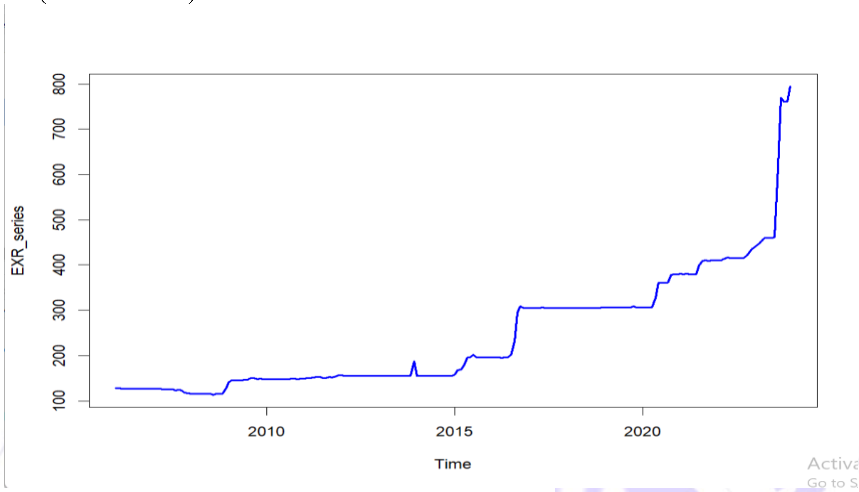

Figure 1: Time plot for the Naira/Dollar Exchange Rates (EXR) series

The plot reveals a significant upward trend in the EXR over time, with occasional stability interrupted by significant spikes, especially in recent years. Figure 2 is the boxplot of the monthly EXR between 2006 and 2023. The median lines for each month show trends in the central tendency of exchange rates as they change. Variations in the height of the whiskers and boxes indicate times when volatility is higher or lower.

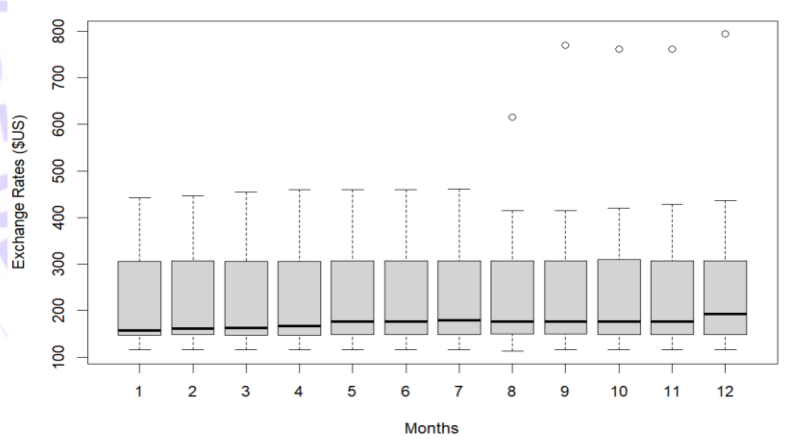

Figure 2: Monthly Naira-Dollar Exchange Rates (EXR) Boxplot (2006 to 2023)

The boxplot also reveals seasonal trends in exchange rates, which may indicate months with regularly higher or lower rates. Fluctuations in the size of the boxes and whiskers over time signal periods of heightened or reduced volatility. Unusual economic events or market conditions that resulted in significant deviations from typical exchange rates can be indicated by outliers in certain months, leading to increased or decreased volatility. This is particularly relevant for December 2023. In general, this boxplot offers a thorough visual overview of the monthly Naira-Dollar exchange rates (EXR) throughout almost 18 years, showcasing central trends, variability, and any notable irregularities in the data. The decomposition plots in Figure 3 show the breakdown of an additive time series representing Exchange Rates (EXR) over time into its components with the observed series.

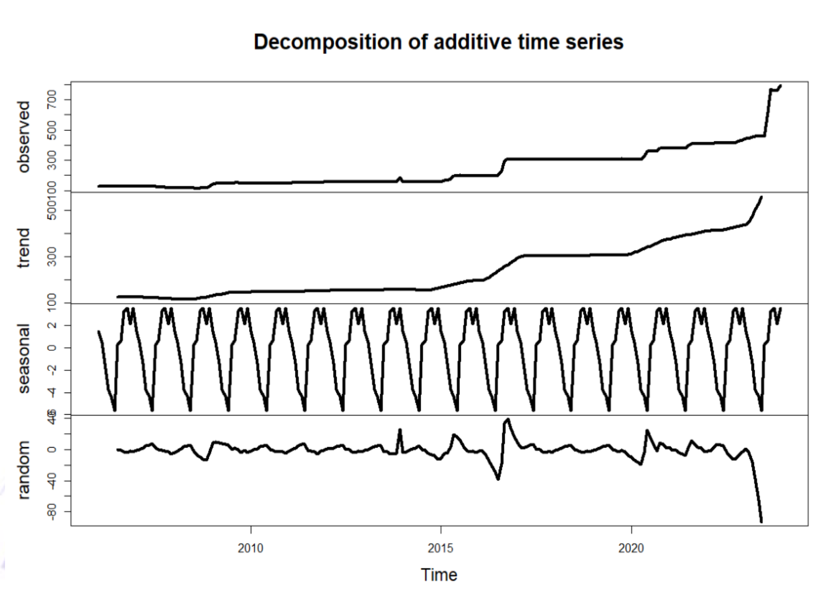

Figure 3: Decomposition of Naira-Dollar Exchange Rates (EXR) series

Generally, on the first panel, there is an upward trend, with notable upticks in the years 2015, and 2020, and a significant surge in 2023. There are intervals of moderate steadiness interrupted by sudden jumps. The second panel illustrates the trend which is the general direction or movement of the series over a long period of time. The trend here reflects the sustained growth in the data, after accounting for seasonal and random variations. There is a consistent upward trajectory over time, with a more noticeable incline in recent years, especially post-2020. The third panel depicts the seasonal element of the time series, which represents recurring patterns or cycles in the data, usually occurring within a year. The seasonal pattern displays a distinct periodic variation, suggesting that the exchange rates adhere to a steady seasonal cycle over the years. The lower panel illustrates the random or residual element, which represents the leftover variability after eliminating the trend and seasonal elements. This plot depicts the irregular fluctuations that cannot be accounted for by the trend or seasonal patterns. The random element seems to show some variation, with intermittent spikes, but it does not demonstrate any consistent pattern.

In Figure 4, a time series plot displays the first logarithmic difference of the EXR series. The purpose of obtaining the first log difference of the series is to eliminate its trend. This implies that the series has been detrended.

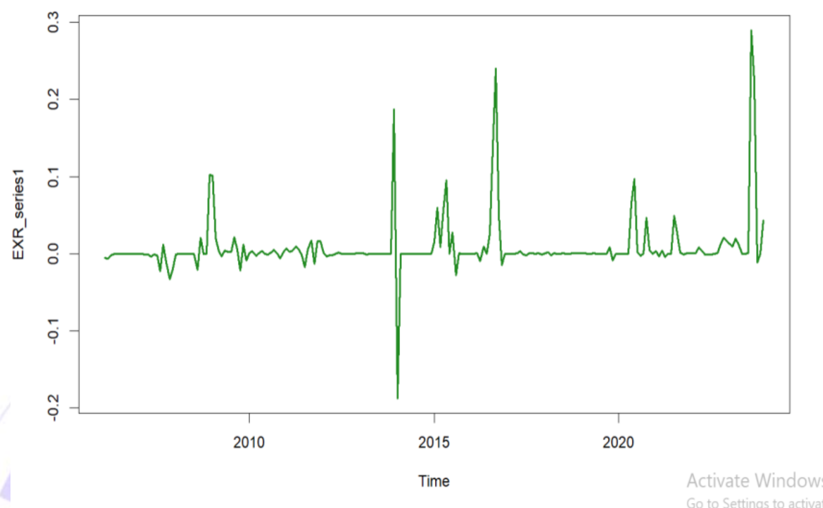

Figure 4: Time Plot of the First Log Difference of the EXR series

The visualization from this plot indicates that the series is time-invariant (stationary) as the mean approaches zero and the variance remains constant throughout the study periods. This suggests a difference process of order one {I (1)}. After establishing that the EXR is an I (1) process, the next step is to eliminate the seasonal components from the series. To accomplish this, we take the first difference in the seasonal lag. Figure 5 presents the time series plots for both the first log difference plus the first difference at the seasonal lag.

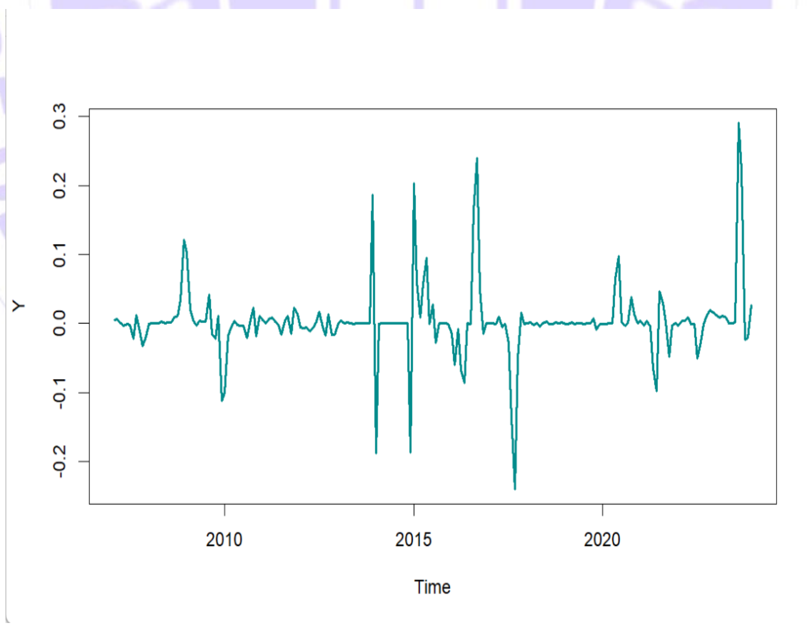

Figure 5: Time series plot of the First Log Difference + First Difference at Seasonal Lag

The time series in Figure 5 indicates that once the first difference is applied, the first log difference and the first difference seasonal lag of the EXR series become stationary. This suggests that both trends and seasonal components have been eliminated from the series. In essence, the transformed series is now sufficiently stationary for modeling. Augmented Dickey-Fuller (ADF) tests was conducted to establish the stationarity of the transformed series as shown in Table 2

Table 2: Stationarity tests for the EXR series after 1st difference

Coefficients	Estimate	Std. Error	t-value	Pr(>|t|)
z.lag.1	-0.77802	0.08574	-9.075	<2e-16 ***
z.diff.lag	0.06068	0.07085	0.857	0.393

According to the ADF results in Table 2, the null hypothesis of non-stationarity in the EXR series (p-value < 0.05) after applying the first log difference was rejected. This means that the series is a stationary process of order one {I (1)} after the first difference. Figure 6 is the correlogram of the EXR series before transformation.

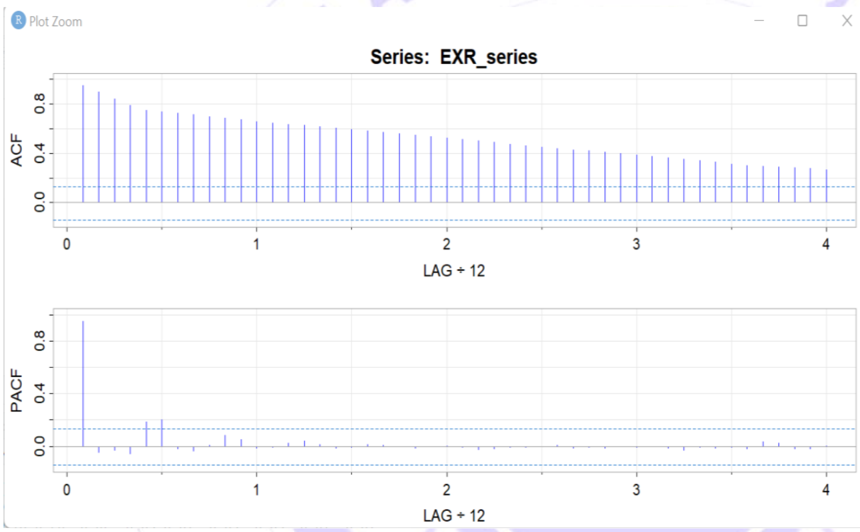

Figure 6: Correlogram of Exchange Rates (EXR) before transformation

Figure 6 was instrumental in predetermining both the stationarity status and the orders p and q of the non-seasonal ARIMA (p, d, q) model. However, our interest here is to pre-determine the non-seasonal orders of the model. Visualizations from this correlogram indicate that the significant spikes of the ACF are decaying exponentially while the only significant spike of the PACF cuts off abruptly at lag 1. This is an indication that the ARIMA (p, d, 0) is suggested to fit the series. Here, the possible orders of p are unclear and further visualizations of the differenced EXR series in Figure 7 are expected to suggest these orders. After differencing, the ACF and PACF plots in Figure 7 were obtained. From Figure 7, the possible p orders are 1, 2, and 3. In other words, the possible competing models as suggested by this correlogram are ARIMA (1, 1, 0), ARIMA (2, 1, 0) and ARIMA (3, 1, 0) respectively.

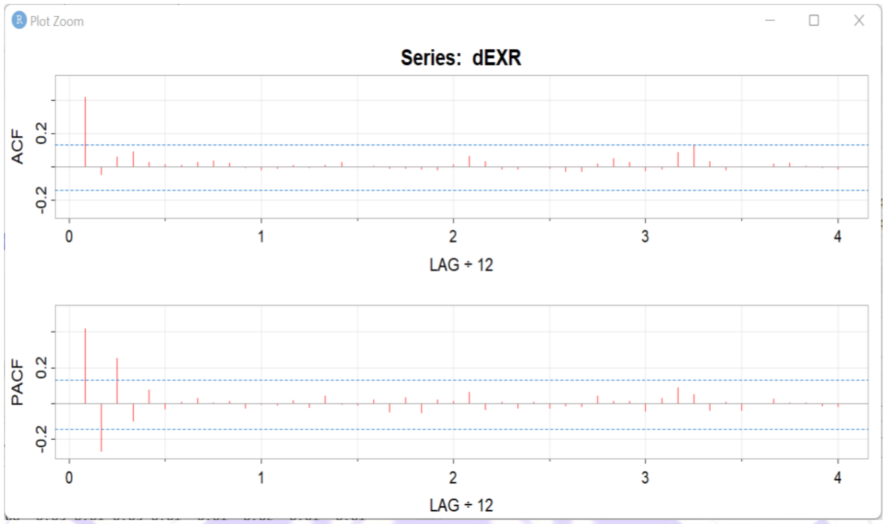

Figure 7: Correlogram of Exchange Rates (EXR) series after the first difference

Figure 8 shows correlogram of the First Log Difference plus First Difference at Seasonal Lag. For orders p and q of the ACF and PACF, two significant spikes cut off abruptly at lag 1, according to the ACF and PACF in the Correlogram of Figure 8.

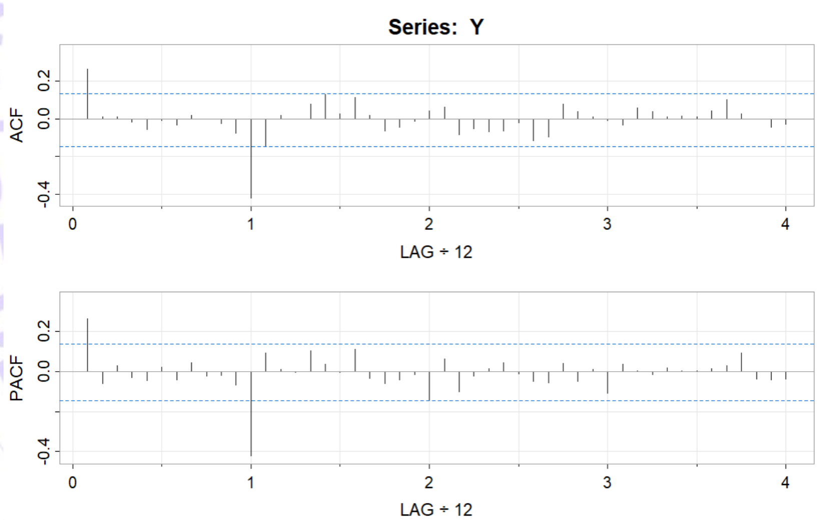

Figure 8: Correlogram plot of the First Log Difference + First Difference at Seasonal Lag

Similarly, the seasonal lags P and Q provide evidence of two significant spikes that cut off abruptly at lag 1, according to the ACF and PACF. It follows that $SARIMA(1,1,0)(1,1,0)_{12}$, $SARIMA(1,1,0)(0,1,1)_{12}$, $SARIMA(0,1,1)(1,1,0)_{12}$ and $SARIMA(0,1,1)(0,1,1)_{12}$ should all be considered as the competing models. By subjecting the competing models to additional selection or evaluation criteria, such as the Akaike Information Criterion (AIC) and other evaluation metrics such as the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE), the optimal model among these competing models would be identified. Results of the selection criteria for seasonal and non-seasonal models are shown in Table 3.

Table 3: Selection Criteria for seasonal and non-seasonal models

S/N	(p, d, q) (P, D, Q)$_{12}$	AIC	RMSE	MAE	(p, d, q)	AIC

1	$(1, 1, 0)(1, 1, 0)_{12}$	-668.8	**0.048*** **	0.0204	(1,1,0)	1,778.95
2	$(1, 1, 0)(0, 1, 1)_{12}$	-710.4	0.061	**0.0158*** **	(2,1,0)	1,766.30
3	$(0, 1, 1)(1, 1, 0)_{12}$	-669	0.049	0.0202	(3,1,0)	**1,750.83*** **
4	$(0, 1, 1)(0, 1, 1)_{12}$	**-711.1*** **	0.063	0.0160	NA	NA

Table 3 showed that the selection criterion (AIC) and evaluation metrics (RMSE and MAE) differ in their selection of the optimal (best) model. The AIC selected the $SARIMA(0,1,1)(0,1,1)_{12}$, as the best model while the RMSE and MAE selected the $SARIMA\ (1,1,0)\ (1,1,0)_{12}$, and $SARIMA\ (1,1,0)\ (0,1,1)_{12}$, as their respective best models. Though, in a time series econometrics framework, the AIC is generally preferred to other model selection or evaluation criteria under a univariate time series framework (Aje et al., 2024; Liew, 2004) but these remaining two competing models will further be examined based on their standard errors. Table 4 presents the estimates of the non-seasonal and seasonal orders of the competing models.

For easy comparison, the estimates of the non-seasonal model {ARIMA (3, 1, 0)} are placed beside that of the other three competing seasonal models {$SARIMA\ (1,1,0)(0,1,1)_{12}$, $SARIMA\ (0,1,1)(0,1,1)_{12}$, and $SARIMA(1,1,0)(1,1,0)_{12}$} as shown in Table 4. Despite that, all the estimated parameters for both the non-seasonal and seasonal models are extremely and statistically significant (p-value < 0.01) only the $SARIMA\ (0,1,1)\ (0,1,1)_{12}$ reported the least value of AIC which indicates that it is the optimal or best model for forecasting the EXR series. However, the $ARIMA\ (3,1,0)$ model produced the largest AIC value which is regarded as an outlier when compared to other AICs in the Table. Consequently, it is not an appropriate model for forecasting the EXR series.

Table 4: Estimates of non-seasonal and seasonal orders of the models

	ARIMA (3,1,0)	SARIMA $(1, 1, 0)\ (0, 1, 1)_{12}$	SARIMA $(0, 1, 1)(0, 1, 1)_{12}$	SARIMA $(1,1,0)(1,1,0)_{12}$
ar1	**0.630(0.066)*** **	**0.273 (0.068)*** **	NA	**0.253 (0.068)*** **
sar1	NA	NA	NA	**-0.494 (0.066)*** **
ar2	**-0.416(0.074)*** **	NA	NA	NA
ar3	**0.281 (0.066)*** **	NA	NA	NA
ma1	NA	NA	**0.283 (0.066)*** **	NA
sma1	NA	**-0.984 (0.357)*** **	**-0.963 (0.173)*** **	NA
Observations	215	203	203	203
Log likelihood	-871.41	358.193	358.545	337.404
sigma2	193.543	0.001	0.001	0.002
AIC.	1,750.83	-710.39	**-711.09*** **	-668.81

Note: *p<0.1; **p<0.05; ***p<0.01

Moreover, we will further confirm the results reported by the AIC using the standard error of forecasts from the three remaining seasonal models. Similarly, Figure 9 displays the time series diagnostics plots that display the standardized residuals, ACF of residuals, and p-values for the Ljung-Box statistic for the $SARIMA\ (0,1,1)\ (0,1,1)_{12}$ model. For the standardized residuals in this plot, the residuals from the $SARIMA\ (0,1,1)\ (0,1,1)_{12}$ model are homoscedastic, which means that the fitted values or points have a constant distribution around zero. Furthermore, the residuals from the fitted $ARIMA\ (0,1,1)\ (0,1,1)_{12}$ model for the ACF in the second panel of the graphs are stationary, indicating that each ACF spike is statistically insignificant and falls within the 95% confidence bounds. Furthermore, when the p-values of the Ljung-Box statistic in the plot exceed the horizontal line, we fail to reject the null hypothesis of no autocorrelation in the residuals of the fitted model.

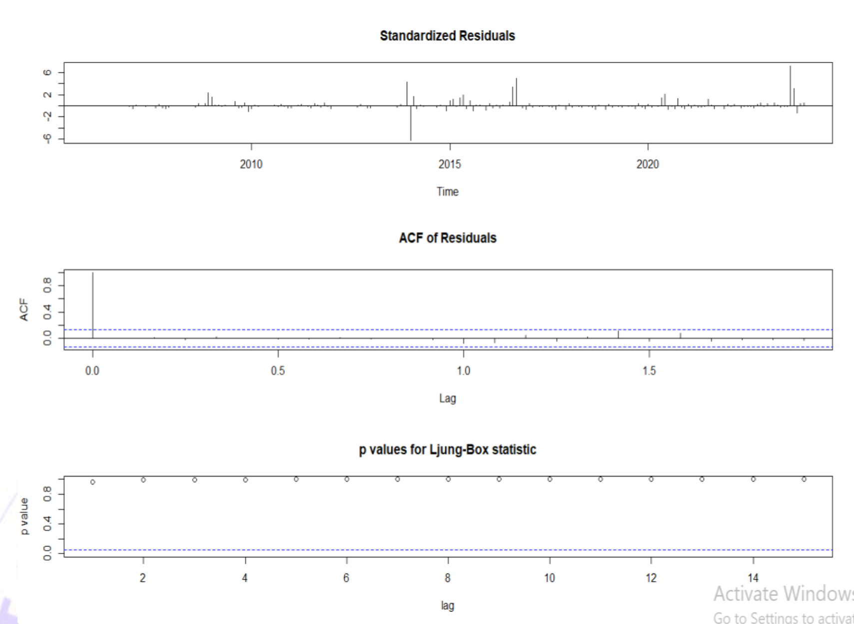

Figure 9: Plots of time series diagnostics for the $SARIMA\ (0,1,1)\ (0,1,1)_{12}$ model

Now that the residuals from the fitted model are autocorrelation-free, stationary, and homoscedastic, they are called "white noise." These results also imply that the $SARIMA(0,1,1)(0,1,1)_{12}$ is expected to project the future values for the EXR series. According to the results of the two-year forecasts in Table 5, of all the seasonal models, only $SARIMA(0,1,1)(0,1,1)_{12}$ consistently shows the smallest standard errors across the entire forecast period from January 2024 to December 2025. This is a confirmation that it is the optimal model as earlier reported by the AIC in Table 4. Therefore, its forecast values would be the best among other forecasted values. Based on the forecasts from SARIMA $(0, 1, 1)\ (0, 1, 1)_{12}$, the value of EXR is projected to rise slightly by 8.674403 % from January 2024 through October 2024. However, it is projected to fall slightly by 0.02610374% between the periods of October and November 2024.

Table 5: Two years forecast from January 2024 to December 2025

			$SARIMA(1,1,0)(0,1,1)_{12}$	$SARIMA(0,1,1)(0,1,1)_{12}$	$SARIMA(1,1,0)(1,1,0)_{12}$
S/N	Year	Month	Forecast	Forecast	Forecast
1	2024	January	798.4641 (0.03915997)	797.8806 (**0.03915126**)	6.694147(0.04552700)
2	2024	February	803.8157 (0.06333281)	801.6555 (**0.06364105**)	6.70141(0.07297958)
3	2024	March	805.839 (0.08238218)	803.4099 (**0.08104039**)	6.716215(0.09444469)
4	2024	April	808.6191 (0.09821867)	806.1042 (**0.09531478**)	6.72424(0.11226017)
5	2024	May	816.8708 (0.11193842)	814.4044 (**0.10771383**)	6.7239(0.12770084)
6	2024	June	821.2208 (0.12417674)	818.9844 (**0.11882605**)	6.723397(0.14148637)
7	2024	July	825.2051 (0.13531907)	823.1587 (**0.12898447**)	6.723322(0.15404782)
8	2024	August	844.5053 (0.14561286)	844.1287 (**0.13839926**)	6.870131(0.16566061)
9	2024	September	869.7597 (0.15522587)	870.4732 (**0.14721316**)	6.983572(0.17651128)
10	2024	October	872.6923 (0.16427724)	873.6659 (**0.15552838**)	6.983591(0.18673257)

11	2024	November	872.318 (0.17285466)	873.4379 (**0.16342104**)	6.993609(0.19642271)
12	2024	December	888.9296 (0.18102374)	889.9106 (**0.17094969**)	7.023954(0.20565677)
13	2025	January	886.8209 (0.18947776)	887.8475 (**0.17886635**)	7.038413(0.22214282)
14	2025	February	891.1404 (0.19769401)	892.0472 (**0.18660195**)	7.046641(0.23960981)
15	2025	March	892.9397 (0.20562557)	894.0002 (**0.19402939**)	7.064001(0.25640806)
16	2025	April	895.8993 (0.21327378)	896.9974 (**0.20118281**)	7.073949(0.27229665)
17	2025	May	905.0082 (0.22066008)	906.2335 (**0.10771383**)	7.073782(0.28733798)
18	2025	June	909.8175 (0.22780781)	911.3308 (**0.21477607**)	7.073523(0.30163742)
19	2025	July	914.2289 (0.23473819)	915.9749 (**0.22125975**)	7.073787(0.31529075)
20	2025	August	935.6112 (0.24146979)	939.3095 (**0.22755877**)	7.291357(0.32837731)
21	2025	September	963.5901 (0.24801869)	6.875878 (**0.23368806**)	7.459332(0.34096207)
22	2025	October	966.839 (0.16427724)	972.1771 (**0.23966064**)	7.453751(0.35309860)
23	2025	November	966.4234 (0.26062190)	971.9234 (**0.24548796**)	7.458632(0.36483163)
24	2025	December	984.827 (0.26669653)	990.2535 (**0.25118013**)	7.495387(0.37619891)

Note: The standard error estimates are indicated in brackets

Likewise, the value of EXR is projected to fall once again by 1.851051% between the periods of November and December 2024. A further fall of 0.232371% is projected between the periods of December 2024 and January 2025. Again, the value of EXR is expected to rise by 8.674304 from January through October 2025. Besides, EXR is expected to fall again by 0.02610288% between October and November 2025. Lastly, a rise of 1.851051 in the value of EXR is expected between November and December 2025. Figure 10 gives the time series plot for the Naira-Dollar Exchange Rates (EXR) series from January 2024 to December 2025.

In summary, the forecast results in the same Table 5 revealed a steady upward trend within the forecasted two-year period with noticeable increases around mid-year and year-end. Noticeably, the forecasts for the year 2024 begin at about 798.46 in January and steadily rise to 888.93 in December.
However, for the year 2025, the forecast results further increase to around 984.83 by the end of 2025, and there is sufficient evidence of variations specifically in later months. Moreover, the most significant forecast jumps are observed around August and September of each year, indicating potential seasonal effects.

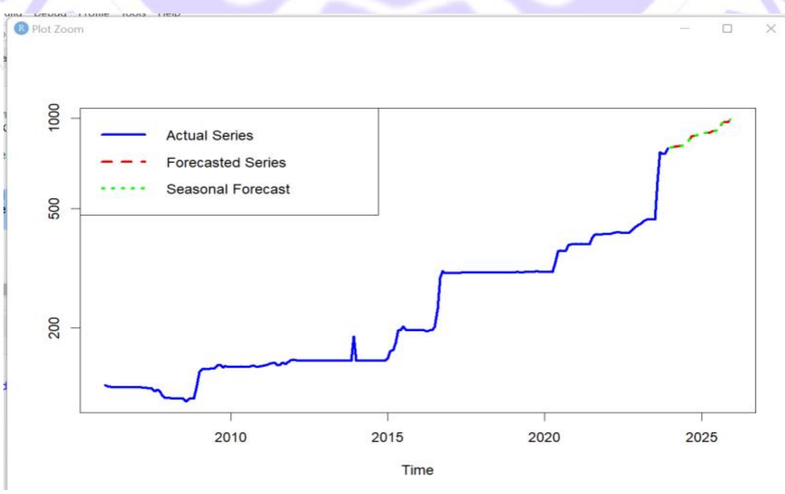

Figure 10: Time Series Forecast plot for the EXR series from January 2024 to December 2025

For the historical data, the section of the plot till the end of 2023 displays the actual observed exchange rates. However, for the predicted exchange rates, the dotted line section represents the EXR forecasts from January 2024 to December 2025. Based on the forecast line, it is seen that the EXR series exhibits a high fluctuation nature within the forecasting period.

4. Summary of Findings

This work compared the forecast performances of non-seasonal and seasonal models in forecasting the future values of the monthly Naira-Dollar Exchange Rates (EXR) series. Pre-tests (descriptive statistics, visualizations), stationarity test, and the use of selection criteria including the Akaike Information Criterion (AIC), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE) were all used to determine the optimal model for both the non-seasonal and seasonal models. Based on the descriptive statistics, significant variation was seen around the average EXR of $US242. Also, the kurtosis value, which is greater than 3, indicates that the distribution is leptokurtic, while a skewness value of 1.53 indicates positive skewness to the right side of the distribution.

Furthermore, the time series plots, box plots, and decomposition plots revealed the existence of seasonality and volatility patterns within the EXR series. With the aid of the Augmented Dickey-Fuller (ADF) test, the series was confirmed to be a difference stationary process of order one {I(1)}. The autocorrelation function (ACF) and partial autocorrelation function (PACF) were utilized to pre-determine the orders of the non-seasonal and seasonal models. For the non-seasonal model, the ACF and PACF suggested ARIMA (1, 1, 0), ARIMA (2, 1, 0), and ARIMA (3, 1, 0) models as the competing models. However, for the seasonal model, the ACF and PACF suggested ARIMA $(1, 1, 0)(1, 1, 0)_{12}$, ARIMA $(1, 1, 0)(0, 1, 1)_{12}$, ARIMA $(0, 1, 1)(1, 1, 0)_{12}$, and ARIMA $(0, 1, 1)(0, 1, 1)_{12}$ as the competing models. Based on the Akaike Information Criterion (AIC), the ARIMA (3, 1, 0) and ARIMA $(0, 1, 1)(0, 1, 1)_{12}$ were selected as the optimal models for forecasting the EXR series. However, based on the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE), the ARIMA $(1, 1, 0)\ (1, 1, 0)_{12}$ and ARIMA $(1, 1, 0)\ (0, 1, 1)_{12}$ were selected as the best models under the seasonal scenario. Based on an AIC value, the ARIMA (3, 1, 0) was inappropriate for forecasting the EXR series, hence it is dropped from the competing models. Of all the seasonal models, only ARIMA $(0, 1, 1)\ (0, 1, 1)_{12}$ consistently displays the smallest standard errors across the entire forecast period (January 2024 to December 2025). As a result, it is the optimal model for forecasting the series.

The post-examination tests conducted on the residuals of the optimal model revealed that it is white noise since it is homoscedastic, stationary, and autocorrelation-free. Thus, the optimal model would be suitable for forecasting the future values of the series. Lastly, the model was used to forecast EXR values from January 2024 to December 2025, predicting continued fluctuations in the exchange rate over the forecast period.

The forecasted fluctuations in the Naira-Dollar exchange rate have significant implications for Nigeria's monetary policy and economic planning. To deflect volatility, the Central Bank of Nigeria will need to adjust interest rates and effectively manage its foreign exchange reserves. Policymakers should focus on trade diversification, reducing import dependence, and implementing inflation control measures to stabilize prices. Encouraging foreign direct investment and maintaining market stability will help boost investor confidence. Additionally, aligning fiscal policies with monetary strategies is essential to mitigate external shocks and ensure economic resilience. Overall, proactive and data-driven policies are necessary to maintain financial stability and sustainable growth.

5. Conclusion

This work offers an ample examination of the Naira-Dollar Exchange Rates (EXR) series forecasting dynamics from January 2006 to December 2023 using the non-seasonal Autoregressive Integrated Moving Average (ARIMA) and Seasonal ARIMA (SARIMA) models. Moreover, our work contributes significantly to the existing literature on univariate forecasting of exchange rates through forecast performance comparisons between ARIMA and SARIMA models, and substantiating the preference of the Akaike Information Criterion (AIC) over other evaluation metrics such as the Root Mean Square Error (RMSE), and Mean Absolute Error (MAE) in selecting the appropriate SARIMA model. Our findings established that ARIMA (3, 1, 0), which is a non-seasonal univariate model, is not adequate for forecasting the EXR series due to its outlier AIC value. Further findings also established the preference of AIC over RMSE and MAE in selecting the optimal ARIMA $(0, 1, 1)\ (0, 1, 1)_{12}$ model for the EXR series, which is in accordance with Liew (2004) work. Based on the forecasts from the optimal ARIMA $(0, 1, 1)\ (0, 1, 1)_{12}$ model, we conclude that Naira is projected to continue to fluctuate and decrease in value against the United States Dollar over the

forecast horizon, which spanned January 2024 to December 2025. Our findings recommend that researchers, policymakers, and business owners utilize the Naira-Dollar fluctuations for planning.
Businesses should adopt strategic measures to mitigate exchange rate volatility by utilizing hedging instruments like forward contracts and currency swaps, diversifying revenue sources, and reducing reliance on imports. Maintaining liquidity buffers and adjusting pricing strategies will help navigate financial uncertainties. Policymakers and financial institutions should provide risk management guidance to ensure economic stability, long-term investment, and sustainable growth.

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