



Mathematical Modelling and Analysis of the Dynamics of Corruption Incorporating Anti – Corruption Agencies

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ABSTRACT

Corruption is a global problem that affects many countries, undermining economic, social, and political development. To better understand the control measures that can reduce corruption transmission, we have formulated and analyzed a mathematical model incorporating anti – corruption agencies. To verify the model's validity, we explored the basic properties of the model in terms of existence, uniqueness, positivity and boundedness, and results showed that the solution exists, is unique, positive and bounded. The basic reproduction number, R_0 , was computed using the next-generation matrix method. Furthermore, the model was studied analytically to explore corruption dynamics. The stability analysis of the formulated model showed that the corruption-free equilibrium is locally and globally asymptotically stable if, $R_0 < 1$ but the corruption-endemic equilibrium is globally asymptotically stable if $R_0 > 1$. A sensitivity analysis of the model parameters with respect to the threshold quantity measuring the spread of corruption was conducted to identify the most influential parameters driving the spread of corruption. The results revealed that reducing the values of parameters with positive indices will contribute in controlling the spread of corruption. Conversely, the sensitivity results suggest that increasing the values of parameters with negative indices will also help to mitigate the spread of corruption.

1. Introduction

Corruption is a social problem that persists in most organizations, particularly in government, where public servants with influence or power are expected to provide services to the citizens they serve (Fori, 2024). Transparency International adopted the technical definition of corruption by Eicher (2009): “Corruption is the abuse of entrusted power for private gain.” Corruption in public service results in revenue losses for government, degeneration of social justice, violation of human rights, and exploitation of vulnerable people (Verma & Sengupta, 2015). Many countries around the world suffer from deep-rooted corruption that hampers economic development, undermines democracy, and damages social justice and the rule of law. It is a complex phenomenon with economic, social, political, and cultural dimensions that cannot easily be eliminated (Hathroubi, 2013). Furthermore, it is clearly a result of an absence of accountability, law, and order. Forms of corruption include bribery, fraud, embezzlement, extortion, nepotism, and favoritism. Therefore, there is need to develop a mathematical model that will help in mitigate this menace of corruption.

Nowadays, mathematical modelers are becoming more realistic based on many social problems. Some mathematical modelers consider socio-economic problems such as rumor spreading, poverty, crime, prostitution,

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corruption, etc. as epidemics (Musa & Fori, 2019). In addition, several studies have proposed mathematical models to understand and analyze the dynamics of corruption (Fori, 2024). For instance, Abdulrahman (2014) developed a mathematical model using a deterministic model approach with constant recruitment rates and a standard incidence rate for the transmission dynamics of corruption as a disease. Legesse and Shiferaw (2018) proposed a mathematical model for corruption by considering awareness created by anti-corruption efforts and counseling in jail. Adeyemi and Victor (2020) proposed and analyzed a mathematical model of the transmission dynamics of corruption among the populace. Alemneh (2020) proposed a nonlinear deterministic model for the dynamics of corruption and analyzed qualitatively using the stability theory of differential equations. Mokaya *at el.* (2021) developed and studied a deterministic model for the spread of corrupt morals that involves a group of people who are going through a counseling and guidance procedure. Zerihum and Abyneh (2022) developed a mathematical model for the dynamics of corruption transmission incorporating media coverage. Abayneh and Zerihun (2022) proposed a deterministic mathematical model that explains the transmission dynamics of corruption by considering social influence on honest individuals and analyzed the model. Rwat *et al.* (2023) developed mathematical models and analyzed it to investigate the transmission of corruption within population. Alope (2023) proposed to combat corruptions in Nigeria system by examine the dynamics of corruption and control measures. The dynamics of the corruption model were described by the susceptible – Exposed – Corrupt – Jail – Honest (SECJH) model using linear ODEs. Alhassan *et al.* (2024) investigate the impact of corruption by developing a mathematical model that describe it dynamics. Fori (2024) proposed a deterministic mathematical model that explained the dynamic of corruption using epidemiological models' approach by considering Loss of Immunity of Ex – Convict.

In this study, we have taken into cognizance the role of anti-corruption agencies that are responsible for detecting and arresting individuals accused of corruption, the trial compartment which helps in identifying corrupt individuals, and the jail compartment that serves as an avenue for punishing corrupt individuals, which make our model to be different.

The rest of the paper is organized as follows: Section 2 is dedicated to model formulation. Section 3 is dedicated to the basic properties of the model, Section 4 is devoted to the equilibrium points of the model. Section 5 is dedicated to sensitivity analysis of the model. Lastly, the conclusion is presented in Section 6.

2. Model Formulation

The total population at time t , denoted by $N(t)$, is divided into eight non-intersecting compartments; population of susceptible individuals, $S(t)$, population of exposed individuals $E(t)$, population of individuals accused of corruption, $C(t)$, population of individuals on trial $T(t)$, population of jailed individuals $J(t)$, population of reformed individuals, $R(t)$, population of honest individuals, $H(t)$ and the population of anti-corruption agencies, $Q(t)$.

In the formulation of the model, the following are the main assumptions:

- i. constant recruitment in both the susceptible compartment and the anti-corruption agencies;
- ii. honest individuals are incorruptible;
- iii. susceptible individual is assumed to be exposed to corruption when there is contact with susceptible and corrupt individual,
- iv. it is assumed that some proportion of the susceptible individuals will become honest after receiving awareness of the dangers and consequences of corruption;
- v. Following awareness initiatives from anti-corruption agencies, it's assumed that a proportion of individuals exposed to corruption will reform;

- vi. individuals accused of corruption remain in the accused compartment until they are detected by law enforcement agency before moving to Trial compartment;
- vii. individuals in the jail compartment are assumed to be reformed after serving their jail time.

2.1 Model diagram

The flow diagram in Figure (1) describe the transition of individuals from one compartment to another. In the diagram, the dotted line represent interaction between anti-corruption agencies and accused corrupt individuals to enable arrest. The solid arrows represent movement of individual from one compartment to another. The flow diagram in Figure 1 describes the transition of individuals from one compartment to another. In the diagram, the dotted line represents the interaction between anti-corruption agencies and accused corrupt individuals to enable arrest. The solid arrows represent the movement of individuals from one compartment to another.

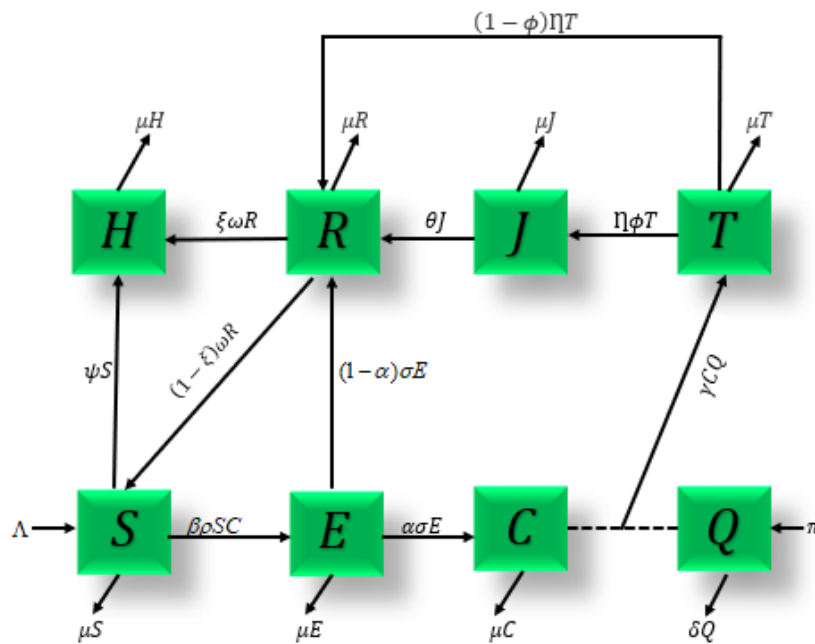


Figure 1: Flow Diagram of the Model

2.2 Model Equations

Based on the aforementioned description of the model in subsection (2), the mathematical model that describe the dynamics of corruption is represented by the following system of first order nonlinear ordinary differential equations.

$$\begin{aligned}
 \frac{dS}{dt} &= \Lambda + (1 - \xi)\omega R - \rho\beta SC - (\psi + \mu)S \\
 \frac{dE}{dt} &= \rho\beta SC - (\sigma + \mu)E \\
 \frac{dC}{dt} &= \alpha\sigma E - \gamma CQ - \mu C \\
 \frac{dT}{dt} &= \gamma CQ - (\eta + \mu)T \\
 \frac{dJ}{dt} &= \eta\phi T - (\theta + \mu)J \\
 \frac{dR}{dt} &= \theta J + (1 - \phi)\eta T + (1 - \alpha)\sigma E - (\omega + \mu)R \\
 \frac{dH}{dt} &= \psi S + \xi\omega R - \mu H \\
 \frac{dQ}{dt} &= \pi - \delta Q
 \end{aligned}
 \tag{1}$$

$$N = S + E + C + T + J + R + H \tag{2}$$

Subject to the initial conditions

$$\begin{aligned}
 S(0) > 0, \quad E(0) \geq 0, \quad C(0) \geq 0, \quad T(0) \geq 0, \\
 J(0) \geq 0, \quad R(0) \geq 0, \quad H(0) \geq 0, \quad Q(0) \geq 0
 \end{aligned}
 \tag{3}$$

Table 1: Parameters of the Modified Model

Parameters	Description
ρ	The probability of corruption transmission per contact between individuals
β	The rate of contact between susceptible individuals and those accused of corruption
α	The proportion of exposed individuals who progress to the accused of corruption subpopulation
σ	Rate at which exposed individuals progress to the accused of corruption subpopulation
γ	Rate at which individuals accused of corruption are arrested for trial
η	The rate at which individuals who are on trial for corruption are sentenced to jail
θ	Rate at which jailed individuals are reformed
ω	Rate at which reformed individual become honest
ξ	Proportion of individuals that move to the honest subpopulation from reformed compartment
ψ	Proportion of individuals that move to the honest subpopulation from susceptible compartment
δ	Decay rate of anti-corruption agencies
μ	Natural death rate
ϕ	The proportion of individuals on trial who are sentenced to jail
Λ	Recruitment rate into the susceptible compartment
π	Influx rate of anti-corruption agencies

3. Basic Properties of the Model

In this subsection, the basic properties of the model will be explored, including the existence and uniqueness of the model solution, invariant region and positivity of the solution.

3.1 Existence and uniqueness of solution of the model

The existence and uniqueness of solution of the model will be established. For the purpose of investigating the existence and uniqueness of the model solution the following representation are made:

$$\text{let } x_1 = S, x_2 = E, x_3 = C, x_4 = T, x_5 = J, x_6 = R, x_7 = H, x_8 = Q$$

So that the model equation given by system (1) can be rewritten in a compact form as follows:

$$\frac{dx}{dt} = f(t, x), x(t_0) = x_0, \quad (4)$$

Where,

$$f = (f_1, f_2, \dots, f_8)^T, \quad x = (x_1, x_2, \dots, x_8)^T, \quad x_0 = (x_{01}, x_{02}, \dots, x_{08})^T$$

Theorem 1

Let \mathfrak{D} denoted the domain

$$|t - t_0| \leq a, \quad \|x - x_0\| \leq b, \quad x = (x_1, x_2, \dots, x_8), \quad x_0 = (x_{01}, x_{02}, \dots, x_{08}), \quad (5)$$

and suppose that $f(t, x)$ satisfies the Lipchitz condition

$$\|f(t, x_1) - f(t, x_2)\| \leq k \|x_1 - x_2\| \quad (6)$$

and whenever the pairs (t, x_1) and (t, x_2) belong to a domain \mathfrak{D} and k is use to represents a positive constant. Then, there exist a constant $\delta > 0$ such that there is a unique (exactly one) continuous vector solution $x(t)$ of the model equation given by system of equation (4) in the interval $|t - t_0| \leq \delta$.

Proof

From the model equation given by system (1), let

$$f_1 = \Lambda - \rho\beta SC + (1 - \xi)\omega R - (\psi + \mu)S$$

$$f_2 = \rho\beta SC - (\sigma + \mu)E$$

$$f_3 = \alpha\sigma E - \gamma CQ - \mu C$$

$$f_4 = \gamma CQ - (\eta + \mu)T$$

$$f_5 = \eta\phi T - (\theta + \mu)J$$

$$f_6 = \theta J + (1 - \phi)\eta T + (1 - \alpha)\sigma E - (\omega + \mu)R$$

$$f_7 = \psi S + \xi\omega R - \mu H$$

$$f_8 = \pi - \delta Q$$

For the functions (f_1, f_2, \dots, f_8) to satisfy Lipchitz condition it suffices to show that the partial derivatives $\frac{\partial f_i}{\partial x_j}, i, j = 1, 2, 3, \dots, 8$ are continuous and bounded in the region \mathfrak{D} . We compute the partial derivatives as follows:

For f_1 we have:

$$\begin{aligned} \left| \frac{\partial f_1}{\partial S} \right| &= |-\rho\beta C - (\psi + \mu)| < \infty, \quad \left| \frac{\partial f_1}{\partial C} \right| = |-\rho\beta S| < \infty, \quad \left| \frac{\partial f_1}{\partial R} \right| = |-(1 - \xi)\omega| < \infty \\ \left| \frac{\partial f_1}{\partial E} \right| &= \left| \frac{\partial f_1}{\partial T} \right| = \left| \frac{\partial f_1}{\partial J} \right| = \left| \frac{\partial f_1}{\partial H} \right| = \left| \frac{\partial f_1}{\partial Q} \right| = 0 < \infty. \end{aligned} \quad (7)$$

For f_2 we have

$$\begin{aligned} \left| \frac{\partial f_2}{\partial S} \right| &= |\rho\beta C| < \infty, \left| \frac{\partial f_2}{\partial E} \right| = |-(\sigma + \mu)| < \infty, \left| \frac{\partial f_2}{\partial C} \right| = |\rho\beta S| < \infty, \\ \left| \frac{\partial f_2}{\partial T} \right| &= \left| \frac{\partial f_2}{\partial J} \right| = \left| \frac{\partial f_2}{\partial R} \right| = \left| \frac{\partial f_2}{\partial H} \right| = \left| \frac{\partial f_2}{\partial Q} \right| = 0 < \infty. \end{aligned} \quad (8)$$

For f_3 we have

$$\begin{aligned} \left| \frac{\partial f_3}{\partial E} \right| &= |(\alpha\sigma)| < \infty, \left| \frac{\partial f_3}{\partial C} \right| = |-(\gamma Q + \mu)| < \infty, \left| \frac{\partial f_3}{\partial Q} \right| = |-\gamma C| < \infty, \\ \left| \frac{\partial f_3}{\partial S} \right| &= \left| \frac{\partial f_3}{\partial T} \right| = \left| \frac{\partial f_3}{\partial J} \right| = \left| \frac{\partial f_3}{\partial R} \right| = \left| \frac{\partial f_3}{\partial H} \right| = 0 < \infty. \end{aligned} \quad (9)$$

For f_4 we have

$$\begin{aligned} \left| \frac{\partial f_4}{\partial C} \right| &= |\gamma Q| < \infty, \left| \frac{\partial f_4}{\partial T} \right| = |-(\eta + \mu)| < \infty, \left| \frac{\partial f_4}{\partial Q} \right| = |\gamma C| < \infty, \\ \left| \frac{\partial f_4}{\partial S} \right| &= \left| \frac{\partial f_4}{\partial E} \right| = \left| \frac{\partial f_4}{\partial J} \right| = \left| \frac{\partial f_4}{\partial R} \right| = \left| \frac{\partial f_4}{\partial H} \right| = 0 < \infty. \end{aligned} \quad (10)$$

For f_5 we have

$$\begin{aligned} \left| \frac{\partial f_5}{\partial T} \right| &= |\eta\theta| < \infty, \left| \frac{\partial f_5}{\partial J} \right| = |-(\theta + \mu)| < \infty, \\ \left| \frac{\partial f_5}{\partial S} \right| &= \left| \frac{\partial f_5}{\partial E} \right| = \left| \frac{\partial f_5}{\partial C} \right| = \left| \frac{\partial f_5}{\partial R} \right| = \left| \frac{\partial f_5}{\partial H} \right| = \left| \frac{\partial f_5}{\partial Q} \right| = 0 < \infty. \end{aligned} \quad (11)$$

For f_6 we have

$$\begin{aligned} \left| \frac{\partial f_6}{\partial E} \right| &= |(1 - \alpha)\sigma| < \infty, \left| \frac{\partial f_6}{\partial T} \right| = |(1 - \phi)\eta| < \infty, \left| \frac{\partial f_6}{\partial J} \right| = |\theta| < \infty, \\ \left| \frac{\partial f_6}{\partial R} \right| &= |-(\omega + \mu)| < \infty, \left| \frac{\partial f_6}{\partial S} \right| = \left| \frac{\partial f_6}{\partial C} \right| = \left| \frac{\partial f_6}{\partial H} \right| = \left| \frac{\partial f_6}{\partial Q} \right| = 0 < \infty. \end{aligned} \quad (12)$$

For f_7 we have

$$\begin{aligned} \left| \frac{\partial f_7}{\partial S} \right| &= |\psi| < \infty, \left| \frac{\partial f_7}{\partial R} \right| = |\xi\omega| < \infty, \left| \frac{\partial f_7}{\partial H} \right| = |-\mu| < \infty, \\ \left| \frac{\partial f_7}{\partial E} \right| &= \left| \frac{\partial f_7}{\partial C} \right| = \left| \frac{\partial f_7}{\partial T} \right| = \left| \frac{\partial f_7}{\partial J} \right| = \left| \frac{\partial f_7}{\partial Q} \right| = 0 < \infty \end{aligned} \quad (13)$$

For f_8 we have

$$\left| \frac{\partial f_8}{\partial Q} \right| = |-\delta| < \infty, \left| \frac{\partial f_8}{\partial S} \right| = \left| \frac{\partial f_8}{\partial E} \right| = \left| \frac{\partial f_8}{\partial C} \right| = \left| \frac{\partial f_8}{\partial T} \right| = \left| \frac{\partial f_8}{\partial J} \right| = \left| \frac{\partial f_8}{\partial R} \right| = \left| \frac{\partial f_8}{\partial H} \right| = 0 < \infty. \quad (14)$$

From equations (7) to (14), it is clear that all the partial derivatives are continuous and bounded in the region \mathfrak{D} . Hence, there exist a constant $\delta > 0$ such that a unique continuous vector solution $x(t)$ of the model equation given by system of equation (4) in the interval $|t - t_0| \leq \delta$ exists.

3.2 The invariant region of the model

The invariant region of the model will be examined through the following theorem:

Theorem 2

The closed set $\mathfrak{D} = \mathfrak{D}_1 \cup \mathfrak{D}_2 \subset \mathbb{R}_+^7 \times \mathbb{R}_+$, where $\mathfrak{D}_1 = \{(S, E, C, T, J, R, H) \in \mathbb{R}_+^7 : N \leq \frac{\Lambda}{\mu}\}$ and $\mathfrak{D}_2 = \{Q \in \mathbb{R}_+ : Q \leq \frac{\pi}{\delta}\}$ is positively invariant and attracting with respect to the model equations given by system (1).

Proof

Consider equation (2), i.e. $N = S + E + C + T + J + R + H$. Differentiating equation (2) with respect to t we have

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dC}{dt} + \frac{dT}{dt} + \frac{dJ}{dt} + \frac{dR}{dt} + \frac{dH}{dt} \quad (15)$$

By substituting the right-hand side of equations (1) in equation (15), and simplifying it, we have

$$\frac{dN}{dt} = \Lambda - \mu(S + E + C + T + J + R + H) \quad (16)$$

substitute equation (2) in equation (16), we have:

$$\frac{dN}{dt} = \Lambda - \mu N \quad (17)$$

Using integrating factor method to solve equation (17), we have

$$N(t) = \frac{\Lambda}{\mu} + \left[N(0) - \frac{\Lambda}{\mu} \right] e^{-\mu t} \quad (18)$$

Now, taking the limit of $N(t)$ as $t \rightarrow \infty$, we have

$$\lim_{t \rightarrow \infty} N(t) \leq \frac{\Lambda}{\mu} \quad (19)$$

Thus, $N(t) \leq \frac{\Lambda}{\mu}$.

Similarly, consider last equation in system (1),

$$\frac{dQ}{dt} = \pi - \delta Q \quad (20)$$

Using integrating factor method to solve equation (20), we have

$$Q(t) = \frac{\pi}{\delta} + \left[Q(0) - \frac{\pi}{\delta} \right] e^{-\delta t} \quad (21)$$

Taking the limit of $Q(t)$ as $t \rightarrow \infty$, we have $Q(t) \leq \frac{\pi}{\delta}$.

3.3 Positivity of the Model

Since, the model monitor human population it is important we show that for a given positive initial data the solution of the model will remain positive for all time $t > 0$.

Theorem 3

Let the initial data of the model equation given by system (1) be positive, then the solution (S, E, C, Q, T, J, R, H) of the model will remain positive for all time $t > 0$.

Proof

Let $t^* = \sup\{t > 0 : S > 0, E > 0, C > 0, T > 0, R > 0, H > 0, Q > 0\}$. Thus, $t^* > 0$.

We assumed that all the state variables are continues. Then, from the system of equation (1), one can easily obtained that

$$\left. \begin{aligned} \frac{dS}{dt} &\geq -(\beta\rho C + \psi + \mu)S \\ \frac{dE}{dt} &\geq -(\sigma + \mu)E \\ \frac{dC}{dt} &\geq -(\gamma Q + \mu)C \\ \frac{dT}{dt} &\geq -(\eta + \mu)T \\ \frac{dJ}{dt} &\geq -(\theta + \mu)J \\ \frac{dR}{dt} &\geq -(\omega + \mu)R \\ \frac{dH}{dt} &\geq -\mu H \\ \frac{dQ}{dt} &\geq -\delta Q \end{aligned} \right\} \quad (22)$$

Solving system (22), we obtained

$$\left. \begin{aligned}
 S(t^*) &\geq S(0)e^{-\left[(\psi+\mu)t^* + \beta\rho\int_0^{t^*} C(u)du\right]} > 0 \\
 E(t^*) &\geq E(0)e^{-(\sigma+\mu)t^*} > 0 \\
 C(t^*) &\geq C(0)e^{-\left[\gamma\int_0^{t^*} Q(u)du + \mu t^*\right]} > 0 \\
 T(t^*) &\geq T(0)e^{-(\eta+\mu)t^*} > 0 \\
 J(t^*) &\geq J(0)e^{-(\theta+\mu)t^*} > 0 \\
 R(t^*) &\geq R(0)e^{-(\omega+\mu)t^*} > 0 \\
 H(t^*) &\geq H(0)e^{-\mu t^*} > 0 \\
 Q(t^*) &\geq Q(0)e^{-\delta t^*} > 0
 \end{aligned} \right\} \quad (23)$$

Since $S(t^*) > 0$, $E(t^*) > 0$, $C(t^*) > 0$, $T(t^*) > 0$, $J(t^*) > 0$, $R(t^*) > 0$, $H(t^*) > 0$, $Q(t^*) > 0$, it implies $S(t) > 0$, $E(t) > 0$, $C(t) > 0$, $T(t) > 0$, $J(t) > 0$, $R(t) > 0$, $H(t) > 0$, $Q(t) > 0$ for all time $t > 0$. Hence, all the state variables were shown to be positive in \mathbb{R}_+^n for all $t > 0$ and hence the proof is complete.

4. Equilibrium points of the model

In this section, we discuss the corruption Free Equilibrium (CFE) and corruption endemic equilibrium points of the model and its analysis.

4.1 Corruption-free equilibrium point (CFE)

The corruption – free equilibrium point represents a situation where there is no indulge in corruption in the population. Thus, the CFE of the model (1) is attained when all the variables and parameters related to corruption are set to zero. Therefore, setting $E^0 = C^0 = T^0 = J^0 = R^0 = 0$ in equation (1) we have.

$$\left. \begin{aligned}
 \Lambda - (\psi + \mu)S^0 &= 0 \\
 \psi S^0 - \mu H^0 &= 0 \\
 \pi - \delta Q^0 &= 0
 \end{aligned} \right\} \quad (24)$$

Solving (24) for S , H and Q , we obtain: $S^0 = \frac{\Lambda}{\psi + \mu}$, $H^0 = \frac{\psi\Lambda}{\mu(\psi + \mu)}$ and $Q^0 = \frac{\pi}{\delta}$

Let $E_1 = (S^*, E^*, C^*, T^*, J^*, R^*, H^*, Q^*)$ denotes the CFE of the model. Then, we have

$E_1 = (S^*, E^*, C^*, J^*, R^*, H^*, Q^*) = \left(\frac{\Lambda}{\psi + \mu}, 0, 0, 0, 0, 0, \frac{\psi\Lambda}{\mu(\psi + \mu)}, \frac{\pi}{\delta} \right)$ as the corruption free – equilibrium point.

4.2 Corruption endemic equilibrium point of the model (CEE)

The corruption endemic equilibrium point represents persistence of corruption in the population. To obtain this equilibrium point we assume that

$$S^0 > 0, E^0 > 0, C^0 > 0, T^0 > 0, J^0 > 0, R^0 > 0, H^0 > 0, Q^0 > 0, \text{ in equation (1).}$$

Let $E_2 = (S^{**}, E^{**}, C^{**}, T^{**}, J^{**}, R^{**}, H^{**}, Q^{**})$ denotes the corruption endemic equilibrium point of the model,

where,

$$S^{**} = \frac{\Lambda + (1 - \xi)\omega R^{**}}{\rho\beta C^{**} + (\psi + \mu)}, \quad E^{**} = \frac{\rho\beta C^{**} S^{**}}{(\sigma + \mu)}, \quad C^{**} = \frac{\alpha\sigma E^{**}}{(\gamma Q^{**} + \mu)}, \quad T^{**} = \frac{\gamma C^{**} Q^{**}}{\eta + \mu},$$

$$J^{**} = \frac{\eta\phi T^{**}}{\theta + \mu}, \quad R^{**} = \frac{\theta J^{**} + (1 - \phi)\eta T^{**} + (1 - \alpha)\sigma E^{**}}{\omega + \mu}, \quad H^{**} = \frac{\psi S^{**} + \xi\omega R^{**}}{\mu}, \quad Q^{**} = \frac{\pi}{\delta}.$$

4.3 Basic reproduction number of the model

We apply the next generation matrix technique by Dickmann *et al.* (2009) to calculate the threshold quantity that determines the spread of corruption and is denoted by R_0 , by considering the corruption related compartments of system (1). Let F_i be the rate of appearance of new corrupted individuals in the i compartment and V_i be the rate of transfer of individuals from i compartment, given the corruption – free equilibrium E_1 , then R_0 is the spectral radius (largest eigen value) of the next generation matrix denoted by $K = FV^{-1}$. We consider the following equations for the dynamics of E, C, T and J :

$$\left. \begin{aligned} \frac{dE}{dt} &= \rho\beta SC - (\sigma + \mu)E \\ \frac{dC}{dt} &= \alpha\sigma E - (\gamma Q - \mu)C \\ \frac{dT}{dt} &= \gamma CQ - (\eta + \mu)T \\ \frac{dJ}{dt} &= \eta\phi T - (\theta + \mu)J \end{aligned} \right\} \quad (25)$$

Now, matrix F_i = rate of appearance of new corrupt individuals into the compartment i , and V_i = rate of transfer into (out) of the compartment i . Thus, we have

$$F_i = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} \beta\rho SC \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$V_i = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} (\sigma + \mu)E \\ -\alpha\sigma E + (\gamma Q + \mu)C \\ -\gamma C Q + (\eta + \mu)T \\ -\eta\phi T + (\theta + \mu)J \end{pmatrix}$$

$$F = \frac{\partial f_i(E_1)}{\partial x_i} = \begin{pmatrix} \frac{\partial f_1(E_1)}{\partial E} & \frac{\partial f_1(E_1)}{\partial C} & \frac{\partial f_1(E_1)}{\partial T} & \frac{\partial f_1(E_1)}{\partial J} \\ \frac{\partial f_2(E_1)}{\partial E} & \frac{\partial f_2(E_1)}{\partial C} & \frac{\partial f_2(E_1)}{\partial T} & \frac{\partial f_2(E_1)}{\partial J} \\ \frac{\partial f_3(E_1)}{\partial E} & \frac{\partial f_3(E_1)}{\partial C} & \frac{\partial f_3(E_1)}{\partial T} & \frac{\partial f_3(E_1)}{\partial J} \\ \frac{\partial f_4(E_1)}{\partial E} & \frac{\partial f_4(E_1)}{\partial C} & \frac{\partial f_4(E_1)}{\partial T} & \frac{\partial f_4(E_1)}{\partial J} \end{pmatrix} = \begin{pmatrix} 0 & \Lambda\beta\rho & 0 & 0 \\ \psi + \mu & & & \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$V = \frac{\partial V_i(E_1)}{\partial x_i} = \begin{pmatrix} \frac{\partial V_1(E_1)}{\partial E} & \frac{\partial V_1(E_1)}{\partial C} & \frac{\partial V_1(E_1)}{\partial T} & \frac{\partial V_1(E_1)}{\partial J} \\ \frac{\partial V_2(E_1)}{\partial E} & \frac{\partial V_2(E_1)}{\partial C} & \frac{\partial V_2(E_1)}{\partial T} & \frac{\partial V_2(E_1)}{\partial J} \\ \frac{\partial V_3(E_1)}{\partial E} & \frac{\partial V_3(E_1)}{\partial C} & \frac{\partial V_3(E_1)}{\partial T} & \frac{\partial V_3(E_1)}{\partial J} \\ \frac{\partial V_4(E_1)}{\partial E} & \frac{\partial V_4(E_1)}{\partial C} & \frac{\partial V_4(E_1)}{\partial T} & \frac{\partial V_4(E_1)}{\partial J} \end{pmatrix} = \begin{bmatrix} \sigma + \mu & 0 & 0 & 0 \\ -\alpha\sigma & \left(\frac{\gamma\pi + \delta\mu}{\delta}\right) & 0 & 0 \\ 0 & -\frac{\gamma\pi}{\delta} & \eta + \mu & 0 \\ 0 & 0 & -\eta\phi & \theta + \mu \end{bmatrix}$$

We now compute V^{-1} using the formula $V^{-1} = \frac{1}{\det(V)} (\text{adj}(V))$, we have

$$V^{-1} = \begin{pmatrix} \frac{(\gamma\pi + \delta\mu)(\eta + \mu)(\theta + \mu)}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & 0 & 0 & 0 \\ \frac{\alpha\sigma\delta(\eta + \mu)(\theta + \mu)}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & \frac{\delta(\sigma + \mu)(\eta + \mu)(\theta + \mu)}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & 0 & 0 \\ \frac{\alpha\sigma\gamma\pi(\theta + \mu)}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & \frac{\gamma\pi(\sigma + \mu)(\theta + \mu)}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & \frac{(\gamma\pi + \delta\mu)(\sigma + \mu)(\theta + \mu)}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & 0 \\ \frac{\alpha\sigma\gamma\pi\eta\phi}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & \frac{\eta\phi\delta(\sigma + \mu)}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & \frac{\mu\eta\phi\delta(\sigma + \mu)}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & \frac{(\gamma\pi + \delta\mu)(\sigma + \mu)(\eta + \mu)}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} \end{pmatrix} \quad (26)$$

It follows that

$$FV^{-1} = \begin{pmatrix} 0 & \frac{\beta\rho S^o}{\psi + \mu} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sigma + \mu} & 0 & 0 & 0 \\ \frac{\alpha\sigma\delta}{(\gamma\pi + \delta\mu)(\sigma + \mu)} & \frac{\delta}{\gamma\pi + \delta\mu} & 0 & 0 \\ \frac{\alpha\sigma\gamma\delta}{(\gamma\pi + \delta\mu)(\sigma + \mu)(\eta + \mu)} & \frac{\gamma\pi}{(\gamma\pi + \delta\mu)(\theta + \mu)} & \frac{1}{\eta + \mu} & 0 \\ \frac{\alpha\sigma\gamma\pi\eta\phi}{(\gamma\pi + \delta\mu)(\theta + \mu)(\sigma + \mu)(\eta + \mu)} & \frac{\gamma\pi\eta\phi}{(\gamma\pi + \delta\mu)(\theta + \mu)(\eta + \mu)} & \frac{\eta\phi}{(\theta + \mu)(\eta + \mu)} & \frac{1}{\theta + \mu} \end{pmatrix} \quad (27)$$

Simplifying equation (27), we have

$$FV^{-1} = \begin{pmatrix} \frac{\alpha\sigma\Lambda\beta\rho\delta}{(\gamma\pi + \delta\mu)(\sigma + \mu)(\mu + \psi)} & \frac{\Lambda\beta\rho\delta}{(\gamma\pi + \delta\mu)(\psi + \mu)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (28)$$

The characteristic equation (28) is

$$\begin{vmatrix} \frac{\alpha\sigma\Lambda\beta\rho\delta}{(\gamma\pi + \delta\mu)(\sigma + \mu)(\psi + \mu)} - \lambda & \frac{\Lambda\beta\rho\delta}{(\gamma\pi + \delta\mu)(\psi + \mu)} & 0 & 0 \\ 0 & 0 - \lambda & 0 & 0 \\ 0 & 0 & 0 - \lambda & 0 \\ 0 & 0 & 0 & 0 - \lambda \end{vmatrix} = 0 \quad (29)$$

Thus, the eigenvalues of equation (29) are as follows:

$$\lambda_1 = \frac{\alpha\sigma\Lambda\beta\rho\delta}{(\gamma\pi + \delta\mu)(\sigma + \mu)(\mu + \psi)}, \quad \lambda_2 = \lambda_3 = \lambda_4 = 0$$

Therefore, the largest eigenvalue is λ_1 . Thus, the basic reproduction number is given by

$$R_0 = \frac{\alpha\sigma\Lambda\beta\rho\delta}{(\gamma\pi + \delta\mu)(\sigma + \mu)(\mu + \psi)} \quad (30)$$

Remark 1

The implication of the threshold quantity for corruption is of two folds.

- i. If $R_0 > 1$, then at least one corrupted person will influence more than one person to engage in corruption, and thus corruption will persist in the population.
- ii. If $R_0 < 1$, then it means that less than one corrupted individual will influence less than one person, and hence, corruption will be controlled.

4.4 Local stability analysis of the corruption-free equilibrium point of the model

The local stability analysis of the corruption-free equilibrium point can be discussed by examining the linearized form of the system of equation (1) at the equilibrium point. To establish the local stability of the corruption-free equilibrium (CFE) of the system (1), we examine the eigenvalues of the Jacobian matrix of the model evaluated at the CFE.

Theorem 4

The corruption free equilibrium point E_i of the model is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$

Proof

To proof this theorem, first we obtain the Jacobian matrix of the system (1) and evaluate it at the corruption free equilibrium point. From the system (1), let

$$\begin{aligned}
 f_1 &= \Lambda + (1 - \xi)\omega R - \rho\beta SC - (\psi + \mu)S \\
 f_2 &= \rho\beta SC - (\sigma + \mu)E \\
 f_3 &= \alpha\sigma E - \gamma CQ - \mu C \\
 f_4 &= \gamma CQ - (\eta + \mu)T \\
 f_5 &= \eta\phi T - (\theta + \mu)J \\
 f_6 &= \theta J + (1 - \phi)\eta T + (1 - \alpha)\sigma E - (\omega + \mu)R \\
 f_7 &= \psi S + \xi\omega R - \mu H \\
 f_8 &= \pi - \delta Q
 \end{aligned}$$

$$\begin{aligned}
 f_1 &= \Lambda + (1 - \xi)\omega R - \rho\beta SC - (\psi + \mu)S \\
 f_2 &= \rho\beta SC - (\sigma + \mu)E \\
 f_3 &= \alpha\sigma E - \gamma CQ - \mu C \\
 f_4 &= \gamma CQ - (\eta + \mu)T \\
 f_5 &= \eta\phi T - (\theta + \mu)J \\
 f_6 &= \theta J + (1 - \phi)\eta T + (1 - \alpha)\sigma E - (\omega + \mu)R \\
 f_7 &= \psi S + \xi\omega R - \mu H \\
 f_8 &= \pi - \delta Q
 \end{aligned} \tag{31}$$

Thus, the Jacobian matrix for the system (31) is obtain as follows:

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial E} & \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial T} & \frac{\partial f_1}{\partial J} & \frac{\partial f_1}{\partial R} & \frac{\partial f_1}{\partial H} & \frac{\partial f_1}{\partial Q} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial E} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial T} & \frac{\partial f_2}{\partial J} & \frac{\partial f_2}{\partial R} & \frac{\partial f_2}{\partial H} & \frac{\partial f_2}{\partial Q} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial E} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial T} & \frac{\partial f_3}{\partial J} & \frac{\partial f_3}{\partial R} & \frac{\partial f_3}{\partial H} & \frac{\partial f_3}{\partial Q} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial E} & \frac{\partial f_4}{\partial C} & \frac{\partial f_4}{\partial T} & \frac{\partial f_4}{\partial J} & \frac{\partial f_4}{\partial R} & \frac{\partial f_4}{\partial H} & \frac{\partial f_4}{\partial Q} \\ \frac{\partial f_5}{\partial S} & \frac{\partial f_5}{\partial E} & \frac{\partial f_5}{\partial C} & \frac{\partial f_5}{\partial T} & \frac{\partial f_5}{\partial J} & \frac{\partial f_5}{\partial R} & \frac{\partial f_5}{\partial H} & \frac{\partial f_5}{\partial Q} \\ \frac{\partial f_6}{\partial S} & \frac{\partial f_6}{\partial E} & \frac{\partial f_6}{\partial C} & \frac{\partial f_6}{\partial T} & \frac{\partial f_6}{\partial J} & \frac{\partial f_6}{\partial R} & \frac{\partial f_6}{\partial H} & \frac{\partial f_6}{\partial Q} \\ \frac{\partial f_7}{\partial S} & \frac{\partial f_7}{\partial E} & \frac{\partial f_7}{\partial C} & \frac{\partial f_7}{\partial T} & \frac{\partial f_7}{\partial J} & \frac{\partial f_7}{\partial R} & \frac{\partial f_7}{\partial H} & \frac{\partial f_7}{\partial Q} \\ \frac{\partial f_8}{\partial S} & \frac{\partial f_8}{\partial E} & \frac{\partial f_8}{\partial C} & \frac{\partial f_8}{\partial T} & \frac{\partial f_8}{\partial J} & \frac{\partial f_8}{\partial R} & \frac{\partial f_8}{\partial H} & \frac{\partial f_8}{\partial Q} \end{pmatrix} \tag{32}$$

$$J(X) = \begin{pmatrix} -\rho\beta C - (\psi + \mu) & 0 & -\rho\beta S & 0 & 0 & (1-\xi)\omega & 0 & 0 \\ \rho\beta C & -(\sigma + \mu) & \rho\beta S & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha\sigma & -(\gamma Q + \mu) & 0 & 0 & 0 & 0 & -\gamma C \\ 0 & 0 & \gamma Q & -(\eta + \mu) & 0 & 0 & 0 & \gamma C \\ 0 & 0 & 0 & \eta\varphi & -(\theta + \mu) & 0 & 0 & 0 \\ 0 & (1-\alpha)\sigma & 0 & (1-\varphi)\eta & \theta & -(\omega + \mu) & 0 & 0 \\ \psi & 0 & 0 & 0 & 0 & \xi\omega & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{pmatrix} \tag{33}$$

Where $X = S, E, C, T, J, R, H, Q$

The Jacobian evaluated at the corruption free equilibrium E_1 is given by

$$J(E_1) = \begin{pmatrix} -(\psi + \mu) & 0 & -\frac{\Lambda\rho\beta}{\psi + \mu} & 0 & 0 & (1-\xi)\omega & 0 & 0 \\ 0 & -(\sigma + \mu) & \frac{\Lambda\rho\beta}{\psi + \mu} & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha\sigma & -\left(\frac{\gamma\pi + \delta\mu}{\delta}\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\gamma\pi}{\delta} & -(\eta + \mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta\phi & -(\theta + \mu) & 0 & 0 & 0 \\ 0 & (1-\alpha)\sigma & 0 & (1-\phi)\eta & \theta & -(\omega + \mu) & 0 & 0 \\ \psi & 0 & 0 & 0 & 0 & \xi\omega & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{pmatrix} \quad (34)$$

Solving the characteristic equation (34), we have

$$\begin{aligned} \lambda_1 = -\delta < 0, \quad \lambda_2 = -\mu < 0, \quad \lambda_3 = -(\psi + \mu) < 0, \quad \lambda_4 = -(\omega + \mu) < 0, \quad \lambda_5 = -(\theta + \mu) < 0 \\ \lambda_6 = -(\eta + \mu) < 0 \end{aligned} \quad (35)$$

Clearly, from equation (35) the first six eigen value satisfy the negativity requirements for local stability of CFE. Now, to obtain the remaining eigenvalues, Routh – Hurwitz criteria is applied. The characteristic polynomial of the Jacobian matrix of the equation is given

$$\lambda^2 + \left(\sigma + \mu + \frac{\pi\gamma + \delta\mu}{\delta} \right) \lambda + \left(\frac{(\sigma + \mu)(\pi\gamma + \delta\mu)}{\delta} - \frac{\alpha\sigma\Lambda\beta\rho}{\psi + \mu} \right) = 0 \quad (36)$$

Rewriting equation (35) in standard quadratic form, we have

$$a_0\lambda^2 + a_1\lambda + a_2 = 0,$$

$$\text{where } a_0 = 1, \quad a_1 = \sigma + \mu + \frac{\pi\gamma + \delta\mu}{\delta}, \quad a_2 = \frac{(\sigma + \mu)(\pi\gamma + \delta\mu)}{\delta} - \frac{\alpha\sigma\Lambda\beta\rho}{\psi + \mu}$$

The Routh – Hurwitz condition that ensure the roots of the characteristic polynomial given in equation (27) have negative real parts are: $a_i > 0$ and $\det(H_i) > 0$ for $i = 1, 2$. According to Routh – Hurwitz criteria for determining stability of equilibrium points for dynamical system, equation (27) has negative real root if $a_1 > 0$, $a_2 > 0$ and $\det(H_1) > 0$, $\det(H_2) > 0$. Now observe, from equation (37), that $a_0 = 1 > 0$, $a_1 = \sigma + \mu + \frac{\pi\gamma + \delta\mu}{\delta} > 0$, since all the model parameters are positive. Moreover, for $a_2 > 0$, we have the following simplification:

$$\begin{aligned}
 a_2 &= \frac{(\sigma + \mu)(\gamma\pi + \delta\mu)}{\delta} - \frac{\alpha\sigma\Lambda\rho\beta}{\psi + \mu} = \frac{(\sigma + \mu)(\gamma\pi + \delta\mu)}{\delta} - \frac{\alpha\sigma\Lambda\rho\beta}{\psi + \mu} \\
 &= \frac{(\sigma + \mu)(\gamma\pi + \delta\mu)}{\delta} \left[1 - \frac{\alpha\sigma\Lambda\rho\beta\delta}{(\gamma\pi + \delta\mu)(\sigma + \mu)(\psi + \mu)} \right] = \frac{(\sigma + \mu)(\gamma\pi + \delta\mu)}{\delta} [1 - R_0]
 \end{aligned}$$

Thus, $a_2 > 0$ if $R_0 < 1$.

Therefore, it follows that: $\det(H_1) = a_1 > 0$ and $\det(H_2) = \begin{vmatrix} a_1 & a_0 \\ 0 & a_2 \end{vmatrix} = a_1 a_2 > 0$, if $R_0 < 1$.

Hence, the corruption free equilibrium point given by E_1 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Remark 2

It shows that, corruption can be curtail in a given population if $R_0 < 1$ and cannot be controlled if $R_0 > 1$.

4.5 Local stability analysis of corruption-endemic equilibrium point of the model

Here, we carry out the local stability analysis of the corruption endemic equilibrium point. To study the stability of the corruption-endemic equilibrium point of the model, we will first compute the Jacobian of the model equations and then evaluate it at the equilibrium point. This will be achieved through the following theorem:

Theorem 5: The corruption endemic equilibrium point, E_2 of the model is locally asymptotically stable in the region \mathcal{D} .

Proof:

Consider the Jacobian matrix of the model equations given by equation (33)

$$J(X) = \begin{pmatrix} -\rho\beta C - (\psi + \mu) & 0 & -\rho\beta S & 0 & 0 & (1-\xi)\omega & 0 & 0 \\ \rho\beta C & -(\sigma + \mu) & \rho\beta S & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha\sigma & -(\gamma Q + \mu) & 0 & 0 & 0 & 0 & -\gamma C \\ 0 & 0 & \gamma Q & -(\eta + \mu) & 0 & 0 & 0 & \gamma C \\ 0 & 0 & 0 & \eta\varphi & -(\theta + \mu) & 0 & 0 & 0 \\ 0 & (1-\alpha)\sigma & 0 & (1-\varphi)\eta & \theta & -(\omega + \mu) & 0 & 0 \\ \psi & 0 & 0 & 0 & 0 & \xi\omega & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{pmatrix} \quad (38)$$

The Jacobian matrix evaluated at the corruption endemic equilibrium point, E_2 is given as

$$J(E_2) = \begin{pmatrix} -\rho\beta C^{**} - (\psi + \mu) & 0 & -\rho\beta S^{**} & 0 & 0 & (1-\xi)\omega & 0 & 0 \\ \rho\beta C^{**} & -(\sigma + \mu) & \rho\beta S^{**} & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha\sigma & -(\gamma Q^{**} + \mu) & 0 & 0 & 0 & 0 & -\gamma C^{**} \\ 0 & 0 & \gamma Q^{**} & -(\eta + \mu) & 0 & 0 & 0 & \gamma C^{**} \\ 0 & 0 & 0 & \eta\phi & -(\theta + \mu) & 0 & 0 & 0 \\ 0 & (1-\alpha)\sigma & 0 & (1-\phi)\eta & \theta & -(\omega + \mu) & 0 & 0 \\ \psi & 0 & 0 & 0 & 0 & \xi\omega & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{pmatrix} \tag{39}$$

Now, from the Jacobian matrix evaluated at the corruption endemic equilibrium point in equation (39), it is clear that

$$\lambda_1 = -\delta, \lambda_2 = -\mu \tag{40}$$

After deleting the seventh row and eighth columns of the Jacobian matrix in equation (39), we have

$$J_2(E_2) = \begin{pmatrix} -\rho\beta C^{**} - (\psi + \mu) & 0 & -\rho\beta S^{**} & 0 & 0 & (1-\xi)\omega \\ \rho\beta C^{**} & -(\sigma + \mu) & \rho\beta S^{**} & 0 & 0 & 0 \\ 0 & \alpha\sigma & -(\gamma Q^{**} + \mu) & 0 & 0 & 0 \\ 0 & 0 & \gamma Q^{**} & -(\eta + \mu) & 0 & 0 \\ 0 & 0 & 0 & \eta\phi & -(\theta + \mu) & 0 \\ 0 & (1-\alpha)\sigma & 0 & (1-\phi)\eta & \theta & -(\omega + \mu) \end{pmatrix} \tag{41}$$

From equation (40), we have $\lambda_1 < 0$ and $\lambda_2 < 0$, since $\mu > 0$ and $\delta > 0$. The sign of the remaining eigenvalues of $J_2(E_2)$ in equation (41) will be determined by Routh–Hurwitz criteria for stability. The characteristic polynomial of the Jacobian matrix in equation (41) is given by

$$\begin{vmatrix} -\rho\beta C^{**} - (\psi + \mu) - \lambda & 0 & -\rho\beta S^{**} & 0 & 0 & (1-\xi)\omega \\ \rho\beta C^{**} & -(\sigma + \mu) - \lambda & \rho\beta S^{**} & 0 & 0 & 0 \\ 0 & \alpha\sigma & -(\gamma Q^{**} + \mu) - \lambda & 0 & 0 & 0 \\ 0 & 0 & \gamma Q^{**} & -(\eta + \mu) - \lambda & 0 & 0 \\ 0 & 0 & 0 & \eta\phi & -(\theta + \mu) - \lambda & 0 \\ 0 & (1-\alpha)\sigma & 0 & (1-\phi)\eta & \theta & -(\omega + \mu) - \lambda \end{vmatrix} = 0 \tag{42}$$

Thus, we obtain the following characteristic polynomial:

$$f(\lambda) = b_0\lambda^6 + b_1\lambda^5 + b_2\lambda^4 + b_3\lambda^3 + b_4\lambda^2 + b_5\lambda + b_6 = 0 \quad (43)$$

Where the coefficients of the characteristic polynomial are presented as follows:

$$b_0 = 1,$$

$$b_1 = \rho\beta C^{**} + \eta + \gamma Q^{**} + 6\mu + \omega + \psi + \sigma + \theta,$$

$$b_2 = (\rho\beta C^{**} + \mu + \psi)(\sigma + 5\mu + \gamma Q^{**} + \eta + \theta + \omega) + (\sigma + \mu)(\gamma Q^{**} + 4\mu + \eta + \theta + \omega) + (\gamma Q^{**} + \mu)(\eta + 3\mu + \theta + \omega) + (\eta + \mu)(\theta + 2\mu + \omega) + (\theta + \mu)(\omega + \mu) - \alpha\sigma\rho\beta S^{**},$$

$$b_3 = [(\beta\rho C^{**} + \mu + \psi)((\sigma + \mu)(\gamma Q^{**} + \mu) + (\sigma + \mu)(\theta + \mu) + (\gamma Q^{**} + \mu)(\omega + \mu) + (\eta + \mu)(\theta + \mu) + (\gamma Q^{**} + \mu)(\theta + \mu) + (\theta + \mu)(\omega + \mu) + (\sigma + \mu)(\eta + \mu) + (\eta + \mu)(\omega + \mu) + (\sigma + \mu)(\omega + \mu) + (\gamma Q^{**} + \mu)(\eta + \mu)) + \alpha\beta^2\rho^2\sigma C^{**}S^{**}(\sigma + \mu)(\gamma Q^{**}(\eta + 3\mu + \theta + \omega) + \mu(\eta + 3\mu + \theta + \omega)) + (\sigma + \mu)(\eta + \mu)(\theta + 2\mu + \omega) + (\gamma Q^{**} + \mu) + (\eta + \mu)(\theta + 2\mu + \omega) + (\theta + \mu) + \gamma Q^{**}(\omega + \mu) + \mu(\omega + \mu) + \eta + \mu + (\sigma(\omega + \mu) + \mu(\omega + \mu))(\omega + \mu)] - [\beta\rho(1-\xi)\omega(1-\alpha)\sigma C^{**} + \alpha\beta\rho\sigma S^{**}(\beta\rho C^{**} + \eta + 4\mu + \omega + \psi + \theta)],$$

$$b_4 = [(\beta\rho C^{**} + \mu + \psi)(\sigma + \mu)(\gamma Q^{**} + \mu)(\eta + 3\mu + \theta + \omega) + (\beta\rho C^{**} + \mu + \psi)(\sigma + \mu)(\eta + \mu)(2\theta + 3\mu + \omega) + (\beta\rho C^{**} + \mu + \psi)(\theta + \gamma Q^{**}\mu + 3\mu^2 + \sigma\mu + 2\mu\theta + \sigma\theta + \mu + \theta) + (\sigma + \mu)(\gamma Q^{**} + \mu)(\eta(\theta + 2\mu + \omega) + \mu(\theta + 2\mu + \omega)) + \theta(\sigma(\omega + \mu) + \mu(\omega + \mu) + (\sigma + \mu)(\eta + \mu) + (\gamma Q^{**} + \mu)(\eta(\omega + \mu) + \mu(\omega + \mu))) + \mu(\sigma(\omega + \mu) + \mu(\omega + \mu) + (\sigma + \mu)(\eta + \mu) + (\gamma Q^{**} + \mu)(\eta(\omega + \mu) + \mu(\omega + \mu))) + \alpha\beta^2\rho^2\sigma C^{**}S^{**}(\eta + 3\mu + \theta + \omega)] - [\rho\sigma(C^{**}(\eta + 3\mu + \theta + \omega)\beta(\eta + 3\mu + \theta + \omega)\rho(\eta + 3\mu + \theta + \omega) + \mu(\eta + 3\mu + \theta + \omega) + \psi(\eta + 3\mu + \theta + \omega)) + \alpha\beta\rho\sigma S^{**}((\eta + \mu)(\theta + \mu) + (\eta + \mu)(\omega + \mu) + (\theta + \mu)(\omega + \mu)) + \beta\rho C^{**}(1-\xi)\omega((\gamma Q^{**} + \mu)(1-\alpha)\sigma + (\theta + \mu)(1-\alpha)\sigma + (\eta + \mu)(1-\alpha)\sigma)],$$

$$b_5 = [\alpha\beta^2\rho^2\sigma C^{**}S^{**}((\eta + \mu)(\theta + \mu) + (\eta + \mu)(\omega + \mu) + (\theta + \mu)(\omega + \mu)) + (\beta\rho C^{**} + \mu + \psi)(\sigma + \mu)((\gamma Q^{**} + \mu)(\eta + \mu)(\omega + \mu)) + (\beta\rho C^{**} + \mu + \psi)(\gamma Q^{**} + \mu)(\eta((\theta + \mu)(\omega + \mu) + \sigma(\eta + \mu) + \mu(\eta + \mu))) + \mu((\theta + \mu)(\omega + \mu) + \sigma(\eta + \mu) + \mu(\eta + \mu)) + (\sigma + \mu)(\gamma Q^{**} + \mu)(\eta + \mu)(\theta + \mu)(\omega + \mu)] -$$

$$\left[\alpha \beta \rho \sigma (C^{**} S^{**} \beta \rho + \mu + \psi) ((\eta + \mu)(\theta + \mu) + (\eta + \mu)(\omega + \mu) + (\theta + \mu)(\omega + \mu)) + \alpha \beta \rho \sigma S^{**} (\eta + \mu) \right. \\ \left. (\theta + \mu)(\omega + \mu) + \beta \rho C^{**} (1 - \xi) \omega ((\gamma Q^{**} + \mu)(\eta + \mu)(1 - \alpha) \sigma + (\gamma Q^{**} + \mu)) \times \right. \\ \left. ((\theta + \mu)(1 - \alpha) \sigma + (\eta + \mu)(\theta + \mu)(1 - \alpha) \sigma) + C \alpha \beta \gamma Q^{**} \rho \sigma (1 - \xi) \omega (1 - \phi) \eta \right]$$

$$b_6 = \left[(\eta + \mu)(\theta + \mu) \left(\omega (C S \alpha \beta^2 \rho^2 \sigma + (\beta \rho C^{**} + \mu + \psi)(\sigma + \mu)(\gamma Q^{**} + \mu)) + \mu (\alpha \beta^2 \rho^2 \sigma C^{**} S^{**} + (\beta \rho C^{**} + \mu + \psi)(\sigma + \mu)(\gamma Q^{**} + \mu)) \right) \right] - \\ \left[\beta \eta \gamma Q^{**} \phi \rho \sigma \theta (1 - \xi) \omega (1 + (\theta + \mu)(1 - \phi) \eta) + S \alpha \beta \rho \sigma (C^{**} \beta \rho + \mu + \psi) (\eta + \mu)(\theta + \mu)(\omega + \mu) + C^{**} \beta \rho (1 - \xi) \right]$$

The necessary and sufficient conditions for the roots of the characteristic polynomial in equation (43) to have a negative real part are the Routh-Hurwitz conditions. Thus, the Routh-Hurwitz conditions for the roots of the characteristic polynomial in equation (43) to have negative real parts are presented as follows:

$$b_0 > 0, b_1 > 0, b_2 > 0, b_3 > 0, b_4 > 0, b_5 > 0, b_6 > 0$$

$$H_1 = b_1 > 0 \tag{44}$$

$$H_2 = \begin{vmatrix} b_1 & 1 \\ b_3 & b_2 \end{vmatrix} = b_1 b_2 - b_3 > 0 \tag{45}$$

$$H_3 = \begin{vmatrix} b_1 & 1 & 0 \\ b_3 & b_2 & b_1 \\ b_5 & b_4 & b_3 \end{vmatrix} = b_1 \begin{vmatrix} b_2 & b_1 \\ b_4 & b_3 \end{vmatrix} - \begin{vmatrix} b_3 & b_1 \\ b_5 & b_3 \end{vmatrix} = b_1 (b_2 b_3 + b_5) - (b_1^2 b_4 + b_3^2) > 0 \tag{46}$$

$$H_4 = \begin{vmatrix} b_1 & 1 & 0 & 0 \\ b_3 & b_2 & b_1 & 1 \\ b_5 & b_4 & b_3 & b_2 \\ 0 & b_6 & b_5 & b_4 \end{vmatrix} = \left. \begin{aligned} & b_1 b_2 b_3 b_4 + b_1^2 b_2 b_6 + 2 b_1 b_4 b_5 + b_2 b_3 b_5 - \\ & (b_1 b_2^2 b_5 + b_1^2 b_4^2 + b_1 b_3 b_6 + b_3^2 b_4 + b_5^2) > 0 \end{aligned} \right\} \tag{47}$$

$$H_5 = \begin{vmatrix} b_1 & 1 & 0 & 0 & 0 \\ b_3 & b_2 & b_1 & 1 & 0 \\ b_5 & b_4 & b_3 & b_2 & b_1 \\ 0 & b_6 & b_5 & b_4 & b_3 \\ 0 & 0 & 0 & b_6 & b_5 \end{vmatrix} = \left. \begin{aligned} & 2 b_1^2 b_2 b_5 b_6 + b_1^2 b_3 b_4 b_6 + b_1 b_2 b_3 b_4 b_5 + 2 b_1 b_4 b_5^2 + b_2 b_3 b_5^2 - \\ & (b_1^3 b_6^2 + b_1^2 b_4^2 b_5 + b_1 b_2 b_3^2 b_6 + 3 b_1 b_3 b_5 b_6 + b_3^2 b_4 b_5 + b_5^3) > 0 \end{aligned} \right\} \tag{48}$$

$$H_6 = \begin{vmatrix} b_1 & 1 & 0 & 0 & 0 & 0 \\ b_3 & b_2 & b_1 & 1 & 0 & 0 \\ b_5 & b_4 & b_3 & b_2 & b_1 & 1 \\ 0 & b_6 & b_5 & b_4 & b_3 & b_2 \\ 0 & 0 & 0 & b_6 & b_5 & b_4 \\ 0 & 0 & 0 & 0 & 0 & b_6 \end{vmatrix} = \left. \begin{aligned} & 2b_1^2b_2b_5b_6^2 + b_1^2b_3b_4b_6^2 + b_1b_2b_3b_4b_5b_6 + 2b_1b_4b_5^2b_6 + b_2b_3b_5^2b_6 - \\ & (b_1^3b_6^3 + b_1^2b_4^2b_5b_6 + b_1b_2b_3^2b_6^2 + 3b_1b_3b_5b_6^2 + b_3^2b_4b_5b_6 + b_5^3b_6) > 0 \end{aligned} \right\} \quad (49)$$

From equation (45), we have $H_2 > 0$, if $b_1b_2 > b_3$. Also, from equation (46), it is clear that $H_3 > 0$, whenever $b_1(b_2b_3 + b_5) > (b_1^2b_4 + b_3^2)$. Moreover, $H_4 > 0$, provided that

$$(b_1(b_2(b_3b_4 + b_1b_6) + 2b_4b_5) + b_2b_3b_5) > (b_1(b_2^2b_5 + b_1b_4^2 + b_3b_6) + b_3^2b_4 + b_5^2)$$

Similarly, from equation (48) we have $H_5 > 0$ if the following inequality is satisfied:

$$(2b_1^2b_2b_5b_6 + b_1^2b_3b_4b_6 + b_1b_2b_3b_4b_5 + 2b_1b_4b_5^2 + b_2b_3b_5^2) > (b_1^3b_6^2 + b_1^2b_4^2b_5 + b_1b_2b_3^2b_6 + 3b_1b_3b_5b_6 + b_3^2b_4b_5 + b_5^3)$$

Finally, observe that $H_6 > 0$, if the following inequality is satisfied:

$$2b_1^2b_2b_5b_6^2 + b_1^2b_3b_4b_6^2 + b_1b_2b_3b_4b_5b_6 + 2b_1b_4b_5^2b_6 + b_2b_3b_5^2b_6 > (b_1^3b_6^3 + b_1^2b_4^2b_5b_6 + b_1b_2b_3^2b_6^2 + 3b_1b_3b_5b_6^2 + b_3^2b_4b_5b_6 + b_5^3b_6)$$

Thus, all the roots of the characteristic polynomial given by equation (43) have negative real parts, provided that these conditions are satisfied. Consequently, the corruption endemic equilibrium point of the model, E_2 is locally asymptotically stable in the region \mathfrak{D} .

4.6 Global stability of corruption-free equilibrium point of the model

To ensure that the stability of the corruption-free equilibrium point of the model is independent of the initial data, the global stability analysis of the corruption-free equilibrium point will be investigated. The method of Castillo-Chavez et al. (2002) will be used for analyzing the global stability of the corruption-free equilibrium point.

Theorem 6:

The corruption-free equilibrium point of the model is globally asymptotically stable (GAS) in the region \mathfrak{D} , provided that $R_0 < 1$ and that the following assumptions (H1) and (H2) are satisfied:

(H1) For $\frac{dX}{dt} = F(X, 0)$, X^* is globally asymptotically stable.

(H2) For $G(X, Z) = AZ - \hat{G}(X, Z)$, $\hat{G}(X, Z) \geq 0$ for all $(X, Z) \in \mathfrak{D}$, where \mathfrak{D} a positively invariant attracting domain and $A = D_Z G(X_0, 0)$ is an M-matrix (the off diagonal elements of A are nonnegative).

Proof

We begin the proof by defining new variables and breaking the model equations given by system (1) into subsystems of the form $\frac{dX}{dt} = F(X, Z)$, $\frac{dZ}{dt} = G(X, Z)$, $G(X, 0) = 0$,

where $X = (S, R, H, Q)$ and $Z = (E, C, T, J)$.

Here, $X \in \mathbb{R}_+^4$ represents (its components) the number of individuals that are not participating in corrupt practices and $Z \in \mathbb{R}_+^4$ denotes (its components) the number of individuals that have link with corruption in one way or the other. The corruption free equilibrium point is now denoted by

$$E_1 = (X_0, 0), \text{ where } X_0 = \left(\frac{\Lambda}{\psi + \mu}, 0, \frac{\Lambda\psi}{\mu(\psi + \mu)}, \frac{\pi}{\delta} \right).$$

We have to prove the two conditions (H1) and (H2) as stated in the Theorem 6.

Now consider the reduced system $\frac{dX}{dt} = F(X, 0)$ as follows:

$$\frac{dX}{dt} = F(X, 0) = \left(\begin{array}{l} \frac{dS}{dt} = \Lambda - (\psi + \mu)S \\ \frac{dR}{dt} = -(\psi + \mu)R \\ \frac{dH}{dt} = \psi S - \mu H \\ \frac{dQ}{dt} = \pi - \delta Q \end{array} \right)$$

Let us consider the equations in the following forms:

$$\left. \begin{array}{l} \frac{dS}{dt} = \Lambda - (\psi + \mu)S \\ \frac{dR}{dt} = -(\psi + \mu)R \\ \frac{dH}{dt} = \psi S - \mu H \\ \frac{dQ}{dt} = \pi - \delta Q \end{array} \right\}$$

(49)

Solving equation (49) we have

$$\begin{aligned} S(t) &= \frac{\Lambda}{(\psi + \mu)} + \left[S(0) - \frac{\Lambda}{(\psi + \mu)} \right] e^{-(\psi + \mu)t}, & R(t) &= R(0) e^{-(\psi + \mu)t}, \\ H(t) &= \frac{\psi\Lambda}{\mu(\psi + \mu)} + \left[H(0) - \frac{\psi\Lambda}{\mu(\psi + \mu)} \right] e^{-\mu t}, & Q(t) &= \frac{\pi}{\delta} + \left[Q(0) - \frac{\pi}{\delta} \right] e^{-\delta t} \end{aligned}$$

It is clear that

$$S \rightarrow \frac{\Lambda}{\psi + \mu}, R \rightarrow 0, H \rightarrow \frac{\psi\Lambda}{\mu(\psi + \mu)} \text{ and } Q \rightarrow \frac{\pi}{\delta} \text{ as } t \rightarrow \infty \text{ irrespective of the values of the initial conditions}$$

$S(0), R(0), H(0)$ and $Q(0)$. For the second condition consider the following:

Condition H2:

$$G(X, Z) = \begin{pmatrix} \frac{dE}{dt} = \rho\beta SC - (\sigma + \mu)E \\ \frac{dC}{dt} = \alpha\sigma E - (\gamma Q + \mu)C \\ \frac{dS}{dt} = \gamma CQ - (\eta + \mu)T \\ \frac{dT}{dt} = \eta\phi T - (\theta + \mu)J \end{pmatrix},$$

$$\text{Now, } \frac{dZ}{dt} = G(X, 0) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = 0.$$

Furthermore,

$$G(X, Z) = AZ - \hat{G}(X, Z),$$

$$\hat{G}(X, Z) = AZ - G(X, Z)$$

With $A = \frac{\partial G}{\partial Z}(X^*, 0) = D_Z(X_0, 0)$ is an M-matrix (the off diagonal elements of A are nonnegative). Now let

$$\frac{dE}{dt} = f_1 = \rho\beta SC - (\sigma + \mu)E$$

$$\frac{dC}{dS} = f_2 = \alpha\sigma E - (\gamma Q + \mu)C$$

$$\frac{dT}{dt} = f_3 = \gamma C Q - (\eta + \mu)T$$

$$\frac{dJ}{dt} = f_4 = \eta\phi T - (\theta + \mu)J$$

$$A = \begin{pmatrix} \frac{\partial f_1(E_0)}{\partial E} & \frac{\partial f_1(E_0)}{\partial C} & \frac{\partial f_1(E_0)}{\partial I} & \frac{\partial f_1(E_0)}{\partial J} \\ \frac{\partial f_2(E_0)}{\partial E} & \frac{\partial f_2(E_0)}{\partial C} & \frac{\partial f_2(E_0)}{\partial I} & \frac{\partial f_2(E_0)}{\partial J} \\ \frac{\partial f_3(E_0)}{\partial E} & \frac{\partial f_3(E_0)}{\partial C} & \frac{\partial f_3(E_0)}{\partial I} & \frac{\partial f_3(E_0)}{\partial J} \\ \frac{\partial f_4(E_0)}{\partial E} & \frac{\partial f_4(E_0)}{\partial C} & \frac{\partial f_4(E_0)}{\partial I} & \frac{\partial f_4(E_0)}{\partial J} \end{pmatrix}$$

$$A = \begin{pmatrix} -(\sigma + \mu) & \rho\beta S^* & 0 & 0 \\ \alpha\sigma & -(\gamma Q^* + \mu) & 0 & 0 \\ 0 & \gamma Q^* & -(\eta + \mu) & 0 \\ 0 & 0 & \eta\phi & -(\theta + \mu) \end{pmatrix}$$

$$\hat{G}(X, Z) = \begin{pmatrix} \hat{G}_1(X, Z) \\ \hat{G}_2(X, Z) \\ \hat{G}_3(X, Z) \\ \hat{G}_4(X, Z) \end{pmatrix}$$

$$= \begin{pmatrix} -(\sigma + \mu) & \rho\beta S^* & 0 & 0 \\ \alpha\sigma & -(\gamma Q^* + \mu) & 0 & 0 \\ 0 & \gamma Q^* & -(\eta + \mu) & 0 \\ 0 & 0 & \eta\phi & -(\theta + \mu) \end{pmatrix} \begin{pmatrix} E \\ C \\ T \\ J \end{pmatrix} = \begin{pmatrix} \rho\beta SC - (\sigma + \mu)E \\ \alpha\sigma E - (\gamma Q + \mu)C \\ \gamma C Q - (\eta + \mu)T \\ \eta\phi T - (\theta + \mu)J \end{pmatrix}$$

$$= \begin{pmatrix} \rho\beta C(S^* - S) \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Since, $0 \leq S \leq S^*$, it is obvious that $\hat{G}(X, Z) \geq 0$. Therefore, the corruption free equilibrium point of the model is globally asymptotically stable in the region \mathfrak{D} .

4.7 Global Stability of Corruption Endemic Equilibrium Point

Consider the following candidate for quadratic Lyapunov function

$$\begin{aligned} V(S, E, C, T, J, R, H, Q) = & \frac{1}{2}(S - S^{**})^2 + \frac{1}{2}(E - E^{**})^2 + \frac{1}{2}(C - C^{**})^2 + \frac{1}{2}(T - T^{**})^2 + \frac{1}{2}(J - J^{**})^2 \\ & + \frac{1}{2}(R - R^{**})^2 + \frac{1}{2}(H - H^{**})^2 + \frac{1}{2}(Q - Q^{**})^2 \end{aligned} \quad (50)$$

Now, differentiating the Lyapunov function along the solution of the model equation given by system (1), we have

$$\begin{aligned} \frac{dV}{dt} = & \left[(S - S^{**}) + (E - E^{**}) + (C - C^{**}) + (T - T^{**}) + (J - J^{**}) + (R - R^{**}) + (H - H^{**}) \right] \frac{d}{dt} (S + \\ & E + C + T + J + R + H) + (Q - Q^{**}) \frac{dQ}{dt} \end{aligned} \quad (51)$$

From the model equation (1), we have

$$\frac{d}{dt} (S + E + C + T + J + R + H) = \Lambda - \mu(S + E + C + T + J + R + H) \quad \text{and} \quad \frac{dQ}{dt} = \pi - \delta Q \quad (52)$$

Substituting equation (52) into equation (51), we have

$$\begin{aligned} \frac{dV}{dt} = & \left[(S - S^{**}) + (E - E^{**}) + (C - C^{**}) + (T - T^{**}) + (J - J^{**}) + (R - R^{**}) + (H - H^{**}) \right] [\Lambda - \\ & \mu(S + E + C + T + J + R + H) + (Q - Q^{**}) [\pi - \delta Q] \end{aligned} \quad (53)$$

At steady state, we have the following:

$$\Lambda = \mu(S^{**} + E^{**} + C^{**} + T^{**} + J^{**} + R^{**} + H^{**}) \text{ and } \pi = \delta Q^{**} \quad (54)$$

Using (54) in (53), we have

$$\begin{aligned} \frac{dV}{dt} = & \left[(S - S^{**}) + (E - E^{**}) + (C - C^{**}) + (T - T^{**}) + (J - J^{**}) + (R - R^{**}) + (H - H^{**}) \right] \left[\mu(S^{**} + \right. \\ & \left. E^{**} + C^{**} + T^{**} + J^{**} + R^{**} + H^{**}) - \mu(S + E + C + T + J + R + H) \right] + (Q - Q^{**}) \left[\delta Q^{**} - \delta Q \right] \end{aligned} \quad (53)$$

Now, equation (53) can be written as

$$\begin{aligned} \frac{dV}{dt} = & \left[(S - S^{**}) + (E - E^{**}) + (C - C^{**}) + (T - T^{**}) + (J - J^{**}) + (R - R^{**}) + (H - H^{**}) \right] \\ & \left[-\mu(S - S^{**}) - \mu(E - E^{**}) - \mu(C - C^{**}) - \mu(T - T^{**}) - \mu(J - J^{**}) - \mu(R - R^{**}) - \mu(H - H^{**}) \right] \\ & + (Q - Q^{**}) \left[-\delta(Q - Q^{**}) \right] \end{aligned} \quad (54)$$

Clearing the square brackets in equation (54), we have

$$\begin{aligned}
\frac{dV}{dt} = & -\mu(S - S^{**})^2 - \mu(S - S^{**})(E - E^{**}) - \mu(S - S^{**})(C - C^{**}) - \mu(S - S^{**})(T - T^{**}) - \\
& \mu(S - S^{**})(J - J^{**}) - \mu(S - S^{**})(R - R^{**}) - \mu(E - E^{**})(S - S^{**}) - \mu(E - E^{**})^2 - \\
& \mu(E - E^{**})(C - C^{**}) - \mu(E - E^{**})(T - T^{**}) - \mu(E - E^{**})(J - J^{**}) - \mu(E - E^{**})(R - R^{**}) - \\
& \mu(E - E^{**})(H - H^{**}) - \mu(C - C^{**})(S - S^{**}) - \mu(C - C^{**})(E - E^{**}) - \mu(C - C^{**})^2 - \\
& \mu(C - C^{**})(T - T^{**}) - \mu(C - C^{**})(J - J^{**}) - \mu(C - C^{**})(R - R^{**}) - \mu(C - C^{**})(H - H^{**}) - \\
& \mu(T - T^{**})(S - S^{**}) - \mu(T - T^{**})(E - E^{**}) - \mu(T - T^{**})(C - C^{**}) - \mu(T - T^{**})^2 - \\
& \mu(T - T^{**})(J - J^{**}) - \mu(T - T^{**})(R - R^{**}) - \mu(T - T^{**})(H - H^{**}) - \mu(J - J^{**})(S - S^{**}) - \\
& \mu(J - J^{**})(S - S^{**}) - \mu(J - J^{**})(E - E^{**}) - \mu(J - J^{**})(C - C^{**}) - \mu(J - J^{**})(T - T^{**}) - \\
& \mu(J - J^{**})^2 - \mu(J - J^{**})(R - R^{**}) - \mu(J - J^{**})(H - H^{**}) - \mu(R - R^{**})(S - S^{**}) - \\
& \mu(R - R^{**})(E - E^{**}) - \mu(R - R^{**})(C - C^{**}) - \mu(R - R^{**})(T - T^{**}) - \mu(R - R^{**})(J - J^{**}) - \\
& \mu(R - R^{**})^2 - \mu(R - R^{**})(H - H^{**}) - \mu(H - H^{**})(S - S^{**}) - \mu(H - H^{**})(E - E^{**}) - \\
& \mu(H - H^{**})(C - C^{**}) - \mu(H - H^{**})(T - T^{**}) - \mu(H - H^{**})(J - J^{**}) - \mu(H - H^{**})(R - R^{**}) - \\
& \mu(H - H^{**})^2 - \delta(Q - Q^{**})^2
\end{aligned} \tag{55}$$

After some simplifications, we obtained

$$\begin{aligned}
\frac{dV}{dt} = & -\mu \left[(S - S^{**})^2 + (E - E^{**})^2 + (C - C^{**})^2 + (T - T^{**})^2 + (J - J^{**})^2 + (R - R^{**})^2 + (H - H^{**})^2 \right] - \\
& \left[\pi(Q - Q^{**})^2 - 2\mu(S - S^{**})(E - E^{**}) + (S - S^{**})(C - C^{**}) + (S - S^{**})(T - T^{**}) + (S - S^{**})(J - J^{**}) + \right. \\
& (S - S^{**})(R - R^{**}) + (S - S^{**})(H - H^{**}) + (E - E^{**})(C - C^{**}) + (E - E^{**})(T - T^{**}) + (E - E^{**})(J - J^{**}) + \\
& (E - E^{**})(R - R^{**}) + (E - E^{**})(H - H^{**}) + (C - C^{**})(T - T^{**}) + (C - C^{**})(J - J^{**}) + (C - C^{**})(R - R^{**}) + \\
& (C - C^{**})(H - H^{**}) + (T - T^{**})(J - J^{**}) + (T - T^{**})(R - R^{**}) + (T - T^{**})(H - H^{**}) + (J - J^{**})(R - R^{**}) + \\
& \left. (J - J^{**})(H - H^{**}) + (R - R^{**})(H - H^{**}) \right]
\end{aligned} \tag{56}$$

It is obvious from equation (56) that $\frac{dV}{dt} \leq 0$ and $\frac{dV}{dt} = 0$ if and only if $S = S^{**}$, $E = E^{**}$, $C = C^{**}$, $T = T^{**}$, $J = J^{**}$, $R = R^{**}$ and $H = H^{**}$. Therefore, the only positively invariant set of the model equation given by system

(1) where $\frac{dV}{dt} = 0$ is the corruption endemic equilibrium point, E_2 . Thus, it follows by Lasalle's invariance principle that E_2 is globally attractive. Hence, E_2 is globally asymptotically stable in the region \mathfrak{D} .

5. Sensitivity Analysis

Sensitivity analysis is a crucial step in understanding the dynamics of complex systems, such as the spread of corruption. By analyzing how changes in individual parameters affect the overall behavior of the system, we can identify the most critical factors that drive the spread of corruption. This information is essential for developing effective strategies to prevent and mitigate corruption. The sensitivity analysis reveals the significance of each parameter in the spread of corruption. Therefore, we performed a sensitivity analysis of the model parameters using the basic reproduction number as a response function. This will help us identify the most important parameters that drive the dynamics of corruption, such as the rate of bribery, the effectiveness of law enforcement, and the level of public awareness. By understanding the relative importance of these parameters, policymakers can focus their efforts on the most critical areas and develop targeted interventions to reduce the spread of corruption. We used the local sensitivity analysis based on the normalized forward sensitivity index, which provides a quantitative measure of the sensitivity of the system to changes in each parameter.

5.1 Computation of sensitivity index of the model parameters

The sensitivity indices of the model parameters with respect to the reproduction number, given in equation (30), are computed using the normalized forward sensitivity index to determine the parameter with the greatest influence on the dynamics of corruption. The normalized forward sensitivity index is a measure of the relative change in the output of a system with respect to a change in one of its parameters. It is a useful tool for identifying the most sensitive parameters in a model and for understanding how changes in these parameters affect the overall behavior of the system. The normalized sensitivity index formula is given by:

$$\Omega_{\sigma}^{R_0} = \frac{\partial R_0}{\partial \sigma} \times \frac{\sigma}{R_0}, \text{ where } R_0 = \frac{\alpha\sigma\Lambda\beta\rho\delta}{(\gamma\pi + \delta\mu)(\sigma + \mu)(\mu + \psi)}.$$

Using the initial values of the model parameters in Table 2, we calculated the sensitivity indices for the model parameters, and the results are summarized in Table 3.

Table 2: Baseline values of the parameters

Parameter	Value	Source
ρ	0.002	Gutema (2024)
β	0.0007	Assumed
α	0.82	Assumed
σ	0.7	Assumed
γ	0.3	Zerihun (2022)
δ	0.3	Alhassan (2024)
π	0.5	Assumed
μ	0.3	Gutema (2024)
Λ	0.82	Assumed
ψ	0.6	Assumed

Table 3: Sensitivity signs of the model parameters with respect to the reproduction, R_0

Parameter	Description	Sensitivity sign
ρ	Probability of corruption transmission per contact between individuals	+ve
β	Rate of contact between susceptible individuals and those accused of corruption	+ve
α	Proportion of exposed individuals who progress to the accused of corruption subpopulation	+ve
σ	Rate at which exposed individuals progress to the accused of corruption subpopulation	+ve
γ	Rate at which individuals accused of corruption are arrested for trial	-ve
δ	Decay rate of anti-corruption agencies	+ve
π	Influx rate of anti-corruption agencies	-ve
μ	Natural death rate	-ve
Λ	Recruitment rate into the susceptible compartment	+ve
ψ	Proportion of individuals that join the honest subpopulation from susceptible compartment	-ve



Table 4: Sensitivity index for the parameters with respect to R_0

Parameter	$\Omega_l^{R_0}$	Sensitivity Index
ρ	1	1
β	1	1
α	1	1
σ	$\frac{\mu}{\sigma + \mu}$	0.3
γ	$-\frac{\pi\gamma}{\pi\gamma + \mu\delta}$	-0.625
δ	$\frac{\pi\gamma}{\pi\gamma + \mu\delta}$	0.625
π	$-\frac{\pi\gamma}{\pi\gamma + \mu\delta}$	-0.625
μ	$\frac{\mu(3\mu^2\delta + \pi\sigma\gamma + 2\pi\gamma\mu + \pi\gamma\psi + 2\sigma\mu\psi + \sigma\delta\psi + 2\mu\delta\psi)}{(\gamma\pi + \delta\mu)(\sigma + \mu)(\mu + \psi)}$	1.475
Λ	1	1
ψ	$-\frac{\psi}{\mu + \psi}$	-0.667

5.2 Interpretation of sensitivity indices

The plus or minus sign indicated against the model parameters in Table 3 provides an indication of the direction and magnitude of the impact of each parameter on the dynamics of corruption. The sensitivity indices result in Table 4, which were obtained through a comprehensive sensitivity analysis, demonstrate that an increase or a decrease in the values of the parameters with a positive sensitivity index will have a corresponding impact on the corruption dynamics. For example, the sensitivity index of the contact rate between susceptible and corrupted individuals β is $\Omega_{R_0}^\beta = 1$, indicating that a 10% increase or decrease in the contact rate will lead to a 10% increase or decrease in the basic reproduction number. On the other hand, the parameters with a negative sign in Table 3 will have a negative effect on the basic reproduction number, leading to a decrease in R_0 as their values increase. Furthermore, a decrease in the values of these parameters will result in an increase in the basic reproduction number, highlighting the importance of these parameters in controlling the spread of corruption. Thus, the sensitivity analysis

results indicate that reducing the values of parameters with positive indices can contribute to controlling the spread of corruption. Additionally, the results suggest that increasing the values of parameters with negative indices can also help mitigate the spread of corruption.

6. Conclusion

This study has successfully developed and analyzed a mathematical model to examine the dynamics of corruption. The model provides valuable insights into the spread of corruption within a population and identifies key parameters that influence its growth. The calculation of the threshold quantity measuring the spread of corruption (basic reproduction number, R_0), the local and global stability analyses of the corruption-free equilibrium point provide a framework for understanding the conditions under which corruption can be controlled.

To show that the developed corruption model is mathematically and epidemiologically meaningful, we conducted a qualitative analysis of the model by showing that the solution of the model is positive and bounded. The stability analysis of the model showed that corruption-free equilibrium is both locally and globally asymptotically stable if $R_0 < 1$, which means corruption can be minimize from population. In addition, if $R_0 > 1$ corruption - endemic equilibrium is locally and globally asymptotically stable, which means corruption will spread in the population. Furthermore, results of the sensitivity analysis of the model parameters revealed that reducing the values of parameters with positive indices will contribute in controlling the spread of corruption. On the other hand, the sensitivity results suggest that increasing the values of parameters with negative indices will also help to mitigate the spread of corruption. Interestingly, this model can serve as a useful tool for policymakers and stakeholders seeking to develop effective strategies for mitigating the spread of corruption. Future research will be built upon this model by considering optimal control strategies.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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