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Assessing the Impact of Hospital and Traditional-based Treatments on Snakebite Envenoming in Kaltungo: A Mathematical Modeling Approach

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ABSTRACT

In Kaltungo, Northeast Nigeria, snakebite envenoming (SBE) poses a substantial public health threat, with both hospital-based and traditional treatments being widely used. To assess the effectiveness of these treatment strategies, this study employs a mathematical modeling approach. A compartmental model is developed, incorporating SBE epidemiological data, treatment patterns, and outcomes to quantify the effectiveness of hospital-based and traditional treatments in Kaltungo. The basic dynamical features of the model were explored and fitted with local data. A qualitative study of the model revealed that it has two equilibrium points, which were shown to be globally asymptotically stable. The numerical findings indicate that hospital-based treatment is more effective in yielding good recovery outcomes and reducing SBE-related mortality than traditional treatment. However, the findings further establish that combining both hospital-based and traditional treatments is the best approach, producing optimal recovery outcomes and the most significant reduction in SBE-induced deaths. This study emphasizes the need for integrated treatment approaches that combine the benefits of hospital-based and traditional treatments. The model provides a valuable tool for evaluating treatment strategies and informing evidence-based decision-making for SBE management in Kaltungo and similar settings.

1. Introduction

Snakebite envenoming (SBE) is considered a serious and potentially life-threatening condition by the World Health Organization (WHO), resulting from the injection of venom from a venomous snake into the human body. SBE poses a significant public health threat in tropical and subtropical regions, with a disproportionate impact on sub-Saharan Africa (WHO, 2019). Globally, snakebites result in substantial morbidity and mortality, with estimated annual incidence rates of 4.5-5.4 million bites, 1.8-2.7 million envenoming cases, and 81,000-138,000 fatalities (Chippaux, 1998; Kasturiratne *et al*., 2008; WHO, 2019). These statistics disproportionately affect rural communities with inadequate access to healthcare.

According to a study by Gutiérrez *et al*., (2017), sub-Saharan Africa experiences a significant burden of snakebite-related mortality, with estimated annual deaths ranging from 20,000 to 32,000. In Nigeria, SBE is a major health concern, particularly in rural areas where snakebites are prevalent due to the presence of venomous snakes and occupational activities (Abubakar *et al*., 2010; Habib *et al*., 2015). Earlier studies on the incidence of snakebite in Nigeria revealed that it has an estimated annual incidence of 497 cases per 100,000 populations, with a 10 – 20% natural mortality (Pugh and Theakston, 1980; Habib *et al*., 2001).

Kaltungo, in Gombe State, Northeast Nigeria, is among the regions severely affected by SBE, with a high incidence of snakebites primarily due to venomous snakes and local population activities (Habib, 2013; Chippaux, 2017). It has been reported that the Snakebite Treatment and Research Hospital (STRH), Kaltungo admits about 12 snakebite victim daily, on the average with more than 3000 cases annually. Despite the severity of the issue, studies using a mathematical modeling approach to evaluate SBE treatment approaches are lacking in this region. Hospital-based treatments, including antivenom therapy and supportive care, are considered the most effective SBE management approach

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(Warrell *et al*., 1977; Habib *et al*., 2001). However, in rural areas like Kaltungo, traditional treatments are often preferred due to cultural beliefs, limited healthcare access, or perceived effectiveness. Although traditional treatments may offer some benefits, their effectiveness and safety are poorly documented and may lead to inadequate treatment, resulting in poor outcomes. Study conducted by Omogbai *et al*., (2002), revealed that only about 8.5% Nigerian snakebite victims sought treatment in hospitals. On the other hand, Chippaux (2011) discovered that majority of snakebite victims first seek treatment at traditional healers, and only visit hospitals when situation deteriorate or there are complications.

Since WHO declared SBE a category A neglected tropical disease in 2017, several mathematical models have been developed and studied (Bravo *et al*., 2019; Kim, 2020; Goldstein *et al*., 2021; Abdullahi *et al*., 2021, 2024 ; Martin *et al*., 2022; Joseph, *et al*., 2024). These models have made valuable contributions by recommending effective control strategies to reduce the burden of SBE in communities. Notably, mathematical modeling provides a valuable tool for evaluating the impact of different treatment strategies. By integrating epidemiological data, treatment patterns, and outcomes, mathematical models can assess the effectiveness of hospital-based and traditional treatments, identify factors influencing treatment outcomes, and inform evidence-based decision-making.

This study aims to develop a mathematical model to evaluate the impact of hospital-based and traditional treatments on SBE outcomes in Kaltungo, Gombe State, in Northeast Nigeria. The model will incorporate epidemiological data, treatment patterns, and outcomes to quantify the effectiveness of different treatment approaches and identify optimal SBE management strategies in this region. The paper is organized as follows: Section 2 formulates the model, while Section 3 analyses it. Section 4 presents the model fittings and numerical simulations. Finally, Sections 5 and 6 discuss the results and conclude the study, respectively.

2. Material and Method

This section presents the study area and the steps involved in formulating the compartmental-based model.

2.1 Study Area

The study area for this research is Kaltungo, a Local Government Area (LGA) in Gombe State, Nigeria. Kaltungo is situated in the north-eastern part of Nigeria, within the Sudan savanna region. The LGA has a total land area of approximately 881 km² and a population of 183,000 people (National Population Commission (NPC) , 2016). Kaltungo is predominantly rural, with most residents engaged in subsistence farming and animal husbandry. The area is characterized by a tropical climate with two distinct seasons: a wet season from May to October and a dry season from November to April. The region's vegetation is mainly composed of grasslands and savannas, providing a habitat for various species of snakes, including venomous ones like puff adders, vipers, and cobras. The healthcare system in Kaltungo is limited, with a few government-owned hospitals and primary healthcare centres, as well as traditional healing canters. Snakebite envenoming is a significant public health concern in the area, with many reported cases and fatalities. While STRH, Kaltungo provides emergency care, including antivenom treatment, accessibility and availability can be limited, especially in remote areas. Consequently, traditional remedies, such as herbal treatments and spiritual healing, remain a common recourse for many residents.

2.2 Model Formulation

The model considers two interacting populations, namely humans and venomous snakes. The human population at time t is denoted by $N_H(t)$ while the venomous snake population at time t is represented by $V_S(t)$. The human population is further subdivided into five mutually exclusive classes, namely: the susceptible class ($S(t)$), the envenomed class ($E(t)$), the class of envenomed individuals receiving traditional-based treatment ($T_T(t)$), the class of envenomed individuals receiving hospital-based treatment ($H_T(t)$), and the recovered class ($R(t)$). Thus,

$$N_H(t) = S(t) + E(t) + T_T(t) + H_T(t) + R(t) \quad (1)$$

Let the force of envenomation be defined as

$$\lambda(t) = \beta V_S \quad (2)$$

In equation (2), β is the effective contact rate between venomous snakes and humans, i.e., the contact capable of causing envenomation.

2.3 Description of the model

The population of susceptible individuals is generated by birth (at a rate Λ) and by those who recovered from snakebite envenoming at a rate α. This population decreases by those who become envenomed following effective contacts with venomous snakes (at a rate λ), it is also decreased due to natural death (at a rate μ), and it is assumed that natural death occurs in all human classes at the same rate. Thus,

$$\frac{dS}{dt} = \Lambda + \alpha R - (\lambda + \mu) S \tag{3}$$

The population of envenomed individuals is generated, by susceptible individuals bitten by venomous snake and become envenomed at the rate λ . It is decreased by those who seek for traditional-based treatment (at a rate τ_T), those who seek for hospital-based treatment (at a rate τ_H) and SBE-induced death (at a rate δ_E), and natural death. So that

$$\frac{dE}{dt} = \lambda S - (\tau_T + \tau_H + \delta_E + \mu) E. \tag{4}$$

The population of envenomed individuals receiving traditional-based treatment is generated by those who seek for traditional-based treatment, at the rate τ_T and by those who revert to traditional treatment from hospital-based treatment (at a rate σ_H). It is decreased by those who revert to seek for hospital-based treatment (at the rate σ_T), those who recovered (at the rate γ_T), SBE-induced death (at the rate δ_T), and natural death. so that

$$\frac{dT_T}{dt} = \tau_T E + \sigma_H H_T - (\sigma_T + \gamma_T + \delta_T + \mu) T_T. \tag{5}$$

The population of envenomed individuals receiving hospital-based treatment is generated by those who seek for hospital-based treatment, (at the rate τ_H) and by those who revert to hospital-based treatment from traditional-based treatment (at a rate σ_T). It is decreased by those who revert to seek for traditional-based treatment (at the rate σ_H), those who recovered (at the rate γ_H), SBE-induced death (at the rate δ_H), and natural death. So that

$$\frac{dH_T}{dt} = \tau_H E + \sigma_T T_T - (\sigma_H + \gamma_H + \delta_H + \mu) H_T \tag{6}$$

The population of recovered individuals is generated by those who recovered from traditional-based treatment (at the rate γ_T) and those who recovered from hospital-based treatment (at the rate γ_H). It diminishes as a result of those who transit to susceptible population (at the rate α) and natural death. Thus,

$$\frac{dR}{dt} = \gamma_T T_T + \gamma_H H_T - (\alpha + \mu) R. \tag{7}$$

The population of venomous snakes is increased due to constant recruitment (at the rate Λ_S) and reduces due to natural death (at a rate μ_S). Thus,

$$\frac{dV_S}{dt} = \Lambda_S - \mu_S V_S \tag{8}$$

2.4 The model equation

The model that describe the dynamics of snakebite envenomation incorporating hospital and traditional-based treatments is govern by the set of first order nonlinear differential equations shown in equation (9). The schematic diagram of the model is depicted in Figure 1. The state variables and parameters of the model are given in Tables 1 and 2, respectively.

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda + \alpha R - (\lambda + \mu) S, \\
\frac{dE}{dt} &= \lambda S - (\tau_T + \tau_H + \delta_E + \mu) E, \\
\frac{dT_T}{dt} &= \tau_T E + \sigma_H H_T - (\sigma_T + \gamma_T + \delta_T + \mu) T_T, \\
\frac{dH_T}{dt} &= \tau_H E + \sigma_T T_T - (\sigma_H + \gamma_H + \delta_H + \mu) H_T, \\
\frac{dR}{dt} &= \gamma_T T + \gamma_H H - (\alpha + \mu) R, \\
\frac{dV_S}{dt} &= \Lambda_S - \mu_S V_S,
\end{aligned} \tag{9}$$

with the following initial conditions

$$S(0) > 0, E(0) > 0, T_T(0) \geq 0, H_T(0) \geq 0, R(0) \geq 0, V_S(0) > 0.$$

Table 1: Description of state variables of the model

State Variable	Description
$S(t)$	Population of susceptible individuals
$E(t)$	Population of envenomed individuals
$T_T(t)$	Population of envenomed individuals receiving traditional-based treatment
$H_T(t)$	Population of envenomed individuals receiving hospital-based treatment
$R(t)$	Population of recovered individuals
$V_S(t)$	Population of venomous snakes

Table 2: Description of parameters of the model

Parameter	Description
Λ	Recruitment rate of susceptible humans
Λ_S	Recruitment rate of venomous snakes
$\mu(\mu_S)$	Natural death rate of humans(natural death rate of venomous snakes)
β	Effective contact rate between venomous snakes and humans
τ_T	Treatment-seeking rate of traditional-based treatment
τ_H	Treatment-seeking rate of hospital-based treatment
γ_T	Recovery rate for traditional-based treatment
γ_H	Recovery rate for hospital-based treatment
$\delta_E, \delta_T,$ and δ_H	SBE-induced death rates in envenomed, individuals receiving traditional-based and hospital-based treatments classes, respectively
σ_T	Rate at which individual receiving traditional-based treatment revert to hospital-based treatment
σ_H	Rate at which individual receiving hospital-based treatment revert to traditional-based treatment
α	Rate at which recovered individuals transit to susceptible class

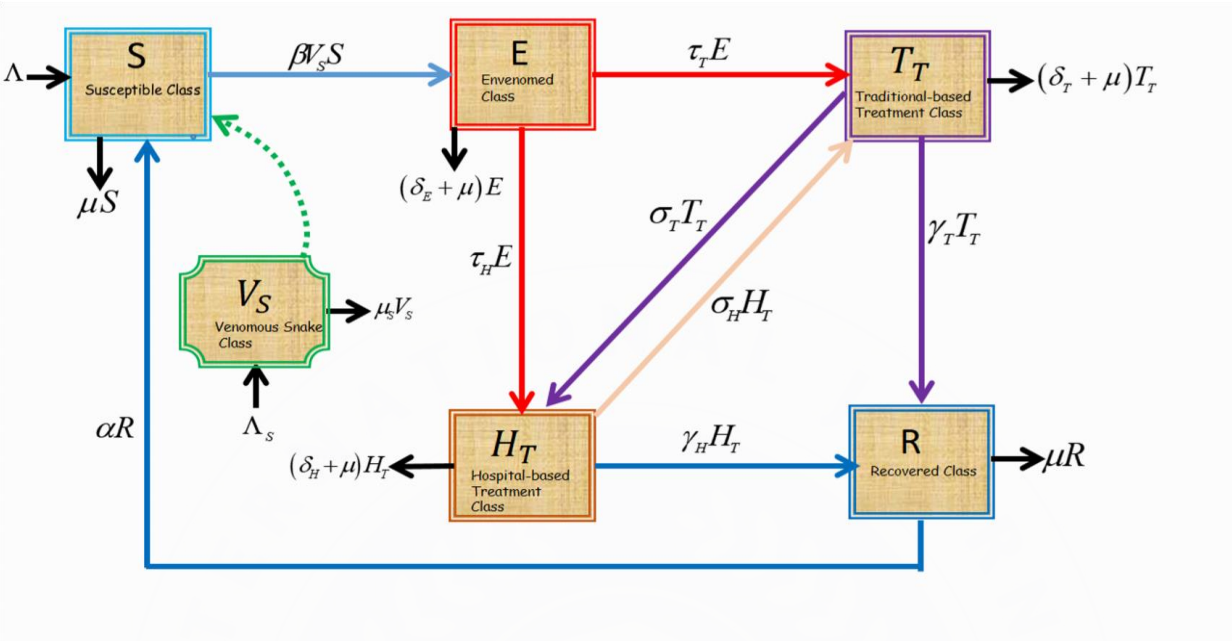


Figure 1: Schematic diagram of the model

3. Analysis of the Model

In this section, the basic dynamical features of the compartmental-based model will be explored through qualitative analysis.

3.1 Positivity of Model Solution

For model (9), it is necessary to show that all the state variables are positive, so that the solutions of the model with positive initial conditions will remain positive for all $t > 0$. Thus, the following theorem is established.

Theorem 1. *If the initial data* $Y(0) > 0$, *where* $Y(t) = (S, E, T_T, H_T, R, V_S)$. *Then the solution* $Y(t)$ *of the model (9) will remain positive for all* $t > 0$.

Proof. This theorem is prove using the method of contradiction, first we assume that there exists a time t_1 such that

$$S(t_1) = 0, \frac{dS(t_1)}{dt} < 0, E(t) > 0, T_T(t) > 0, H_T(t) > 0, R(t) > 0, V_S(t) > 0, 0 < t < t_1. \quad (10)$$

There exists a time t_2 such that

$$E(t_2) = 0, \frac{dE(t_2)}{dt} < 0, S(t) > 0, T_T(t) > 0, H_T(t) > 0, R(t) > 0, V_S(t) > 0, 0 < t < t_2. \quad (11)$$

There exists a time t_3 such that

$$T_T(t_3) = 0, \frac{dT_T(t_3)}{dt} < 0, S(t) > 0, E(t) > 0, H_T(t) > 0, R(t) > 0, V_S(t) > 0, 0 < t < t_3. \quad (12)$$

There exists a time t_4 such that

$$H_T(t_4) = 0, \frac{dH_T(t_4)}{dt} < 0, S(t) > 0, E(t) > 0, T_T(t) > 0, R(t) > 0, V_S(t) > 0, 0 < t < t_4. \quad (13)$$

There exists a time t_5 such that

$$R(t_5) = 0, \frac{dR(t_5)}{dt} < 0, S(t) > 0, E(t) > 0, T_T(t) > 0, H_T(t) > 0, V_S(t) > 0, 0 < t < t_5. \quad (14)$$

There exists a time t_6 such that

$$V_S(t_6)=0, \frac{dV_S(t_5)}{dt}<0, S(t)>0, E(t)>0, T_T(t)>0, H_T(t)>0, R(t)>0, 0<t<t_6. \quad (15)$$

Applying the assumption in equation (10) in the first equation of model (9) we have

$$\frac{dS(t_1)}{dt}=\Lambda+\alpha R>0. \quad (16)$$

This contradict the assumption in equation (10) that $\frac{dS(t_1)}{dt}<0$. Thus, $S(t) > 0$ for $t > 0$. Similarly, applying the assumption in equation (11) in the second equation of model (9) we have

$$\frac{dE(t_2)}{dt}=\lambda S>0. \quad (17)$$

This again contradict the assumption in equation (11) that $\frac{dE(t_2)}{dt}<0$. Thus, $E(t)>0$ for $t>0$. Using similar approach it can be shown that $T_T(t)>0, H_T(t)>0, R(t)>0$ and $V_S(t)>0$, for all $t > 0$. Therefore, the solution $Y(t)$, of the model (9) will remain positive for all $t > 0$. □

3.2 Invariant Region

Consider the following biologically feasible region

$$\Psi=\Psi_H\cup\Psi_S\subset\mathbb{R}_+^5\times\mathbb{R}_+, \quad (18)$$

with

$$\Psi_H=\left\{S,E,T_T,H_T,R: N_H\le\frac{\Lambda}{\mu}\right\}, \Psi_S=\left\{V_S: V_S\le\frac{\Lambda_S}{\mu_S}\right\} \quad (19)$$

Lemma 2. *The region* $\Psi\subset\mathbb{R}_+^6$ *is positively-invariant for the model (9) for every nonnegative initial condition in* $\mathbb{R}_+^6$.

Proof. Adding the first five equations of model (9) and considering the sixth equation we obtain

$$\frac{dN_H}{dt}=\Lambda-\mu N_H-\left(\delta_E E+\delta_T T+\delta_H H\right),$$
$$\frac{dV_S}{dt}=\Lambda_S-\mu_S V_S. \quad (20)$$

Applying the method of integrating factors and comparison theorem in Lakshmikanthan and Leela (1969) it can be established that

$$N_H(t)\le N_H(0)e^{-\mu t}+\frac{\Lambda}{\mu}\left[1-e^{-\mu t}\right],$$
$$V_S(t)\le V_S(0)e^{-\mu_S t}+\frac{\Lambda_S}{\mu_S}\left[1-e^{-\mu_S t}\right]. \quad (21)$$

Therefore, $\limsup_{t\to\infty} N_H(t)\le\frac{\Lambda}{\mu}$ and $\limsup_{t\to\infty} V_S(t)\le\frac{\Lambda_S}{\mu_S}$. Thus, the region Ψ is positively invariant. □

Hence, the dynamics of the flow generated by model (9) can be studied in Ψ. In this region, the model is epidemiological and mathematically well-posed.

3.3 Existence and Stability of Equilibrium Points

Here, the equilibrium points of the model are calculated by setting the right hand sides of model (9) to zero and then solve for the state variables of the model. Furthermore, the global stability analysis of each equilibrium point is analysed. Hence, the following results are established:

3.3.1 Snakebite envenoming-free equilibrium (SBE-Free)

The model (9) has a SBE-free equilibrium point, given by

$$\Theta_0 = \left(S^0, E^0, T_T^0, H_T^0, R^0, V_S^0\right) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, \frac{\Lambda_S}{\mu_S}\right) \tag{22}$$

Note that the SBE-free equilibrium point of the model represents a case where population under study will be free from snakebite envenomation. Biologically speaking, such situation can occur either because the venomous snakes do not exists in the population (which is not of interest) or there is no contact between humans and venomous snakes (i.e. $\beta = 0$).

3.3.2 Global Stability Analysis of SBE-free Equilibrium

To investigate whether effective control of SBE in a given population is independent of the initial data of the sub-populations of the model (9), it is essential to show that the SBE-free equilibrium is globally-asymptotically stable (GAS).

Theorem 3. *The SBE-free, equilibrium point of the model (9), is GAS in the region* Ψ.

Proof. Consider the following candidate Lyapunov function

$$\mathcal{W}(t) = g_1 E + g_2 T_T + g_3 H_T + g_4 V_S \tag{23}$$

Where $g_i, (i = 1, 2, \ldots 4)$ are some positive constants to be determined. The corresponding Lyapunov derivative along the solution of model (9) is given by $\dot{\mathcal{W}}(t)$ (where a dot represents differentiation with respect to time *t*).

$$\dot{\mathcal{W}}(t) = g_1 \dot{E} + g_2 \dot{T}_T + g_3 \dot{H}_T + g_4 \dot{V}_S \tag{24}$$

Substituting the RHS of model (9) in equation (24) we obtain

$$\begin{aligned} \dot{\mathcal{W}}(t) = {} & g_1\left[\beta V_S S - (\tau_T + \tau_H + \delta_E + \mu)E\right] + g_2\left[\tau_T E + \sigma_H H_T - (\sigma_T + \gamma_T + \delta_T + \mu)T_T\right] \\ & + g_3\left[\tau_H E + \sigma_T T_T - (\sigma_H + \gamma_H + \delta_H + \mu)H_T\right] + g_4\left[\Lambda_S - \mu_S V_S\right]. \end{aligned} \tag{25}$$

Choosing $g_1 = g_2 = g_3 = g_4 = 1$ and using them in equation (25) the following is obtain:

$$\dot{\mathcal{W}}(t) = \beta V_S S - (\delta_E + \mu)E - (\gamma_T + \delta_T + \mu)T_T - (\gamma_H + \delta_H + \mu)H_T + \Lambda_S - \mu_S V_S. \tag{26}$$

Using the following results in equation (26):

$$S \le \frac{\Lambda}{\mu}, V_S \le \frac{\Lambda_S}{\mu_S}. \tag{27}$$

We obtain

$$\dot{\mathcal{W}}(t) \le -(\delta_E + \mu)E - (\gamma_T + \delta_T + \mu)T_T - (\gamma_H + \delta_H + \mu)H_T + \frac{\beta\Lambda\Lambda_S}{\mu\mu_s} \tag{28}$$

Since $\beta = 0$, then equation (28) reduces to

$$\dot{\mathcal{W}}(t) \le -(\delta_E + \mu)E - (\gamma_T + \delta_T + \mu)T_T - (\gamma_H + \delta_H + \mu)H_T. \tag{29}$$

Therefore, since all the model parameters are positives then $\dot{\mathcal{W}}(t) \le 0$ and $\dot{\mathcal{W}}(t) = 0$ provided $E = T_T = H_T = 0$. Thus, it follows from LaSalle's in-variance principle in LaSalle, (1976) that the maximal invariant set contained in the region Ψ is the singleton Θ_0. Hence, the SBE-free equilibrium point is GAS in Ψ. □

3.4 Snakebite envenoming endemic equilibrium point (SBE-Endemic)

Let $\beta > 0$, so that model (9) has a SBE-endemic equilibrium point, given by

$$\Theta_1 = \left(S^*, E^*, T_T^*, H_T^*, R^*, V_S^*\right) = \left(\frac{\Lambda + \alpha R^*}{\lambda^* + \mu}, \frac{\lambda^* S^*}{W_1}, \frac{\tau_T E^* + \sigma_H H_T^*}{W_2}, \frac{\tau_H E^* + \sigma_T T_T^*}{W_3}, \frac{\gamma_T T^* + \gamma_H H^*}{W_4}, \frac{\Lambda_S}{\mu_S}\right), \tag{30}$$

where

$$W_1 = \tau_T + \tau_H + \delta_E + \mu, W_2 = \sigma_T + \gamma_T + \delta_T + \mu, W_3 = \sigma_H + \gamma_H + \delta_H + \mu, W_4 = \alpha + \mu. \tag{31}$$

3.4.1 Global stability analysis of SBE-endemic equilibrium

Theorem 4. *The SBE-endemic equilibrium point* Θ_1 *of model (*9*), is globally asymptotically stable in the region* Ψ.

Proof. Consider the following quadratic Lyapunov function:

$$\mathcal{F}(Y) = \frac{1}{2}\left[\left(S - S^*\right) + \left(E - E^*\right) + \left(T_T - T_T^*\right) + \left(H_T - H_T^*\right) + \left(R - R^*\right) + \left(V_S - V_S^*\right)\right]^2 \tag{32}$$

Where $\mathrm{Y} = \left(S, E, T_T, H_T, R, V_S\right)$. The Lyapunov derivative with respect to time is

$$\frac{d\mathcal{F}}{dt} = \left[\left(S - S^*\right) + \left(E - E^*\right) + \left(T_T - T_T^*\right) + \left(H_T - H_T^*\right) + \left(R - R^*\right)\right] \times \left[\frac{d\left(S + E + T_T + H_T + R\right)}{dt}\right] \tag{33}$$

$$+\left(V_S - V_S^*\right)\frac{dV_S}{dt}$$

According to model (9), it is implied that

$$\frac{d\left(S + E + T_T + H_T + R\right)}{dt} = \Lambda - \mu\left(S + E + T_T + H_T + R\right) - \delta_E E - \delta_T T - \delta_H H,$$
$$\frac{dV_S}{dt} = \Lambda_S - \mu_S V_S. \tag{34}$$

Substituting equation (34) into equation (33) yields

$$\frac{d\mathcal{F}}{dt} = \left[\left(S - S^*\right) + \left(E - E^*\right) + \left(T_T - T_T^*\right) + \left(H_T - H_T^*\right) + \left(R - R^*\right)\right]$$
$$\times\left[\Lambda - \mu\left(S + E + T_T + H_T + R\right) - \delta_E E - \delta_T T - \delta_H H\right] + \left(V_S - V_S^*\right)\left(\Lambda_S - \mu_S V_S\right). \tag{35}$$

It can be established from model (9) that, at the endemic steady state

$$\Lambda = \mu\left(S^* + E^* + T_T^* + H_T^* + R^*\right) + \delta_E E^* + \delta_T T^* \delta_H H^*,$$
$$\Lambda_S = \mu_S V_S^*. \tag{36}$$

Substituting equation (36) into equation (35) produces

$$\frac{d\mathcal{F}}{dt} = \left[\left(S - S^*\right) + \left(E - E^*\right) + \left(T_T - T_T^*\right) + \left(H_T - H_T^*\right) + \left(R - R^*\right)\right]$$
$$\times\left[\mu\left(S^* + E^* + T_T^* + H_T^* + R^*\right) + \delta_E E^* + \delta_T T^* \delta_H H^* - \mu\left(S + E + T_T + H_T + R\right)\right. \tag{37}$$
$$\left.-\delta_E E - \delta_T T - \delta_H H\right] + \left(V_S - V_S^*\right)\left[\mu_S V_S^* - \mu_S V_S\right].$$

Simplifying equation (37) reduces to

$$\frac{d\mathcal{F}}{dt}=\left[\left(S-S^*\right)+\left(E-E^*\right)+\left(T_T-T_T^*\right)+\left(H_T-H_T^*\right)+\left(R-R^*\right)\right]$$
$$\times\left[-\mu\left(S-S^*\right)-\mu\left(E-E^*\right)-\mu\left(T_T-T_T^*\right)-\mu\left(H_T-H_T^*\right)-\mu\left(R-R^*\right)\right. \quad (38)$$
$$\left.-\delta_E\left(E-E^*\right)-\delta_T\left(T_T-T^*\right)-\delta_H\left(H-H^*\right)-\mu_S\left(V_S-V_S^*\right)\left(V_S-V_S^*\right)\right].$$

Expanding equation (38) yields

$$\frac{d\mathcal{F}}{dt}=-\mu\left(S-S^*\right)^2-\left(\mu+\delta_E\right)\left(E-E^*\right)^2-\left(\mu+\delta_T\right)\left(T_T-T_T^*\right)^2-\left(\mu+\delta_H\right)\left(H_T-H_T^*\right)^2$$
$$-\mu\left(R-R^*\right)^2-\mu_S\left(V_S-V_S^*\right)^2-\left(2\mu+\delta_E\right)\left(S-S^*\right)\left(E-E^*\right)-\left(2\mu+\delta_T\right)\left(S-S^*\right)$$
$$\times\left(T_T-T_T^*\right)-\left(2\mu+\delta_H\right)\left(S-S^*\right)\left(H_T-H_T^*\right)-2\mu\left(S-S^*\right)\left(R-R^*\right)-\left(2\mu+\delta_E+\delta_T\right) \quad (39)$$
$$\times\left(E-E^*\right)\left(T_T-T_T^*\right)-\left(2\mu+\delta_E+\delta_H\right)\left(E-E^*\right)\left(H_T-H_T^*\right)-\left(2\mu+\delta_T+\delta_H\right)\left(T_T-T_T^*\right)$$
$$\times\left(H_T-H_T^*\right)-\left(2\mu+\delta_E\right)\left(E-E^*\right)\left(R-R^*\right)-\left(2\mu+\delta_T\right)\left(T_T-T_T^*\right)\left(R-R^*\right)$$
$$-\left(2\mu+\delta_H\right)\left(H_T-H_T^*\right)\left(R-R^*\right)$$

Since, $S\le S^*, E\le E^*, T_T\le T_T^*, H_T\le H_T^*, R\le R^*, V_S\le V_S^*$, then $\frac{d\mathcal{F}}{dt}\le 0$ in equation (39) and $\frac{d\mathcal{F}}{dt}=0$, provided that $S=S^*, E=E^*, T_T=T_T^*, H_T=H_T^*, R=R^*, V_S=V_S^*$. It follows that $\mathcal{F}$ is a Lyapunov function on Ψ. By LaSalle's Invariance Principle in LaSalle, (1976), it is established that all solutions of the model (9) converge asymptotically to the unique SBE-endemic equilibrium point as $t\to\infty$.

4. Model Fittings and Numerical Simulations

For the purpose of model fittings and numerical simulations, we further derive the following equation to describe the cumulative number of SBE-induced mortality. Let *D* denotes the number of SBE-induced mortality such that

$$\frac{dD}{dt}=\delta_E E_C+\delta_T T+\delta_H H_T. \quad (40)$$

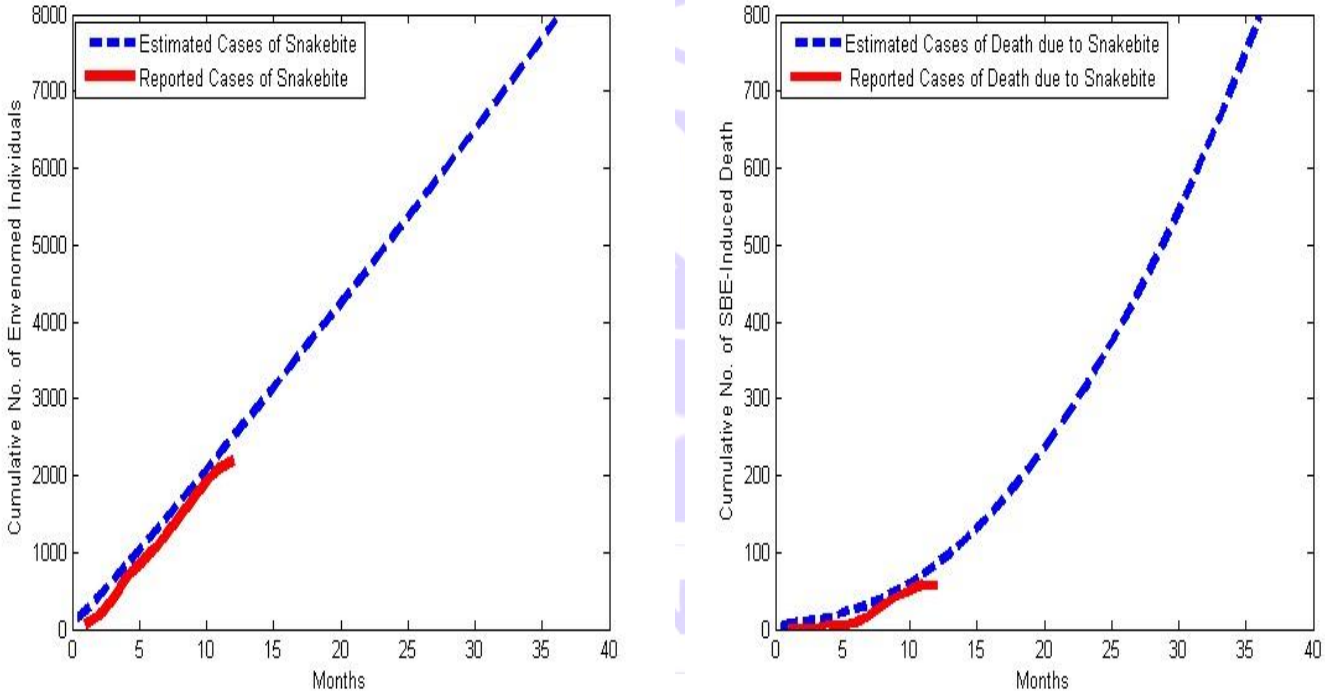
4.1 Model Fitting

To fit and validate the model, we utilized cumulative monthly data on SBE cases and SBE related mortality from January to December 2024, obtained from STRH in Kaltungo, Gombe State, Nigeria (Table 3). The human recruitment rate and natural death rate are parameterized as follows: The total population of Kaltungo is 183,000 (NPC, 2016). The Life expectancy of Nigeria as at 2021 is 63.4 years (WHO, 2024). Thus, $\mu=\frac{1}{63.4}\times\frac{1}{365}=4.3213\times10^{-5}$ per day. Also, using the relation, $N_H\le\frac{\Lambda}{\mu}$, with $N_H=183,000$, it follows that $\Lambda=183,000\times4.3213\times10^{-5}\simeq 8$ per day.

Furthermore, we employed nonlinear least squares technique in MatLab programming software to optimize model parameters, yielding the best fit. The simulation results, illustrating the cumulative number of SBE cases and SBE-induced mortality, are displayed in Figure 2. The root mean squared error (RMSE) was employed to evaluate the model's performance in terms of goodness-of-fit and predictive accuracy. The resulting RMSE value of 0.29 indicates a strong alignment between the model's projections and the observed data, as values closer to zero signify better fit. This outcome suggests that the model is capable of accurately capturing the dynamics of SBE and can be reliably used for predictive purposes and further studies on SBE within communities.

Table 3: Reported Data on SBE from STRH, Kaltungo, Gombe State, 2024.

Month	SBE Cases	Cumulative SBE	SBE Deaths	Cumulative SBE Deaths
January	69	69	0	0
February	95	164	1	1
March	213	377	0	1
April	282	659	5	6
May	180	839	0	6
June	183	1022	3	9
July	198	1220	8	17
August	241	1461	15	32
September	237	1698	10	42
October	229	1927	9	51
November	173	2100	7	58
December	92	2192	0	58



(a) (b)

Figure 2: Graph showing the results of the model fitting using reported data collected from STRH, Kaltungo, Gombe State Nigeria **(a)** Cases of SBE **(b)** Cases of SBE-induced mortality. In Figures (a) and (b) the red lines represent the cumulative monthly reported data, while the blue dotted lines indicate the estimated values predicted by the model.

Table 4: Initial values for state variables of the model

State variable	Initial Value	Reference
$S(0)$	1.4075×10^4	Fitted
$E(0)$	69	STRH, Kaltungo, Table3
$T_T(0)$	25	Fitted
$H_T(0)$	40	Fitted
$R(0)$	10	Fitted
$D(0)$	0	STRH, Kaltungo, Table 3
$V_S(0)$	96	Fitted

Table 5: Initial values of parameters of the model

Parameter	Baseline value (per day)	Reference
Λ	8	NPC, 2016; WHO, 2024
Λ_S	41	Fitted
μ	4.3213×10^{-5}	WHO, 2024
μ_S	2.283×10^{-4}	Abdullahi *et al.*, 2021
β	0.0742	Abdullahi *et al.*, 2021
$\tau_T(\tau_H)$	0.302(0.8681)	Fitted
$\delta_E(\delta_T)\delta_H$	0.05(0.04)0.0101	Fitted
$\gamma_T(\gamma_H)$	0.6962(0.9403)	Fitted
$\sigma_T(\sigma_H)$	0.9152(0.0783)	Fitted
α	6.8192×10^{-4}	Fitted

4.2 Numerical Simulations of the Model

The numerical simulations of the model were carried out using MatLab programming software to assess the effectiveness of the hospital and traditional-based treatments taking into account the SBE recovery outcomes and SBE-induced mortality as health benefits. The initial values of state variables and parameters in Tables 4 and 5 respectively, were employed in the simulations.

4.1.1 Assessing the impact of hospital and traditional-based treatments on SBE recovery

Figure 3, illustrates the effectiveness of hospital and traditional-based treatments on the recovery of individuals from SBE.

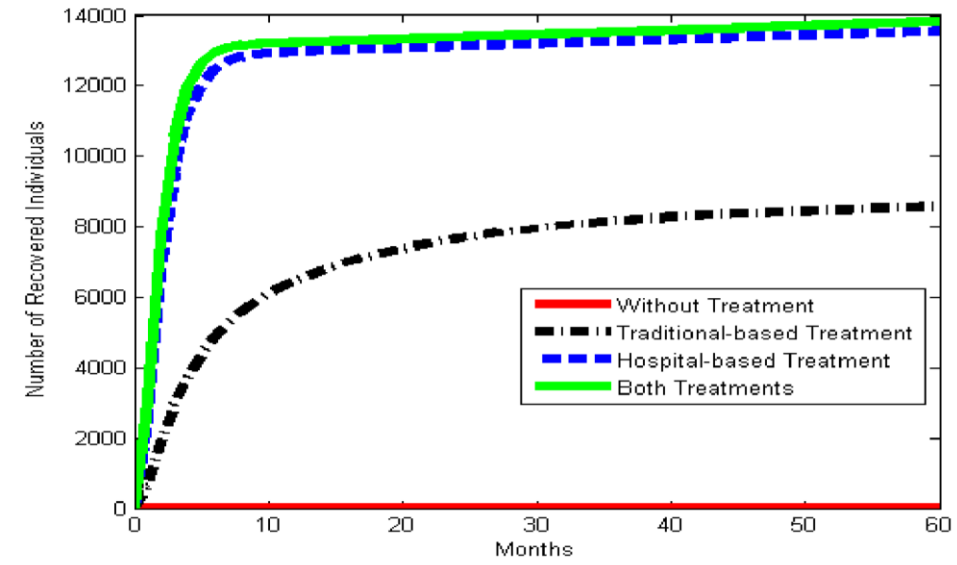


Figure 3: Simulation results showing the impact of hospital and traditional-based treatments on SBE recovery

4.1.2 Evaluating the impact of hospital and traditional-based treatments on SBE-induced deaths

This simulation assesses the effectiveness of hospital and traditional-based treatments in reducing SBE-induced deaths. The number of averted deaths is considered a key health benefit in evaluating the impact of these treatment strategies. The results of the simulation are presented in Figure 4.

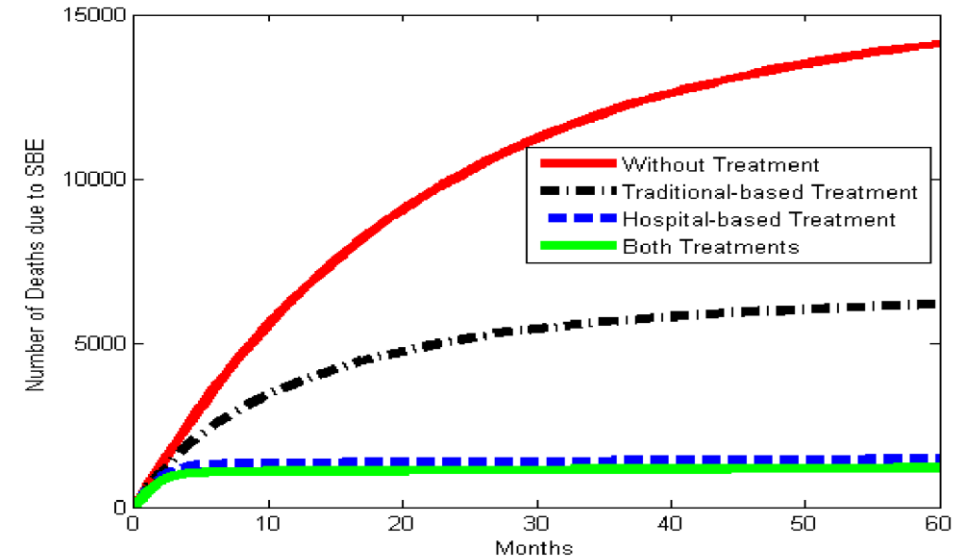


Figure 4: Simulation results showing the impact of hospital and traditional-based treatments on SBE-induced deaths

5. Discussion

The simulation results in Figure 3 illustrate the effectiveness of hospital and traditional-based treatments on the recovery of individuals from SBE. Notably, a combined approach, utilizing both hospital and traditional-based treatments, yields the most favourable outcome, resulting in the highest number of recovered individuals. The main findings of this simulation highlight the benefits of a combined treatment approach, integrating hospital and traditional-

based treatments, which produces the best recovery outcomes. This underscores the potential benefits of a holistic treatment strategy. Hospital treatment alone is the second-most effective approach, emphasizing the importance of modern medical interventions in SBE recovery. While traditional-based treatment alone is less effective, it still demonstrates some benefits, suggesting a potential role for traditional practices in SBE management. These findings imply that a comprehensive treatment strategy, incorporating both hospital and traditional-based approaches, may be the most effective way to improve recovery outcomes for SBE patients. On the other hand, Figure 4 presents the simulation results assessing the effectiveness of hospital and traditional-based treatments in reducing SBE-induced deaths. The results demonstrate that combining hospital and traditional-based treatments yields the most significant reduction in SBE-induced deaths, highlighting the benefits of a holistic treatment approach. Hospital treatment alone is the second-most effective strategy in preventing SBE-induced deaths, emphasizing the critical role of modern medical interventions. Although traditional-based treatment is less effective, it still contributes to reducing SBE-induced deaths, suggesting a complementary role for traditional practices. The implications of these findings suggest that an integrated treatment strategy, combining hospital and traditional-based approaches, may be the most effective way to reduce SBE-induced deaths. This integrated approach could potentially lead to better health outcomes, reduced disability, and mortality rates associated with SBE.

6. Conclusion

A mathematical model for assessing the impact of hospital and traditional-based treatment on the dynamics of snakebite envenoming has been developed and analysed. The model incorporates SBE epidemiological data, treatment patterns, and outcomes to quantify the effectiveness of different treatment approaches and identify optimal SBE management strategies. The basic properties of the model were explored, and the model was also fitted using reported data on SBE cases collected from STRH Kaltungo, Gombe State. Some of the main theoretical and numerical findings of the study are presented as follows:

(i) The mathematical model formulated in this work has two positive equilibrium points: the SBE-free and SBE-endemic equilibriums.

(ii) The SBE-free equilibrium is globally asymptotically stable, provided that the contact rate between venomous snakes and humans is set to zero. This implies that a community free of SBE depends on measures taken to reduce contact between venomous snakes and humans.

(iii) The SBE-endemic equilibrium is globally asymptotically stable, provided that the contact rate between venomous snakes and humans is greater than zero. This implies that when venomous snakes interact with humans, SBE will persist in the community.

(iv) The combination of both hospital and traditional-based treatments produces the best recovery outcomes and yields the most significant reduction in SBE-induced deaths. This highlights the potential benefits of integrating traditional-based treatment into SBE management.

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