International Journal of Development Mathematics Vol 2 Issue 1 Page No. 145 - 155

INTERNATIONAL JOURNAL OF DEVELOPMENT MATHEMATICS

ISSN: 3026-8656 (Print) | 3026-8699 (Online)

journal homepage: https://ijdm.org.ng/index.php/Journals

A Mathematical Model of Indoor Air Pollution and Its Effects on Human Respiratory Health

Lubem M. Kwaghkor[a], Hycienth O. Orapine[a] and Usama B. Salihu[a]

[a]*Department of Mathematics, Nigerian Army University Biu, Borno State, Nigeria*

ARTICLE INFO

Article history:
Received 10 January 2025
Received in revised form 10 March 2025
Accepted 27 March 2025

Keywords:
Mathematical Model, Indoor Air Quality, Pollution, Stability, Simulation.

MSC 2020 Subject classification:
34A30, 34A34, 92B05, 92-10

ABSTRACT

Indoor air quality (IAQ) refers to the quality of air within and around buildings and structures, which is known to affect the comfort and well-being of the building occupants. Research on the urban population has confirmed that people spend more than 90% of their daily life span in indoor environments. This study aims to formulate a mathematical model that can help study indoor air quality dynamics and its impact on the human respiratory system. The formulated seven linear differential equations of first order were found to be uniform and asymptotically stable and the model has a unique solution using the Picard – Lindelof Method. Numerical simulations were carried out to study the effect of indoor pollutants on the human respiratory system and the results were graphed. The results indicate that this model can be used to study the effect of indoor pollutants on the human respiratory system perfectly and hence recommended.

1. Introduction

The quality of air within and around buildings has become a subject of research recently. Researches by Jones (1999), Sisask *et al.,* (2014) and Vilcekova *et al.,* (2017) indicated that the quality of air within and around buildings has become a subject of research recently because the quality of air within and around buildings is a major cause of health and wellbeing of human. Also, people globally spent 90% of their everyday life inside (residential, commercial, institutional, industrial etc) building environment (Klepeis *et al*., 2001; Mannan and Al-Ghamdi, 2021). As a result of improvement in standard of living today, several chemicals and synthetic materials are used for the construction of the inside building decoration which may have negative impacts on the human respiratory system. Also, some materials such as air fresheners, Carbon monoxide from cooking, cleaning agents etc are other sources of indoor air pollution which has several effects on the human respiratory system (Klepeis *et al*., 2001; Sundell, 2004; Prajakta, *et al*., 2013; Lone *et al*., 2021; Mannan and Al-Ghamdi, 2021). However, most environmental researches are focused on outdoor air quality (Sundell, 2004), while indoor air quality (Pollution) has received little attention. Recently, United States Environmental Protection Agency (USEPA, 2013) has shown that indoor air is more adulterated than outdoor air. WHO (2007) reported that more than 1.5 million people died due to indoor air quality (pollution). It is established that indoor air quality (pollution) is acknowledged as the third main cause of disability-accustomed life worldwide (Apte, 2016).

The human respiratory system plays a central role in transporting oxygen (O_2) and carbon dioxide (CO_2) between tissues in the body. Human tissues can survive for many days without food but cannot survive without the exchange of oxygen and carbon dioxide (Lone *et al.*, 2021). The human respiratory system is a process were the oxygen which is available free in the environment is inhaled (about 260 ml/min if oxygen under normal conditions) into the Arterial blood through the Alveolar tissue and then to the tissues where it is used (Salathe *et al*., 1980; Nunn, 1987; Guyton and Hall, 2011). As a bye product at the tissues, carbon dioxide (about 160 ml/min of carbon hydrate under normal conditions) is transported through to the Venous blood to the Alveolar tissue and then to the environment (Nunn, 1987; Ottesen *et al.*, 2004; Guyton and Hall, 2011).

Mathematics is a vital tool in the hand of scientists for solving real-world problems (Khanday and Najar, 2015a; Khanday and Najar, 2015b; Kwaghkor and Luga, 2016; Kwaghkor *et al.,* 2018; Kwaghkor *et al.,* 2019; Kwaghkor *et al.,* 2021; Kwaghkor *et al.,* 2022; Orapine *et al.*, 2023 ; Kwaghkor *et al.,* 2024; Lone *et al*., 2021. Krogh (1919) first presented an oxygen transport model where oxygen from a capillary circulates only into a tissue tube concentric with the capillary. An extension and modification of Krogh model was done by Blum (1960) where he added the capillary

* Corresponding author. Tel.: +23408061263095
E-mail address: lubem.kwaghkor@naub.edu.ng (Lubem M. Kwaghkor.)
https://doi.org/10.62054/ijdm/0201.11

wall resistance. After this, several researchers extended and studied the oxygen transport and carbon dioxide transport models (see Salathe *et al*., 1980; Lin, 1976; Simpson and Ellery, 2012; Clark *et al*., 2006; Cherniack *et al*. 1966). Recently, Lone *et al*., (2021) formulated oxygen and carbon dioxide transport model through blood in the body where he estimated the concentration profiles of oxygen and carbon dioxide over alveolar tissue, arterial blood, tissue and venous blood compartments. The model was a four-compartmental one with alveolar tissue, arterial blood, tissue and venous blood as the compartment. The transportation of oxygen and carbon dioxide in human body was found to have partial pressure as a main driving force. In this study, we shall formulate a mathematical model of indoor air quality (pollutant) and the human respiratory system by adding a first order differential equation of indoor air quality (pollutant concentration) to the mathematical model by Lone *et al*., (2021).

2. Methods

The Model

The formulation of this model shall be done in two stages: (1) the indoor concentration of air pollutant and, (2) the human respiratory system.

The Indoor Concentration of Air Pollutant Model (ICAPM)

The Indoor Concentration of Air Pollutant Model (ICAPM) is presented in equation (1) based on Figure 1 and the following assumptions: A house or room in a house or other enclosed space is Envisage as a simple box, the contents of the box are well mixed, the rate of inflow into the box is not equal to the rate of outflow, inflow rate depends on outdoor air concentration, outflow rate depends on indoor air concentration, there is an indoor source and sink of pollutant, indoor air concentration follows a simple mass balance principle.

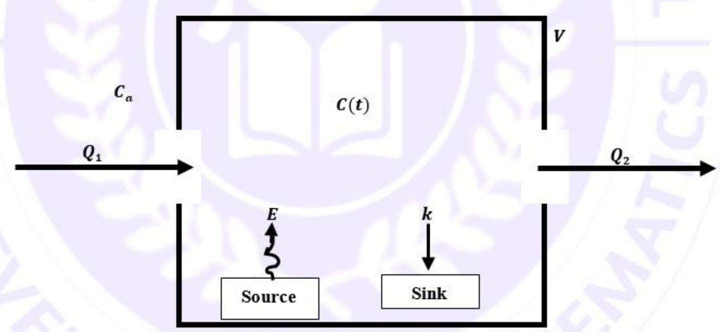

Figure 1: Flow diagram for the ICAPM

The Indoor Concentration of Air Pollutant Model (ICAPM) Equation

The Indoor Concentration of Air Pollutant Model (ICAPM) Equation is presented as

$$V\frac{dC(t)}{dt} = Q_1 C_a + E - Q_2 C(t) - kC(t)V$$

Which is equivalent to

$$\frac{dC(t)}{dt} = \frac{Q_1 C_a}{V} + \frac{E}{V} - \left(\frac{Q_2}{V} + k\right)C(t), \qquad C(0) = C_0 \qquad (1)$$

From equation (1), V is the volume of the box, $C(t)$ is the concentration of pollutant in the box at time t, C_a is the outdoor air pollutant concentration around the box, C_0 is the Initial indoor air pollutant, Q_1 is the inflow rate of pollutant, Q_2 is the outflow rate of pollutant, E is the indoor emission rate of pollutant (Source) and k is the pollutant decay coefficient (Sink).

The Human Respiratory System Model (HRSM)

Here, a four-compartment model of human respiratory system consisting of alveolar tissue, arterial blood and tissue with concentration of oxygen where considered. Also considered are the tissue, venous blood and alveolar tissue with concentration of carbon dioxide. A schematic overview of the four-compartment model of oxygen and carbon dioxide transport via blood in the human body is presented in Figure 2. The HRSM is presented in equations (2) – (7)

based on Figure 2 and the following assumptions: The concentrations of both inhaled and exhaled air at any compartment follows a simple mass balance principle, the blood flowing through the capillaries is treated as a homogeneous mixture of Plasma and Red Blood Cells, the interaction of the inhaled air (O_2) and exhaled air (CO_2) are taken constant, the production of carbon dioxide in the tissue is proportional to the consumption of oxygen and the mass exchange between compartments is represented by concentration gradient.

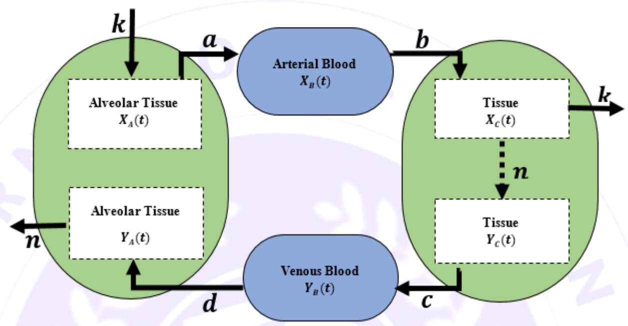

Figure 2: Flow diagram for the HRSM

The Human Respiratory System Model (HRSM) Equations

Based on the assumptions, variables, parameters and the flow diagram in Figure 2, the model equations of the human respiratory system is given as

$$\frac{dX_A(t)}{dt} = kC(t) - aX_A(t), \qquad X_A(0) = X_0 \qquad (2)$$

$$\frac{dX_B(t)}{dt} = aX_A(t) - bX_B(t), \qquad X_B(0) = \sigma \qquad (3)$$

$$\frac{dX_C(t)}{dt} = bX_B(t) - kX_C(t), \qquad X_C(0) = \delta \qquad (4)$$

$$\frac{dY_C(t)}{dt} = nbX_B(t) - cY_C(t), \qquad Y_C(0) = Y_0 \qquad (5)$$

$$\frac{dY_B(t)}{dt} = cY_C(t) - dY_B(t), \qquad Y_B(0) = \sigma^* \qquad (6)$$

$$\frac{dY_A(t)}{dt} = dY_B(t) - nY_A(t), \qquad Y_A(0) = \delta^* \qquad (7)$$

Equation (1) is describing indoor air quality concentrations. Equations (2) – (4) is describing Oxygen (O_2) transport from inhalation through the Alveolar Tissues, then through Arterial Blood to Tissues. While equations (5) – (7) is describing carbon dioxide (CO_2) transport from Tissues, then through Venous Blood, then through the Alveolar Tissue to exhalation. Equations (1) – (7) is the required mathematical model equations that can be used to study the effects of indoor air pollutants concentration on the human respiratory system.

3. Model Analysis

Stability of the Model
Theorem 1: Stability Criteria

Suppose that the n – square matrix A has eigenvalues λ_i $(i = 1, 2, \ldots, n)$. Then the stability of a solution of the linear system $X' = Ax$ is determined according to the following criteria:

(i) If $Re(\lambda_i) < 0$, for all i, then there is uniform asymptotic stability,
(ii) If $Re(\lambda_i) \leq 0$, for any i or algebraic multiplicity equals geometric multiplicity whenever $\lambda_i = 0$, for any i, then there is uniform stability,
(iii) If $Re(\lambda_i) > 0$, for any i or algebraic multiplicity is greater than the geometric multiplicity whenever $\lambda_i = 0$, for any i, then there is instability.

Proof

The matrix form of the model equations (1) – (7) is given as

$$\begin{pmatrix} C'(t) \\ X_A'(t) \\ X_B'(t) \\ X_C'(t) \\ Y_C'(t) \\ Y_B'(t) \\ Y_A'(t) \end{pmatrix} = \begin{pmatrix} -\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ k & -a & 0 & 0 & 0 & 0 & 0 \\ 0 & a & -b & 0 & 0 & 0 & 0 \\ 0 & 0 & b & -k & 0 & 0 & 0 \\ 0 & 0 & bn & 0 & -c & 0 & 0 \\ 0 & 0 & 0 & 0 & c & -d & 0 \\ 0 & 0 & 0 & 0 & 0 & d & -n \end{pmatrix} + \begin{pmatrix} C(t) \\ X_A(t) \\ X_B(t) \\ X_C(t) \\ Y_C(t) \\ Y_B(t) \\ Y_A(t) \end{pmatrix} = \begin{pmatrix} \frac{Q_1 C_a}{V} + \frac{E}{V} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (8)$$

$$X'(t) = AX(t) + B \quad (9)$$

where $X'(t) = \begin{pmatrix} C'(t) \\ X_A'(t) \\ X_B'(t) \\ X_C'(t) \\ Y_C'(t) \\ Y_B'(t) \\ Y_A'(t) \end{pmatrix}$, $A = \begin{pmatrix} -\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ k & -a & 0 & 0 & 0 & 0 & 0 \\ 0 & a & -b & 0 & 0 & 0 & 0 \\ 0 & 0 & b & -k & 0 & 0 & 0 \\ 0 & 0 & bn & 0 & -c & 0 & 0 \\ 0 & 0 & 0 & 0 & c & -d & 0 \\ 0 & 0 & 0 & 0 & 0 & d & -n \end{pmatrix}$,

$X(t) = \begin{pmatrix} C(t) \\ X_A(t) \\ X_B(t) \\ X_C(t) \\ Y_C(t) \\ Y_B(t) \\ Y_A(t) \end{pmatrix}$, $B = \begin{pmatrix} \frac{Q_1 C_a}{V} + \frac{E}{V} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and $\alpha = \frac{Q_2}{V} + k$

Using $|A - \lambda I| = 0$, the eigenvalues of (8) are given as $\lambda_1 = -(\frac{Q_2}{V} + k)$, $\lambda_2 = -a$, $\lambda_3 = -b$, $\lambda_4 = -c$, $\lambda_5 = -d$, $\lambda_6 = -k$, $\lambda_7 = -n$. Since the eigenvalues are all negative, the model (1) – (7) has uniform asymptotic stability. Hence the proof. This means that the model can be used to study the effect of indoor air pollutant on the human respiratory system perfectly.

Existence and Uniqueness Solution of the Model

Theorem 1: (Picard-Lindelof Theorem). Let $f: R \to R$ be a continuous function and let $(\tau, A) \in \mathbb{R}$. If $\exists$ a constant $L \geq 0$ satisfying the Lipschits condition $|f(t, y_2) - f(t, y_1)| \leq L|y_2 - y_1|$, for all $(t, y_2), (t, y_1) \in \mathbb{R}$, then the initial value problem (IVP) $y' = f(t, y)$, $y(0) = x_0$ has a unique solution $x = x(t)$ on the interval τ.

Proof

The functions $f_1(t, C(t)) = \frac{Q_1 C_a}{V} + \frac{E}{V} - \left(\frac{Q_2}{V} + k\right) C(t)$, $f_2(t, X_A(t)) = kC(t) - aX_A(t)$, $f_3(t, X_B(t)) = aX_A(t) - bX_B(t)$, $f_4(t, X_C(t)) = bX_B(t) - kX_C(t)$, $f_5(t, Y_C(t)) = n - cY_C(t)$, $f_6(t, Y_B(t)) = cY_C(t) - dY_B(t)$ and $f_7(t, Y_A(t)) = dX_B(t) - nY_A(t)$ are all polynomial functions which implies continuity. Now, by Lipschitz condition

$$|f_1(t, C_2(t)) - f_1(t, C_1(t))| = \left| -\left(\frac{Q_2}{V} + k\right)(C_2(t) - C_1(t)) \right| \quad (10)$$

$$\left|f_2\left(t, X_{A_2}(t)\right) - f_2\left(t, X_{A_1}(t)\right)\right| = \left|-a\left(X_{A_2}(t) - X_{A_1}(t)\right)\right| \quad (10)$$
$$\left|f_3\left(t, X_{B_2}(t)\right) - f_3\left(t, X_{B_1}(t)\right)\right| = \left|-b\left(X_{B_2}(t) - X_{B_1}(t)\right)\right| \quad (11)$$
$$\left|f_4\left(t, X_{C_2}(t)\right) - f_4\left(t, X_{C_1}(t)\right)\right| = \left|-k\left(X_{C_2}(t) - X_{C_1}(t)\right)\right| \quad (12)$$
$$\left|f_5\left(t, Y_{C_2}(t)\right) - f_5\left(t, Y_{C_1}(t)\right)\right| = \left|-c\left(Y_{C_2}(t) - Y_{C_1}(t)\right)\right| \quad (13)$$
$$\left|f_6\left(t, Y_{B_2}(t)\right) - f_6\left(t, Y_{b_1}(t)\right)\right| = \left|-d\left(Y_{B_2}(t) - Y_{B_1}(t)\right)\right| \quad (14)$$
$$\left|f_7\left(t, Y_{A_2}(t)\right) - f_7\left(t, Y_{A_1}(t)\right)\right| = \left|-n\left(Y_{A_2}(t) - Y_{A_1}(t)\right)\right| \quad (13)$$

Applying Cauchy's inequality on (9) – (12) gives

$$\left|f_1(t, C_2(t)) - f_1(t, C_1(t))\right| \leq \beta\left|\left(C_2(t) - C_1(t)\right)\right| \quad (14)$$
$$\left|f_2\left(t, X_{A_2}(t)\right) - f_2\left(t, X_{A_1}(t)\right)\right| \leq \gamma\left|\left(X_{A_2}(t) - X_{A_1}(t)\right)\right| \quad (15)$$
$$\left|f_3\left(t, X_{B_2}(t)\right) - f_3\left(t, X_{B_1}(t)\right)\right| \leq \delta\left|\left(X_{B_2}(t) - X_{B_1}(t)\right)\right| \quad (16)$$
$$\left|f_4\left(t, X_{C_2}(t)\right) - f_4\left(t, X_{C_1}(t)\right)\right| \leq \theta\left|\left(X_{C_2}(t) - X_{C_1}(t)\right)\right| \quad (17)$$
$$\left|f_5\left(t, Y_{C_2}(t)\right) - f_5\left(t, Y_{C_1}(t)\right)\right| \leq \vartheta\left|\left(Y_{C_2}(t) - Y_{C_1}(t)\right)\right| \quad (18)$$
$$\left|f_6\left(t, Y_{B_2}(t)\right) - f_6\left(t, Y_{b_1}(t)\right)\right| \leq \rho\left|\left(Y_{B_2}(t) - Y_{B_1}(t)\right)\right| \quad (19)$$
$$\left|f_7\left(t, Y_{A_2}(t)\right) - f_7\left(t, Y_{A_1}(t)\right)\right| \leq \sigma\left|\left(Y_{A_2}(t) - Y_{A_1}(t)\right)\right| \quad (20)$$

where $\beta = \left|-\left(\frac{Q_2}{V} + k\right)\right|$, $\gamma = |-a|, \delta = |-b|, \theta = |-k|, \vartheta = |-c|, \rho = |-d|$ and $\sigma = |-K_e|$. Since the Lipchitz condition is satisfied, the model (1) has a unique solution. Hence, the system (1) – (4) satisfied the Lipschitz conditions. So the system has a unique solution.

4. Results

Numerical Simulation

Various simulations were carried out using the values of Table 1 and the results are presented in the graphs below.

Table 1: Values of parameters/variables

Variable/Parameters	Value	Units	Source
a	0.9375	S^{-1}	Lone et al, (2021)
b	0.625	S^{-1}	Lone *et al*, (2021)
c	0.9375	S^{-1}	Lone *et al*, (2021)
d	0.625	S^{-1}	Lone *et al*, (2021)
k	3.72×10^{-8}	S^{-1}	Lone *et al*, (2021)
n	2.72×10^{-8}	S^{-1}	Lone *et al*, (2021)
E	50	μgs^{-1}	Assumed
V	200	μgm^{-3}	Assumed
C_0	100	μgm^{-3}	Assumed
C_a	100	μgm^{-3}	Assumed
Q_1	0.50	S^{-1}	Assumed
Q_2	0.40	S^{-1}	Assumed
X_0	3.72×10^{-6}	mol/cm^3	Lone *et al*, (2021)
Y_0	3.72×10^{-6}	mol/cm^3	Lone *et al*, (2021)

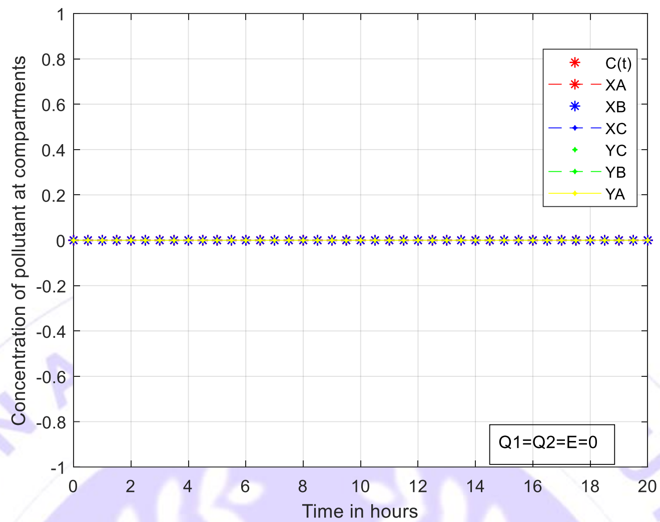

Figure 3: Pollutant concentration when $Q_1 = Q_2 = E = 0$

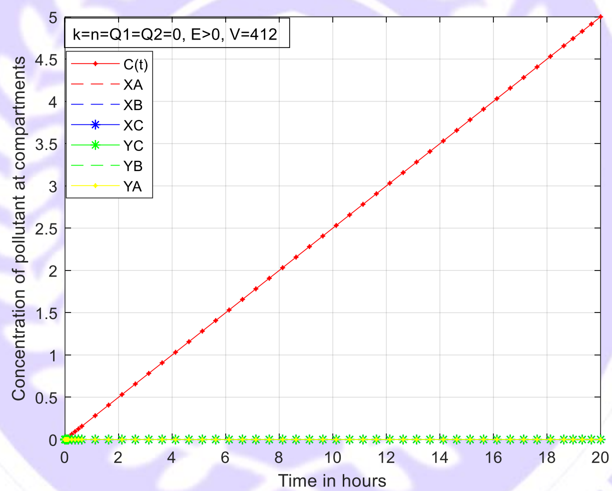

Figure 4: Pollutant concentration when $k = n = Q_1 = Q_2 = 0,\ E > 0, V = 412m^3$

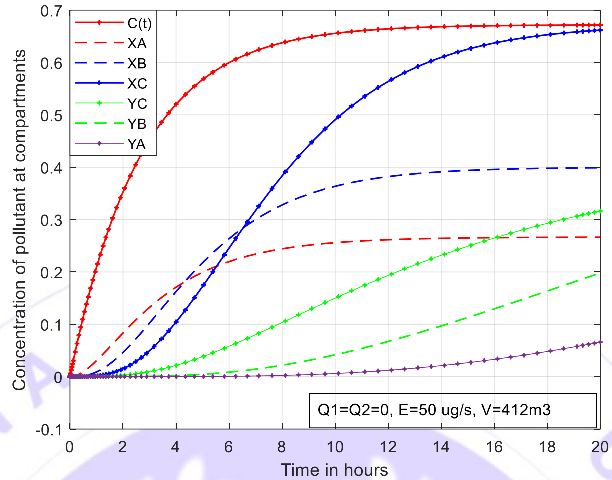

Figure 5: Pollutant concentration when $k > 0, n > 0, Q_1 = Q_2 = 0,\ E > 0$ and $V = 412m^3$

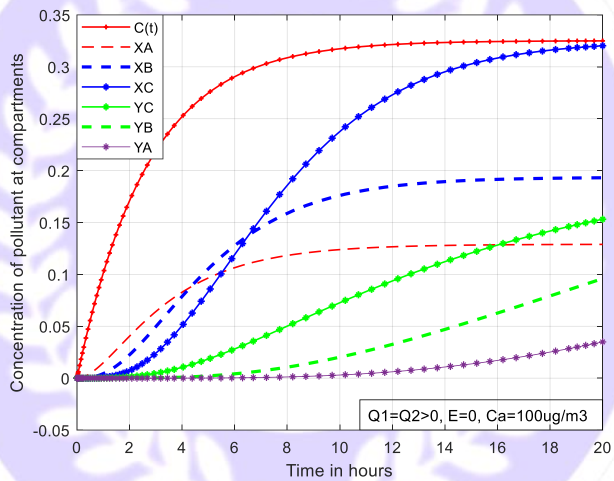

Figure 6: Pollutant concentration when $Q_1 = Q_2 > 0,\ E = 0,\ C_a = 100$ and $V = 412m^3$

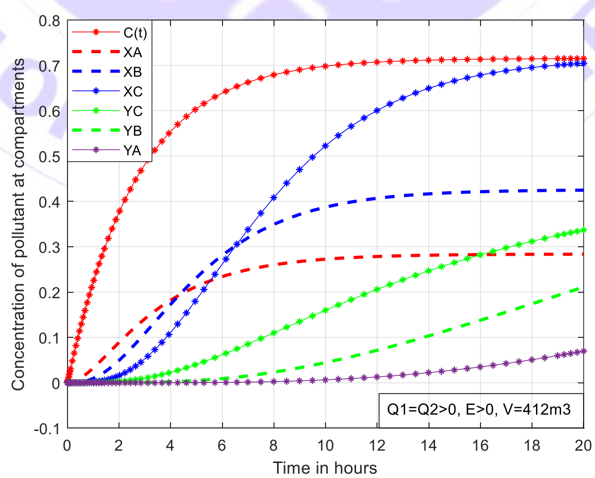

Figure 7: Pollutant concentration when $Q_1 = Q_2 > 0,\ E > 0$, and $V = 412m^3$

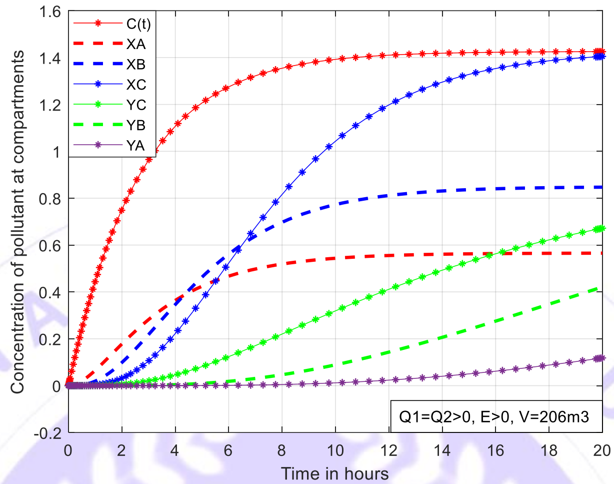

Figure 8: Pollutant concentration when $Q_1 = Q_2 > 0,\ E > 0$, and $V = 206m^3$

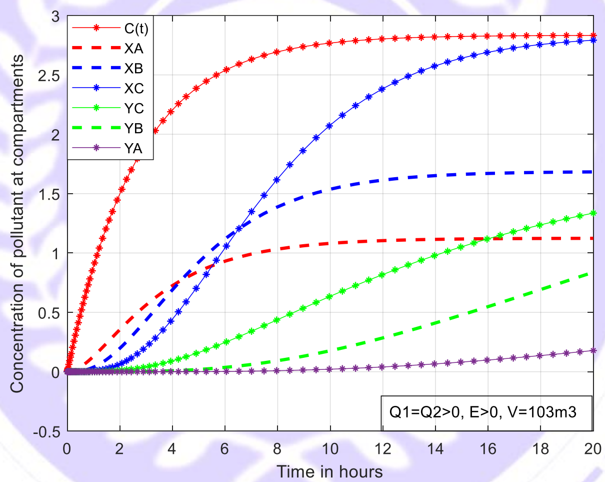

Figure 9: Pollutant concentration when $Q_1 = Q_2 > 0,\ E > 0$, and $V = 103m^3$

5. Discussion

Graph of Figure 3 assumed a situation where there is no pollutant in the room, so there will be zero pollutant concentration at the various compartments. This shows that people within the room will not have issue with breathing. Figure 4 indicates a situation where pollutant concentration in the room is only generated through internal emission (source). But because $k = 0$, the concentration of pollutant at the various compartments will be zero also. This probably mean that no person is within the room. Figure 5 shows that as the concentration of pollutant in the room is increasing due to indoor emission of pollutant ($E > 0$), the concentration at various compartments will begin to raise within one hour and as time progresses, the concentration at the tissue is almost being equal to the concentration in the room. This shows that the more you stay in a room with pollutant concentration, the more complications you will have with your breathing (respiratory System). Figure 6 shows that as the concentration of pollutant in the room is increasing due to outdoor (surrounding) pollutant ($C_a = 100\mu gm^{-3}$), the concentration at various compartments will begin to increase within one hour and as time progresses, the concentration at the tissue is almost being equal to the concentration in the room. But the volume of pollutant here (about $0.33\mu gm^{-3}$) is less compared to that of Figure 5 (about $0.68\mu gm^{-3}$). This shows that the indoor air pollutant has more effect on the human respiratory System than the surrounding pollutant if the room is well ventilated. In Figure 7 where the source of the room pollutant concentration is both from within and the surrounding, the volume of the concentration in the room (about $0.72\ \mu gm^{-3}$) and at the compartment is higher compared to that of Figure 5 (about $0.68\ \mu gm^{-3}$) and 6 (about $0.33\ \mu gm^{-3}$). Any person in this room will have more difficulty in breathing. In Figure 8, the volume of pollutant concentration in the room is high (about $1.41\ \mu gm^{-3}$) probably because of the size of the room. Here the room half of the ones of Figure 3 to Figure 7. In Figure 9, the concentration of pollutant in the room in this case (which is half of the room in Figure 8) is higher (about $2.85\mu gm^{-3}$). The results of Figure 8 and Figure 9 indicates that the smaller the room, the more complication people living inside will have with the breathing (respiratory system). The results here shows that this model can be used to study the effect of indoor air pollution on the human respiratory system perfectly and hence recommended.

6. Conclusion

In this study, we formulated a mathematical model of indoor air quality (pollutant) and the human respiratory system by adding a first order differential equation of indoor air quality (pollutant concentration) to the mathematical model by Lone *et al*., (2021). The formulated seven linear differential equations of first order were found to be uniform and asymptotically stable, and the model has a unique solution using the Picard – Lindelof Method. Numerical simulations were carried out to study the effect of indoor pollutants on the human respiratory system and the results were graphed. The results indicate that this model can be used to study the effect of indoor pollutants on the human respiratory system perfectly and hence recommended.

References

Apte, K.; Salvi, S. (2016). Household air pollution and its effects on health. F1000Research 5, 2593.

Blum J. J. (1960). Concentration profiles in and around capillaries. *Am. J. Physiology*, 198, 991-8.

Clark A. R., Stokes Y. M., Lane M., Thompson J. G. (2006). Mathematical modelling of oxygen concentration in bovine and murine cumlus-oocyte complexes, *Reproduction*, 131, 999-1006.

Guyton A. C. and Hall, J. E. (2011). Text Book of Medical Physiology, W. B. Saunders, 12th ed. 465-523.

Jones, A.P. (1999). Indoor air quality and health. *Atmos. Environ*. 33, 4535–4564.

Khanday M. A., Najar A. A. (2015a). Mathematical model for the transport of oxygen in the living tissue through capillary bed, *Journal of Mechanics in Medicine and Biology,* 15(4):1550055.

Khanday M. A., Najar A. A. (2015b). Maclaurin's series approach for the analytical solution of oxygen transport to the biological tissues through capillary bed, *Journal of Medical Imaging and Health Informatics*, 5(5):959-963.

Klepeis, N.E.; Nelson, W.C.; Ott, W.R.; Robinson, J.P.; Tsang, A.M.; Switzer, P.; Behar, J.V.; Hern, S.C.; Engelmann, W.H. (2001). The National Human Activity Pattern Survey (NHAPS): A resource for assessing exposure to environmental pollutants. *J. Expo. Anal. Environ. Epidemiol.* 11, 231–252.

Krogh A. (1919). The supply of oxygen to the tissue and the regulation of the capillary circulation, *J. Physiology*, 52, 457-464.

Kwaghkor L. M., Onah E. S. and Bassi I. G. (2018). Stochastic Modelling of the Effect of Deforestation in Nigeria. *Journal of the Nigerian Association of Mathematical Physics.* 45 (1). Pp.379 – 387.

Kwaghkor L. M., Onah E. S., Aboiyar, T. and Ikughur, J. A. (2019). Derivation of a Stochastic Labour Market

Model from a Semi – Markov Model. *International Journal of Mathematical Analysis and Optimization: Theory and Applications.* 2019(2). Pp.610-630.

Kwaghkor, L. M. and Luga, T. (2016). Mathematical Model for the Detection and Control of Diabetes. *Journal of the Nigerian Association of Mathematical Physics.* 35 (2). Pp.253 – 260.

Kwaghkor, L. M., Adamu, S., Mohammed, A. and Suleiman, M. (2024). A Nonlinear Mathematical Model for the Effect of Diabetes Population on a Community. *International Journal of Development Mathematics*, 1(1), Pp. 172-185. http://doi.org/10.62054/ijdm/0101.13

Kwaghkor, L. M., Onah, E. S., Bassi, I. G. and Danjuma, T. (2021). Stochastic Transmission Dynamics of Covid-19 within a Density Dependent Population. *FUDMA Journal of Science*. Vol.5(2). Pp. 567 – 573. https://doi.org/10.33003/fjs-2021-0502-569

Kwaghkor, L.M., Mohammed, A & Nyamtswam, E. V. (2022). A Mathematical Model for Diabetes Management. *FUDMA Journal of Science*. Vol. 6 (5). Pp 36 – 40. https://doi.org/10.33003/fjs-2022-0605-1091

Lin S. H. (1976). Oxygen diffusion in a spherical cell with non-linear oxygen uptake kinetics, *J. Theoret. Biol.*, 60, 449-457.

Lone, A. U. H.; Khanday, M. A. and Mubarak, S. (2021). A four-compartment model to estimate oxygen and carbon dioxide exchange concentrations via blood using eigenvalue approach. *South east Asian J. of Mathematics and Mathematical Sciences.* 17(2):367-384.

Mannan, M. and Al-Ghamdi, S. G. (2021). Indoor Air Quality in Buildings: A Comprehensive Review on the Factors Influencing Air Pollution in Residential and Commercial Structure. *Int. J. Environ. Res. Public Health,*18, 3276.

Nunn J. F. (1987). Applied Respiratory Physiology, Elsevier, 3rd ed.

Orapine, H. O., Baidu, A. A., and Kwaghkor, L. M. (2023). The Cauchy Problem for Nonlinear Higher Order Partial Differential Equations Using Projected Differential Transform Method. *International Journal of Mathematical Sciences and Optimization: Theory and Applications*, 9(2), Pp. 74-89. https://doi.org/10.5281/zenodo.10202651

Ottesen J. T., Olufsen M. S., Larsen J. K. (2004). Applied Mathematical Models in Human Physiology, *SIAM,* 13-34.

Prajakta, P. (2013). Shrimandilkar, Indoor Air Quality Monitoring for Human Health. Ijmer 3, 891–897. Available online: http://www.ijmer.com/papers/Vol3_Issue2/BV32891897.pdf

Salathe E. P., Wang T. C., Gross G. F. (1980). Mathematical analysis of oxygen transport to tissues, Mathematical Bioscience, 51, 89-115.

Simpson M. J. and Ellery A. J. (2012). An analytic solution for diffusion and non-linear uptake of oxygen in a spherical cell, *Applied Mathematical Modelling*, 36, 3329-3334.

Sisask, M.; Värnik, P.; Värnik, A.; Apter, A.; Balazs, J.; Balint, M.; Bobes, J.; Brunner, R.; Corcoran, P.; Cosman, D.; *et al*. (2014). Teacher satisfaction with school and psychological well-being affects their readiness to help children with mental health problems. *Health Educ. J.* 73, 382–393.

Sundell, J. (2004). On the history of indoor air quality and health. *Indoor Air Suppl*. 14, 51–58.

USEPA. (2013). Indoor Air Pollution and Health. Report Series No. 104. 2013. Available online:https://www.epa.ie/pubs/reports/research/health/IndoorAirPollutionandHealth.pdf

Vilˇceková, S.; Apostoloski, I.Z.; Meˇciarová, L'.; Burdová, E.K.; Kisel'ák, J. (2017). Investigation of indoor air quality in houses of Macedonia. *Int. J. Environ. Res. Public Health* 14, 37.

World Health Organization. (2007). Indoor Air Pollution: National Burden of Disease Estimates; WHO: Geneva, Switzerland. Available online: https://www.who.int/airpollution/publications/indoor_air_national_burden_estimate_revised.pdf?ua=1.