INTERNATIONAL JOURNAL OF DEVELOPMENT MATHEMATICS
ISSN: 3026-8656 (Print) | 3026-8699 (Online)
journal homepage: https://ijdm.org.ng/index.php/Journals

Control Strategies to Curtail Transmission of Corruption using Mathematical Modelling Approach

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ARTICLE INFO

Article history:
Received 01 January 2025
Received in revised form 04 Aprilr 2025
Accepted 28 May 2025

Keywords:
Mathematical model, Corruption, Pontryagin maximum principle, Optimal control, Numerical simulation.

MSC 2020 Subject classification:
97M10

ABSTRACT

In this study, we propose a deterministic compartmental model to study the behavior of corruption transmission under different control strategies. In the model, reproduction number is formulated to analyze the accurate transmission dynamics of the corruption and equilibrium points of the model were computed. Optimal control theory is applied to the model to demonstrate the impact of various strategies, including control effort on anti – corruption sensitization program u_1, control effort on the arrest and prosecutions of individual accused of corruption u_2 and control effort for punishing individuals found guilty of corruption which will serve as consequences of corruption u_3. The results demonstrated that implementing all three strategies, u_1, u_2 and u_3 simultaneously appear to be the most effective way to curb corruption. Furthermore, the effect of control strategies on the model is analyzed graphically by simulating the model numerically.

1. Introduction

Research on corruption dynamics has been a focal point for many scholars, accumulating extensive expertise and highlights. Through the development of rigorous mathematical models, they proposed effective controlled measures to mitigate the spread of corruption. An extensive research has been undertaken by scholars to investigate strategies for curtailing corruption, including law enforcement and awareness campaign, highlighting the need for additional control measures to mitigate its spread. Alhasan *et al.* (2024) developed a mathematical model on the dynamics of corruption menace with control strategies. Two time – dependent control factors were incorporated to provide an ideal control that will help to stop the corruption from spreading across population. A public education campaign against corruption and punishment of those found guilty of corruption are the two controls that were examined for effectiveness using Pontrygin's maximum principle. It has been noted that implementing the controls either separately or both can significantly reduce the spread of corruption. Gutema *et al.* (2024) developed a mathematical analysis of the corruption dynamics model with optimal control strategy. The method of deterministic model approach was use to study the effect of corruption in the population base on the corrupt status. A qualitative analysis of the model was conducted by showing that the solution of the model is positive and bounded. Hence, the developed model is mathematically and epidemiology meaningful. Furthermore, the model was analyzed and extended into optimal control strategy by incorporating two control variables, i.e. mass education and low enforcement. The necessary conditions for optimal controls were investigated with the help of Pontrygin's Maximum Principles. Lastly, numerical simulations of the optimal model were examined by considering individually and combining the controls. Rwat *et al.* (2023) developed mathematical models and analyzed it to investigate the transmission of corruption within pulation. Aloke (2023) proposed to combat corruptions in Nigeria system by examine the dynamics of corruption and

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https://doi.org/10.62054/ijdm/0202.05

three control measures. Zerihun and Abyneh (2022) developed a mathematical model for the dynamics of corruption transmission incorporating media coverage. Abayneh and Zerihun (2022) proposed a deterministic mathematical model that explain the transmission dynamics of corruption by considering social influence on honest individuals and analyze the model. Mokaya *et al.* (2021) developed and studied a deterministic model for the spread of corrupt morals that involves a group of people who are going through a counseling and guidance procedure. Abdulrahman (2014) proposed a mathematical model with constant recruitment rate and standard incidence for the transmission dynamics of corruption as a disease. Legesse and Shiferaw (2018) proposed a mathematical model for corruption by considering awareness created by anti-corruption and counseling in jail.

In this work, a mathematical model is developed to study the transmission of corruption across population. Optimal control theory is introduced and applied to the development in section 3 and 4, the model is simulated numerically to observe the effect of control strategies on the model.

2. Model Formulation

To analyze transmission dynamics of corruption, a compartmental model is constructed. The model consists of all possible corruption dynamics. Corruption is seen as contagious diseases in nature, and cases have been rising in most of the developing countries in the world. The model is divided into eight compartments: susceptible $S(t)$, Exposed $E(t)$, Corrupt $C(t)$, Trial $T(t)$, Jailed $J(t)$, Reformed $S(t)$ $R(t)$, Honest individual $H(t)$ and anti – corruption agencies $Q(t)$. The dynamics of the corruption are described graphically in Figure 1. Parameters used in the model are described in Table 1.

2.1 Flow diagram of the model

The flow diagram in Figure (1) describe the transition of individuals from one compartment to another. In the diagram, the dotted line represent interaction between anti – corruption agencies and the corrupted individuals to enable arrest. The solid arrows represent movement of individual from one compartment to another.

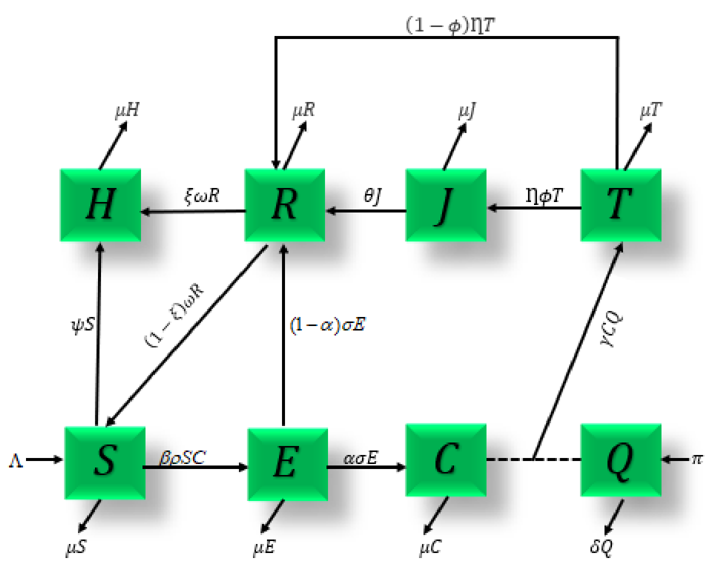

Figure 1: Flow Diagram of the Model

Using the above depiction, a dynamical system of set of nonlinear differential equations for the model is formulated as

follows:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda + (1-\xi)\omega R - \rho\beta SC - (\psi+\mu)S \\
\frac{dE}{dt} &= \rho\beta SC - (\sigma+\mu)E \\
\frac{dC}{dt} &= \alpha\sigma E - \gamma CQ - \mu C \\
\frac{dT}{dt} &= \gamma CQ - (\eta+\mu)T \\
\frac{dJ}{dt} &= \eta\varphi T - (\theta+\mu)J \\
\frac{dR}{dt} &= \theta J + (1-\varphi)\eta T + (1-\alpha)\sigma E - (\omega+\mu)R \\
\frac{dH}{dt} &= \psi S + \xi\omega R - \mu H \\
\frac{dQ}{dt} &= \pi - \delta Q
\end{aligned}\right\} \tag{1}$$

Subject to the initial conditions

$$S(0) > 0,\ E(0) \geq 0,\ C(0) \geq 0,\ T(0) \geq 0,\ J(0) \geq 0,\ R(0) \geq 0,\ H(0) \geq 0,\ Q(0) \geq 0 \tag{2}$$

Since we are dealing with human population, we assumed that all parameters used in the model are non – negative. Consider the feasible region as follows:

$\mathfrak{D} = \mathfrak{D}_1 \cup \mathfrak{D}_2 \subset \mathbb{R}_+^7 \times \mathbb{R}_+,$

where,

$$\mathfrak{D}_1 = \left\{(S, E, C, T, J, R, H) \in \mathbb{R}_+^7 : N \leq \frac{\Lambda}{\mu}\right\} \text{ and } \mathfrak{D}_2 = \left\{Q \in \mathbb{R}_+ : Q \leq \frac{\pi}{\delta}\right\} \tag{3}$$

The region where $\mathfrak{D}$ is positively invariant; all the solutions of system (1) remain in the feasible region (3), (Musa & Fori, 2019).

Table 1. Parameters used in the model

Parameters	Description
ρ	Corruption transmission probability per contact
β	Contact rate between susceptible and corrupted individuals
α	Proportion of individuals that join the corrupted subpopulation from the exposed compartment
σ	Rate at which exposed individuals become corrupted
γ	Rate at which corrupted individuals are arrested
η	Rate at which corrupted individuals on trial are taken to jail
θ	Rate at which jailed individuals are reformed
ω	Rate at which reformed individual become honest
ξ	Proportion of individuals that join the honest subpopulation from reformed compartment
ψ	Proportion of individuals that join the honest subpopulation from susceptible compartment
δ	Decay rate of anti–corruption agencies
μ	Natural death rate
ϕ	Proportion of individuals on trial that will go to jail
Λ	Recruitment rate into the susceptible compartment
π	Influx rate of anti-corruption agencies

2.2 ***Equilibrium points***. By solving system (1), we obtained Corruption Free Equilibrium Point and Corruption Endemic Equilibrium Point as follows:

$$E_1 = \left(\frac{\Lambda}{\psi + \mu}, 0, 0, 0, 0, 0, \frac{\psi \Lambda}{\mu(\psi + \mu)}, \frac{\pi}{\delta} \right) \tag{4}$$

$$E_2 = \left(S^{**}, E^{**}, C^{**}, T^{**}, J^{**}, R^{**}, H^{**}, Q^{**} \right) \tag{5}$$

where,

$$S^{**} = \frac{\Lambda + (1-\xi)\omega R^{**}}{\rho\beta C^{**} + (\psi + \mu)}, \quad E^{**} = \frac{\rho\beta C^{**} S^{**}}{(\sigma + \mu)}, \quad C^{**} = \frac{\alpha\sigma E^{**}}{(\gamma Q^{**} + \mu)}, \quad T^{**} = \frac{\gamma C^{**} Q^{**}}{\eta + \mu},$$

$$J^{**} = \frac{\eta\phi T^{**}}{\theta + \mu}, \quad R^{**} = \frac{\theta J^{**} + (1-\phi)\eta T^{**} + (1-\alpha)\sigma E^{**}}{\omega + \mu}, \quad H^{**} = \frac{\psi S^{**} + \xi\omega R^{**}}{\mu}, \quad Q^{**} = \frac{\pi}{\delta}.$$

2.3 Basic Reproduction Number. The basic reproduction (R_0) for the model was established following the next generation matrix approach by Diekmann, *at el.* (2009). It is obtained as the spectral radius of the matrix (FV^{-1}) at the disease – free equilibrium point, where F and V are as follows:

$$F = \begin{pmatrix} 0 & \frac{\Lambda\beta\rho}{\psi+\mu} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad and \quad V = \begin{bmatrix} \sigma+\mu & 0 & 0 & 0 \\ -\alpha\sigma & \left(\frac{\gamma\pi+\delta\mu}{\delta}\right) & 0 & 0 \\ 0 & -\frac{\gamma\pi}{\delta} & \eta+\mu & 0 \\ 0 & 0 & -\eta\varphi & \theta+\mu \end{bmatrix} \tag{6}$$

Thus, the basic reproduction number is

$$R_0 = \rho\left(FV^{-1}\right) = \frac{\alpha\sigma\Lambda\beta\rho\delta}{(\gamma\pi+\delta\mu)(\sigma+\mu)(\mu+\psi)} \tag{7}$$

3. Optimal control theory

Control measures play a significant role in controlling the spread of corruption. In this control theory, three possible control variables are used as three possible control strategies. Since corruption is seen as contagious diseases, it can infect any person who come in contact with a corrupt individual. Therefore, there is need to extend the model into optimal control problem.

3.1 Extension of the model into optimal control

Optimal control is applied to corruption transmission dynamics to determine the most effective intervention strategies for controlling corruption. Therefore, the optimal level that would be need to control corruption is minimizing the number of exposed and corrupted population.

The optimal control interventions for effective controlling threat of corruption is presented in this section. It observed that some parameters in the model have impact on the basic reproduction number of corruptions R_0. To address it, some controls are incorporated in the developed model, including the following:

i. u_1: control effort on anti – corruption sensitization program to prevent corruption cases

ii. u_2: control effort on the arrest and prosecution of individuals accused of corruption

iii. u_3: control effort on punitive deterrent of individuals found guilty of corruption which will serve as consequences of corruption

Adding the three controls of the model equation (1), the optimal control model is given by

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda + (1-\xi)\omega R - (1-u_1)\rho\beta SC - (\psi+\mu)S \\
\frac{dE}{dt} &= (1-u_1)\rho\beta SC - (\sigma+\mu)E \\
\frac{dC}{dt} &= \alpha\sigma E - \gamma u_2 CQ - \mu C \\
\frac{dT}{dt} &= \gamma u_2 CQ - (\eta u_3 + \mu)T \\
\frac{dJ}{dt} &= \eta\phi u_3 T - (\theta+\mu)J \\
\frac{dR}{dt} &= \theta J + (1-\phi)\eta u_3 T + (1-\alpha)\sigma E - (\omega+\mu)R \\
\frac{dH}{dt} &= \psi S + \xi\omega R - \mu H \\
\frac{dQ}{dt} &= \pi - \delta Q
\end{aligned}\right\} \qquad (8)$$

To explore the optimal level of efforts that would be required to control corruption in society, we constructed an objective functional $J(u_1, u_2, u_3)$, whose goal is to minimize the number of exposed and individuals accused of corruption. Thus, the objective functional corresponding to the optimal control model in equation (8) is given by

$$J = \min_{u_1, u_2, u_3} \int_0^{t_f} \left(a_1 E + a_2 C + \frac{\chi_1 u_1^2}{2} + \frac{\chi_2 u_2^2}{2} + \frac{\chi_3 u_3^2}{2} \right) dt \qquad (9)$$

Subject to the constraint given by system (8).

Where t_f is the final time, a_1 and a_2 are weight constants of the exposed and corrupted population, respectively, while χ_1, χ_2, and χ_3 are weight coefficients for each individual control measure. The term $\frac{\chi_1 u_1^2}{2}$, $\frac{\chi_2 u_2^2}{2}$ and $\frac{\chi_3 u_3^2}{2}$ are costs associated with u_1, u_2 and u_3. We choose nonlinear cost function on the control based on the assumption that cost takes nonlinear function, (Alemne, 2020). Our aim is to search for optimal control function (u_1^*, u_2^*, u_3^*) such that,

$$J\left(u_1^*,u_2^*,u_3^*\right)=\min\left\{J\left(u_1,u_2,u_3\right)|\left(u_1,u_2,u_3\right)\in U\right\} \tag{10}$$

where $U=\left\{\left(u_1,u_2,u_3\right)|\, u_i(t) \text{ is a Lebesgue measurable on } \left[0,t_f\right], 0\le u_1\le 1, i=1,2,3\right\}$ is a closed set, that's means U is the control set system of equation given by (8).

3.2 The Hamiltonia and optimal system

The optimal control model must satisfy the necessary conditions that are formulated by Pontryangin's maximum principle (Pontryagin,1962). The principle converts the system of equation (8) and (9) into a problem of minimizing point – wise a Hamilton $\left(H_T\right)$ with respect to $u_1(t), u_2(t)$ and $u_3(t)$ as

$$H_T=a_1E+a_2C+\frac{1}{2}\sum_{i=1}^{3}\left(\chi_i u_i^2\right)+\lambda_S\frac{dS}{dt}+\lambda_E\frac{dE}{dt}+\lambda_c\frac{dC}{dt}+\lambda_T\frac{dT}{dt}+\lambda_J\frac{dJ}{dt}+\lambda_R\frac{dR}{dt}+\lambda_H\frac{dH}{dt}+\lambda_Q\frac{dQ}{dt}$$

$$\left.\begin{aligned}
H_T &= a_1E+a_2C+\frac{1}{2}\left(\chi_1u_1^2+\chi_2u_2^2+\chi_3u_3^2\right)\\
&+\lambda_S\left[\Lambda+(1-\xi)\omega R-(1-u_1)\rho\beta SC-(\psi+\mu)S\right]\\
&+\lambda_E\left[(1-u_1)\rho\beta SC-(\sigma+\mu)E\right]\\
&+\lambda_C\left[\alpha\sigma E-\gamma u_2CQ-\mu C\right]\\
&+\lambda_T\left[\gamma u_2CQ-(\eta u_3+\mu)T\right]\\
&+\lambda_J\left[\eta\phi u_3T-(\theta+\mu)J\right]\\
&+\lambda_R\left[\theta J+(1-\phi)\eta u_3T+(1-\alpha)\sigma E-(\omega+\mu)R\right]\\
&+\lambda_H\left[\psi S+\xi\omega R-\mu H\right]\\
&+\lambda_Q\left[\pi-\delta Q\right]
\end{aligned}\right\} \tag{11}$$

where $\lambda_S,\lambda_E,\lambda_C,\lambda_T,\lambda_J,\lambda_R,\lambda_H,\lambda_Q$, are adjoint variables to be determined. The adjoint system and the control characterization is presented in the following theorem:

Theorem 1

Let u_1^*, u_2^* and u_3^* be optimal controls and $S^*,E^*,C^*,T^*,J^*,R^*,H^*,Q^*$ be the solutions of the optimal control problems of system (11) that minimize $J\left(u_1,u_2,u_3\right)$ over U, then there exists adjoint variables $\lambda_S,\lambda_E,\lambda_C,\lambda_T,\lambda_J,\lambda_R,\lambda_H,\lambda_Q$ satisfying

$$\left.\begin{aligned}
\frac{d\lambda_S}{dt} &= (\lambda_E-\lambda_S)\left((1-u_1)\rho\beta C\right)+(\lambda_S-\lambda_H)\psi+\lambda_S\mu \\
\frac{d\lambda_E}{dt} &= -a_1+(\lambda_E-\lambda_R)\sigma+(\lambda_R-\lambda_C)\alpha\sigma+\lambda_E\mu \\
\frac{d\lambda_C}{dt} &= -a_2+(\lambda_S-\lambda_E)\left((1-u_1)\rho\beta S\right)+(\lambda_C-\lambda_T)\gamma u_2 Q+\lambda_C\mu \\
\frac{d\lambda_T}{dt} &= (\lambda_T-\lambda_J-\lambda_R)\eta u_3+\eta u_3\lambda_R+\lambda_T\mu \\
\frac{d\lambda_J}{dt} &= (\lambda_J-\lambda_R)\theta+\lambda_J\mu \\
\frac{d\lambda_R}{dt} &= \left(\lambda_S(1-\varepsilon)+\lambda_R-\lambda_H\varepsilon\right)\omega+\lambda_R\mu \\
\frac{d\lambda_H}{dt} &= \lambda_H\mu \\
\frac{d\lambda_Q}{dt} &= (\lambda_C-\lambda_T)\gamma C u_2+\lambda_Q\delta
\end{aligned}\right\} \tag{12}$$

where λ_i for $i = S, E, C, T, J, R, H, Q$ are adjoint variables and controls u_1^*, u_2^* and u_3^* obey the optimality conditions, such that

$$\left.\begin{aligned}
u_1^* &= \max\left\{0, \min\left(1, \frac{\rho\beta CS(\lambda_E-\lambda_S)}{\chi_1}\right)\right\} \\
u_2^* &= \max\left\{0, \min\left(1, \frac{\gamma CQ(\lambda_C-\lambda_T)}{\chi_2}\right)\right\} \\
u_3^* &= \max\left\{0, \min\left(1, \frac{\eta T\left(\lambda_R(1-\phi)+(\lambda_T-\lambda_J)\right)}{\chi_3}\right)\right\}
\end{aligned}\right\} \tag{13}$$

Proof

The adjoint system is obtained using standard result by Pontryagin's Principle. The differential equations governing the adjoint variables are obtained by the differentiation of Hamiltonian equation (11) with respect to the state variables. S, E, C, T, J, R, H, Q respectively. Then, we obtain the adjoin equation as

$$\frac{d\lambda_S}{dt}=-\frac{\partial H_T}{\partial S},\qquad \frac{d\lambda_E}{dt}=-\frac{\partial H_T}{\partial E},\qquad \frac{d\lambda_C}{dt}=-\frac{\partial H_T}{\partial C},\qquad \frac{d\lambda_T}{dt}=-\frac{\partial H_T}{\partial T},$$

$$\frac{d\lambda_J}{dt}=-\frac{\partial H_T}{\partial J},\qquad \frac{d\lambda_R}{dt}=-\frac{\partial H_T}{\partial R},\qquad \frac{d\lambda_H}{dt}=-\frac{\partial H_T}{\partial H},\qquad \frac{d\lambda_Q}{dt}=-\frac{\partial H_T}{\partial Q}$$

The result obtained is evaluated at the optimal controls and the corresponding state variables. Thus, the adjoint system presented in equation (12) is obtained. The computation is presented as follows:

$$\begin{aligned}\frac{d\lambda_S}{dt}&=-\frac{\partial H_T}{\partial S}\\ &=-\left[\lambda_s\left(\left(1-u_1\right)\rho\beta C-\left(\psi+\mu\right)\right)-\lambda_E\left(\left(1-u_1\right)\rho\beta C\right)+\lambda_H\psi\right]\\ &=\left(\lambda_E-\lambda_S\right)\left(\left(1-u_1\right)\rho\beta C\right)+\left(\lambda_S-\lambda_H\right)\psi+\lambda_S\mu\end{aligned}$$

$$\begin{aligned}\frac{d\lambda_E}{dt}&=-\frac{\partial H_T}{\partial E}\\ &=-\left[a_1-\lambda_E\left(\sigma+\mu\right)+\lambda_C\alpha\sigma-\left(1-\alpha\right)\sigma\lambda_R\right]\\ &=-a_1+\left(\lambda_E-\lambda_R\right)\sigma+\left(\lambda_R-\lambda_C\right)\alpha\sigma+\mu\lambda_E\end{aligned}$$

$$\begin{aligned}\frac{d\lambda_C}{dt}&=-\frac{\partial H_T}{\partial C}\\ &=-\left[a_2-\lambda_S\left(1-u_1\right)\rho\beta S+\lambda_E\left(1-u_1\right)\rho\beta S-\lambda_C\gamma u_2Q-\lambda_c\mu+\lambda_T\gamma u_2Q\right]\\ &=-a_2+\left(\lambda_S-\lambda_E\right)\left(\left(1-u_1\right)\rho\beta S\right)+\left(\lambda_C-\lambda_T\right)\gamma u_2Q+\lambda_C\mu\end{aligned}$$

$$\begin{aligned}\frac{d\lambda_T}{dt}&=-\frac{\partial H_T}{\partial T}\\ &=-\left[-\lambda_T\left(\eta u_3+\mu\right)+\lambda_J\left(\eta\phi u_3\right)+\lambda_R\left(1-\theta\right)\eta u_3\right]\\ &=\left(\lambda_T\eta u_3+\lambda_T\mu-\lambda_J\eta u_3\phi-\lambda_R\eta u_3+\lambda_R\eta u_3\phi\right)\eta u_3+-\theta J\\ &=\left(\lambda_T-\lambda_R\right)\eta u_3+\left(\lambda_R-\lambda_J\right)\eta\phi u_3+\lambda_T\mu\end{aligned}$$

$$\frac{d\lambda_J}{dt} = -\frac{\partial H_T}{\partial J}$$
$$= -\left[-\lambda_J\left(\theta+\mu\right)+\lambda_R\theta\right] = \lambda_J(\theta+\mu)-\lambda_R\theta = \left(\lambda_J-\lambda_R\right)\theta+\lambda_J\mu$$

$$\frac{d\lambda_R}{dt} = -\frac{\partial H_T}{\partial R}$$
$$= -\left[-\lambda_S\left(1-\xi\right)\omega-\lambda_R\left(\omega+\mu\right)+\lambda_H\xi\omega\right]$$
$$= \lambda_S\left(1-\xi\right)\omega+\lambda_R\left(\omega+\mu\right)-\lambda_H\xi\omega = \left(\lambda_S\left(1-\varepsilon\right)+\lambda_R-\lambda_H\varepsilon\right)\omega+\lambda_R\mu$$

$$\frac{d\lambda_H}{dt} = -\frac{\partial H_T}{\partial H} = -\left[-\lambda_H H\right] = \lambda_H H$$

$$\frac{d\lambda_Q}{dt} = \frac{\partial H_T}{\partial Q} = -\left[-\lambda_C\left(\gamma u_2 C\right)+\lambda_T\gamma u_2 C-\lambda_Q\delta\right]$$
$$= \lambda_C\left(\gamma u_2 C\right)-\lambda_T\gamma u_2 C-\lambda_Q\delta$$
$$= \left(\lambda_C-\lambda_T\right)\gamma u_2 C+\lambda_Q\delta$$

The zero final time is given by

$$\lambda_S\left(t_f\right) = \lambda_E\left(t_f\right) = \lambda_C\left(t_f\right) = \lambda_T\left(t_f\right) = \lambda_J\left(t_f\right) = \lambda_R\left(t_f\right) = \lambda_H\left(t_f\right) = \lambda_Q\left(t_f\right) = 0 \tag{14}$$

Furthermore, using the optimality condition, we can find the value of optimal control functions $u_1^*, u_2^*\ and\ u_3^*$ for $t \in \left[0, t_f\right]$. So, we can find characterization equation as follow:

$$\left.\begin{aligned}
&\frac{\partial H_T}{\partial u_1} = 0,\ at\, u_1 = u_1^*\\
&\frac{\partial H_T}{\partial u_1} = \frac{\partial H}{\partial u_1^*} = \chi_1 u_1 + \lambda_S \rho\beta SC - \lambda_E \rho\beta SC = 0\\
&\qquad u_1^* = \frac{\rho\beta SC\left(\lambda_E - \lambda_S\right)}{\chi_1}\\
&\frac{\partial H_T}{\partial u_2} = 0,\ at\, u_2 = u_2^*\\
&\frac{\partial H_T}{\partial u_2} = \frac{\partial H}{\partial u_2^*} = \chi_2 u_2 - \lambda_C \gamma u_2 CQ + \lambda_T \gamma u_2 CQ = 0\\
&\qquad u_2^* = \frac{\gamma CQ\left(\lambda_C - \lambda_T\right)}{\chi_2}\\
&\frac{\partial H_T}{\partial u_3} = 0,\ at\, u_3 = u_3^*\\
&\frac{\partial H_T}{\partial u_3} = \frac{\partial H}{\partial u_3^*} = \chi_3 u_3 + \lambda_J \eta\phi u_3 T + \lambda_R\left(1-\phi\right)\eta u_3 T - \lambda_T\left(\eta u_3 + \mu\right)T = 0\\
&\qquad u_3^* = \frac{\eta T\left(\lambda_R\left(1-\phi\right) + \left(\lambda_T - \lambda_J\right)\right)}{\chi_3}
\end{aligned}\right\} \tag{15}$$

Moreover, by using the boundary condition and simplifying the solution of (16), we obtain the following optimal controls:

$$u_1^* = \max\left\{0, \min\left(1, \frac{\rho\beta SC\left(\lambda_E - \lambda_S\right)}{\chi_1}\right)\right\} \tag{16}$$

$$u_2^* = \max\left\{0, \min\left(1, \frac{\gamma CQ\left(\lambda_C - \lambda_T\right)}{\chi_2}\right)\right\} \tag{17}$$

$$u_3^* = \max\left\{0, \min\left(1, \frac{\eta T\left(\lambda_R\left(1-\phi\right) + \left(\lambda_T - \lambda_J\right)\right)}{\chi_3}\right)\right\} \tag{18}$$

As described by Joshi (2002), the constraint is typical in control problem. Hence, the optimal control function is described, and we can use the simulation of optimality system to determine the best strategies that minimize corruption dynamics.

4. Numerical Simulation

In this section, numerical simulation for the analysis carried out in section 3 are presented. The optimal systems were solved using forward - backward sweep to solve the state and adjoint system in order to obtain the optimal strategy. The variables and parameters values used in the simulation are displayed in Table 2 and 3.

Table 2: Variables for the model

Variables	Values
$S(t)$	2000
$E(t)$	1000
$C(t)$	500
$T(t)$	300
$J(t)$	95
$R(t)$	20
$H(t)$	30
$Q(t)$	2

Table 3: Parameters for the Model

Parameter	Value	Sources
ρ	0.36	Abdulrahman (2014)
β	0.234	Lemecha (2018)
α	0.3	Lemecha (2018)
σ	0.007	Alemneh (2020)
γ	0.21	Assumed
η	0.01	Assumed
θ	0.35	Assumed
ω	0.35	Alemneh (2020)
ξ	0.1	Assumed
ψ	0.03	Lemecha (2018)
δ	0.01	Assumed
μ	0.016	Lemecha (2018)
ϕ	0.03	Assumed
Λ	85	Alemneh (2020)
π	1.0	Assumed

The impact of implementing strategy A in controlling the spread of corruption

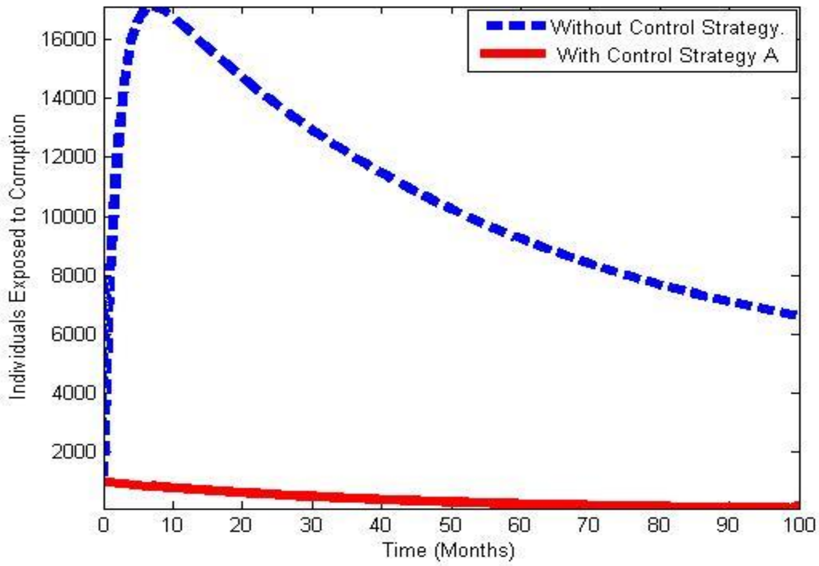

Figure 2: Simulation results of the optimal control model showing the effectiveness of strategy A in averting the number of individuals exposed to corruption against without control strategy.

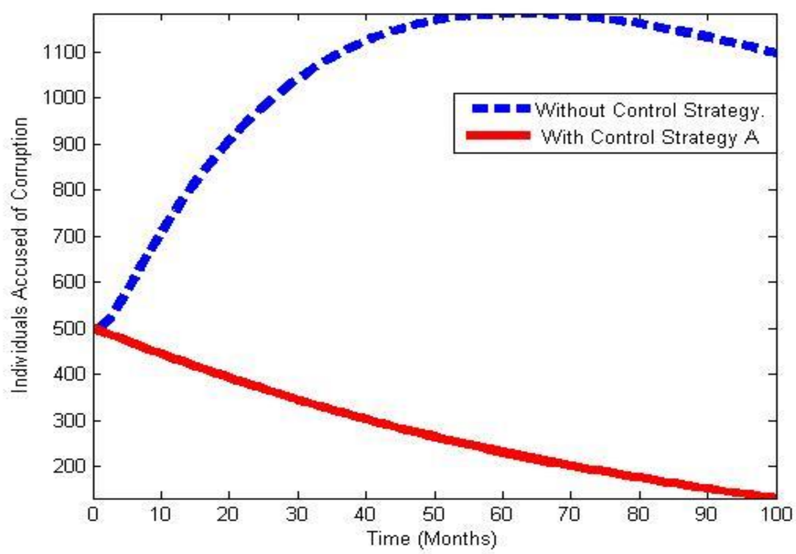

Figure 3: Simulation results of the optimal control model showing the effectiveness of strategy A in averting the number of individuals accused of corruption against without control strategy.

The impact of implementing strategy B in controlling the spread of corruption

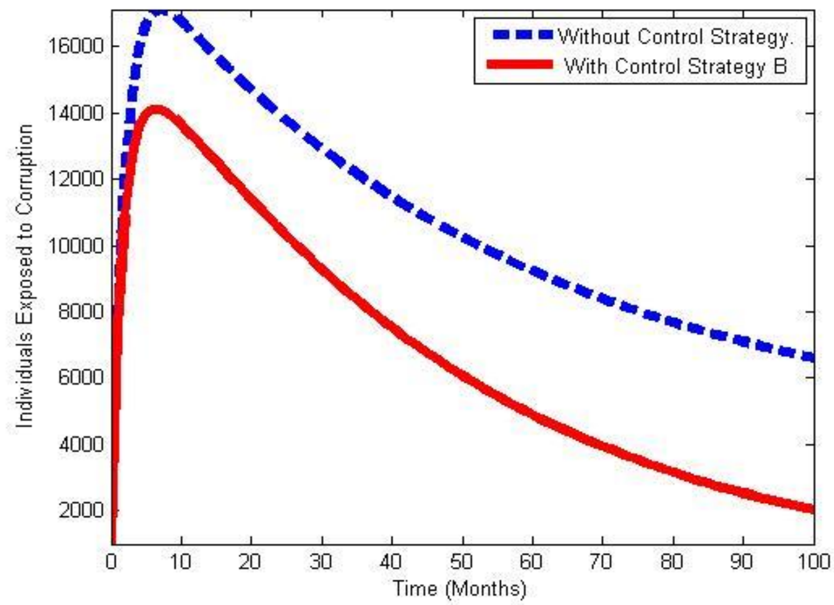

Figure 4: Simulation results of the optimal control model showing the effectiveness of strategy B in averting the number of individuals exposed to corruption against without control strategy.

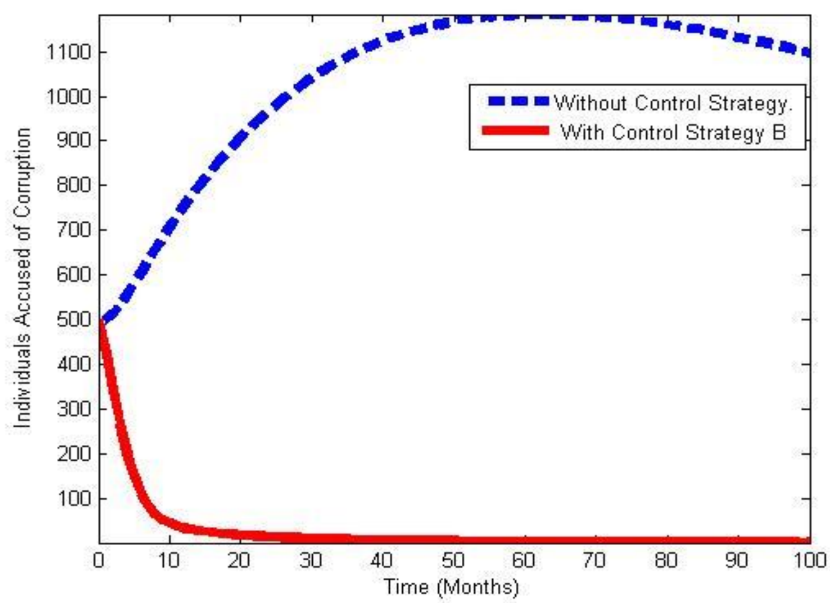

Figure 5: Simulation results of the optimal control model showing the effectiveness of strategy B in averting the number of individuals accused of corruption against without control strategy.

The impact of implementing strategy C in controlling the spread of corruption

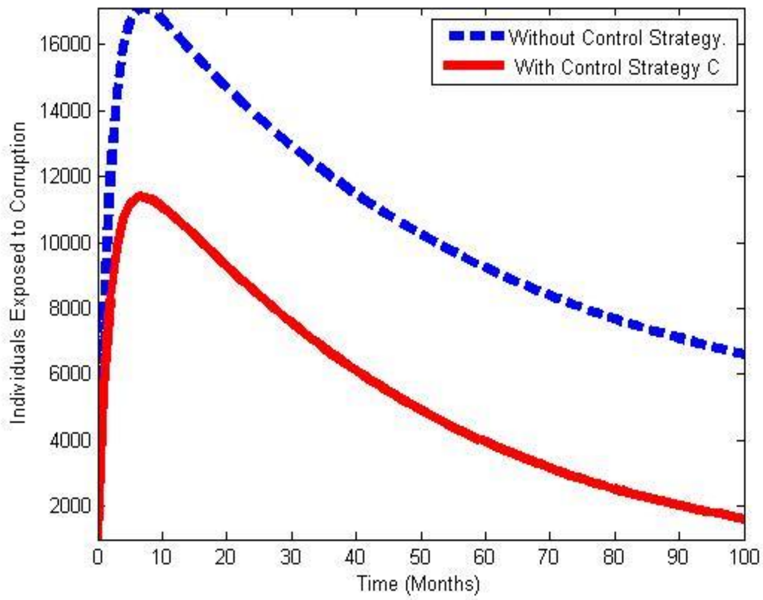

Figure 6: Simulation results of the optimal control model showing the effectiveness of strategy C in averting the number of individuals exposed to corruption against without control strategy.

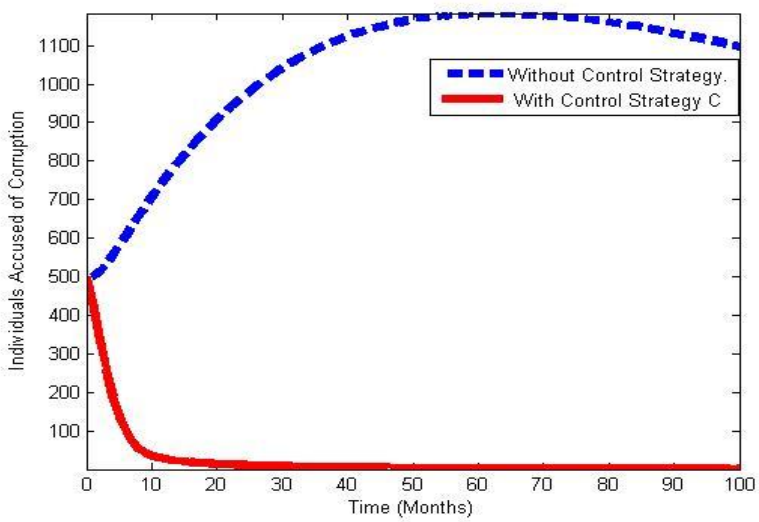

Figure 7: Simulation results of the optimal control model showing the effectiveness of strategy C in averting the number of individuals accused of corruption against without control strategy.

The impact of implementing strategy D in controlling the spread of corruption

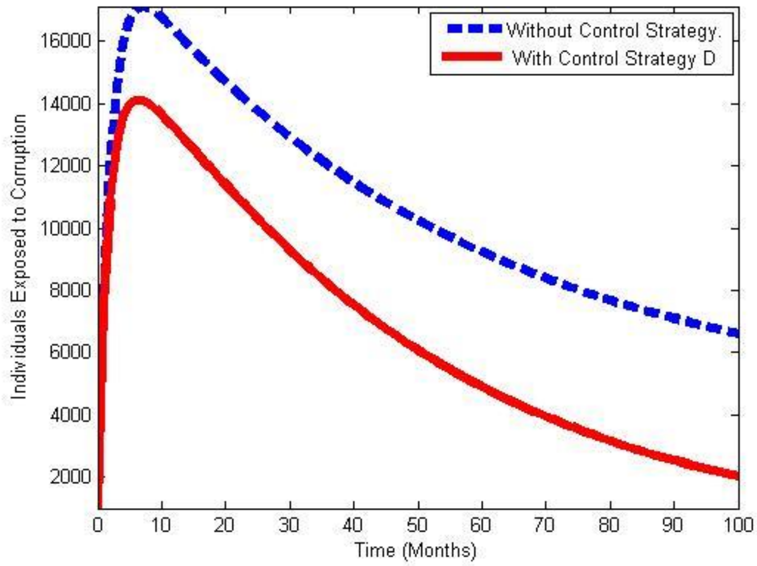

Figure 8: Simulation results of the optimal control model showing the effectiveness of strategy D in averting the number of individuals exposed to corruption against without control strategy.

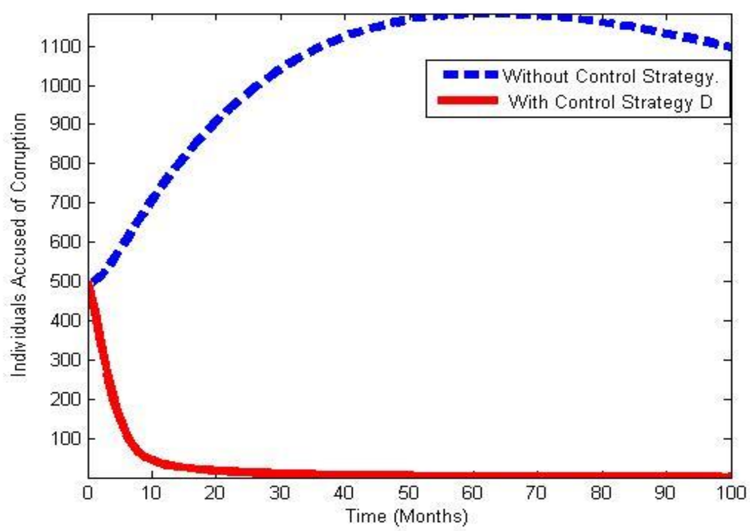

Figure 9: Simulation results of the optimal control model showing the effectiveness of strategy D in averting the number of individuals accused of corruption against without control strategy.

The impact of implementing strategy E in controlling the spread of corruption

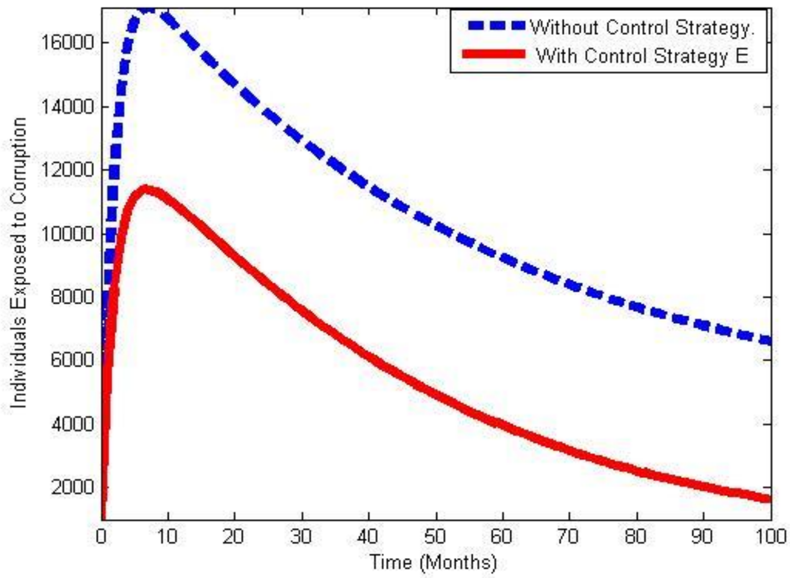

Figure 10: Simulation results of the optimal control model showing the effectiveness of strategy E in averting the number of individuals exposed to corruption against without control strategy.

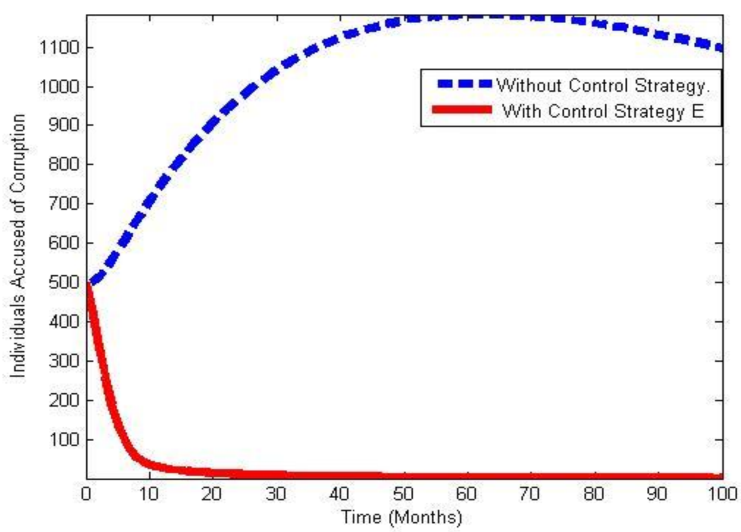

Figure 11: Simulation results of the optimal control model showing the effectiveness of strategy E in averting the number of individuals accused of corruption against without control strategy E.

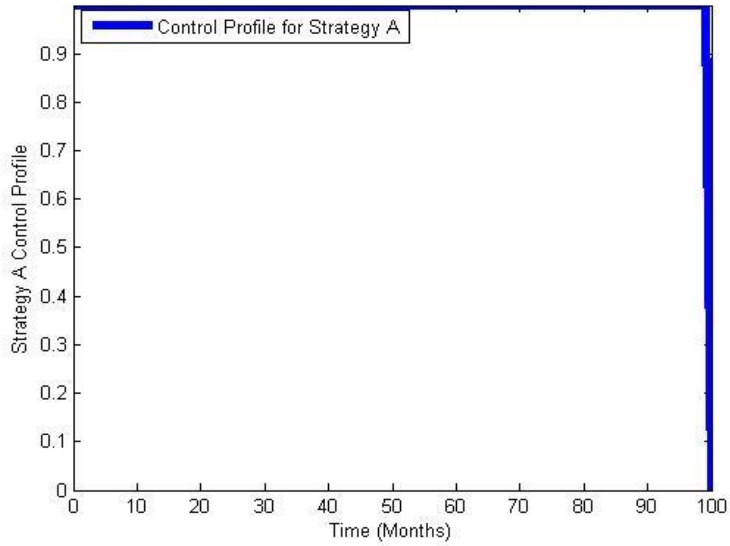

Figure 12: Graph showing the control profile of strategy A.

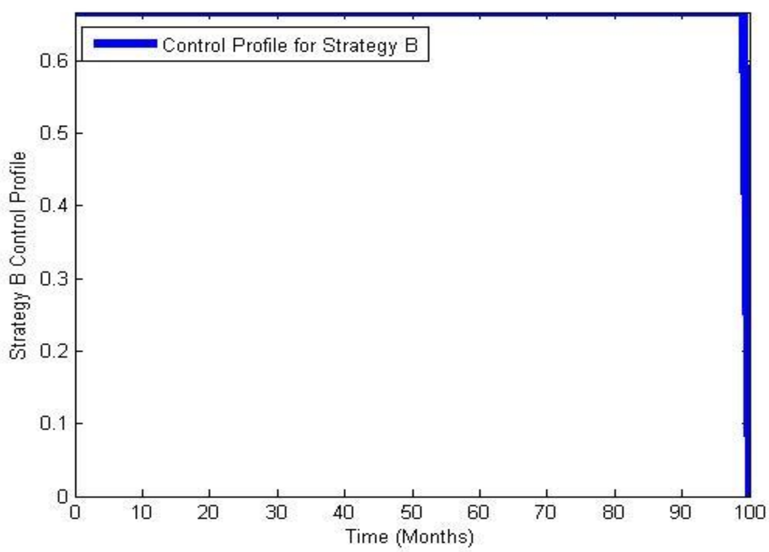

Figure 13: Graph showing the control profile of strategy B

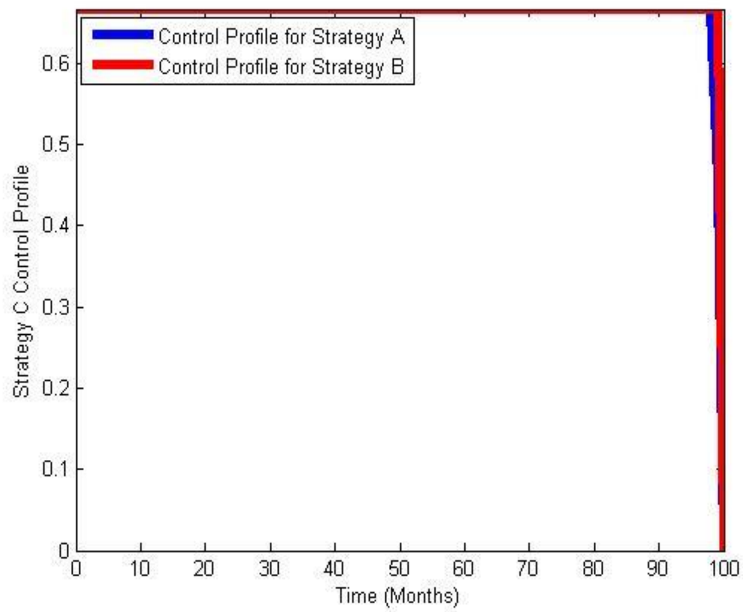

Figure 14: Graph showing the control profile of strategy C.

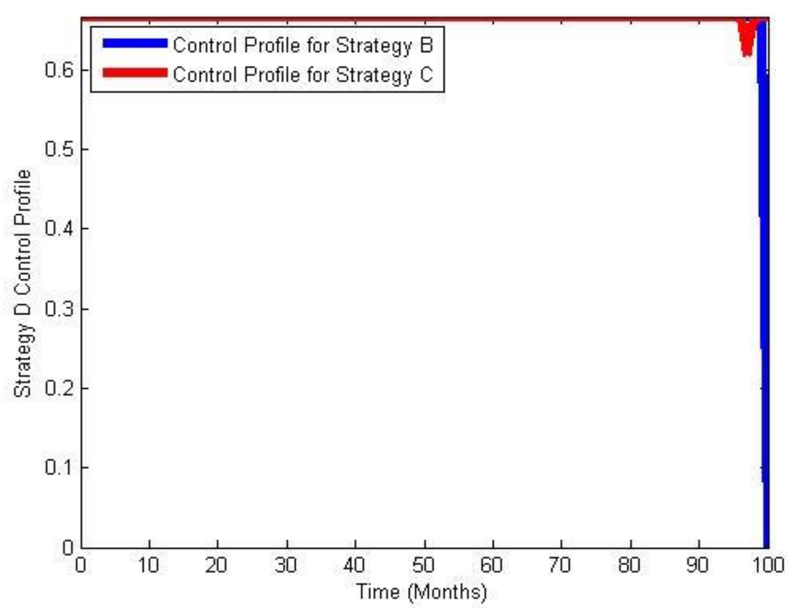

Figure 15: Graph showing the control profile of strategy D.

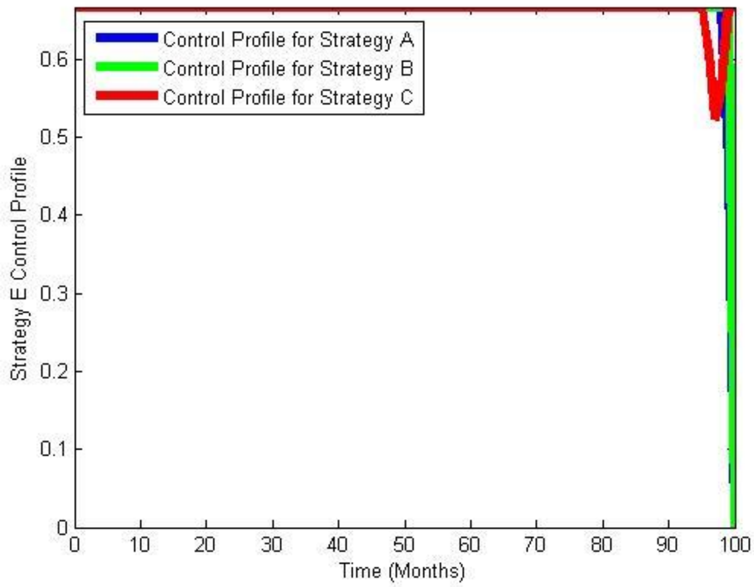

Figure 16: Graph showing the control profile of strategy E.

The impact of implementing the control strategies in preventing the cases of corruption

The number of individuals exposed to corruption and those accused of corruption prevented as a result of implementing the strategies are presented in Tables 4 and 5, respectively.

Table 4: Number of individuals exposed to corruption prevented

Strategy	Number of individuals Exposed to Corruption
A $(u_1\ only)$	1.0404×10^6
B $(u_2\ only)$	3.9271×10^5
C $(u_1\ \&\ u_2\ only)$	5.2486×10^5
D $(u_2\ \&\ u_3\ only)$	3.9209×10^5
E $(u_1, u_2, u_3\ only)$	5.2453×10^5

Table 5: Number of individuals accused of corruption prevented

Strategy	Number of individuals Accused of Corruption
A $(u_1\ only)$	7.6334×10^4
B $(u_2\ only)$	1.0193×10^5
C $(u_1\ \&\ u_2\ only)$	1.0214×10^5
D $(u_2\ \&\ u_3\ only)$	1.0193×10^5
E $(u_1, u_2, u_3\ only)$	1.0214×10^5

4.1 Discussion of numerical results

To identify the most effective control strategy for mitigating the spread of corruption, numerical simulations of the optimal control model were performed based on five different control strategies (A, B, C, D, and E). The results are discussed in the following subsection. The simulation results revealed the impact of implementing these control strategies on the dynamics of corruption over a period of 100 months. As observed from Figure 2 to Figure 11, implementing any of these control strategies results in a significant reduction in the population of individuals exposed to corruption and those accused of corruption. Conversely, failing to implement any of these control strategies leads to a substantial increase in the population of exposed individuals and those accused of corruption.

Figures 12, 13, 14, 15, and 16 depict the control profiles of strategies A, B, C, D, and E, respectively. The control profiles of these strategies reveal that to control the spread of corruption, efforts must be consistently maintained throughout the implementation period. Furthermore, the simulation results of the optimal control model presented in Tables 4 and 5 indicate that strategy A is the most effective in preventing the number of individuals exposed to corruption. In contrast, strategies C and E are the most effective in preventing the number of individuals accused of corruption.

5. Conclusion

In this study, a compartmental model is constructed to examine transmission of corruption within population. Moreover, the basic reproduction number is formulated to calculate the threshold value of the corruption. In order to develop strategies to counteract corruption, optimal control theory is applied to the model. In addition, in order to advance this control theory, three control variables are introduced in the model in the form of control strategies. This strategy includes sensitization program to prevent corruption cases by anti-corruption, arrest and prosecution, and punishing of individuals found guilty of corruption. Individual and combined effects of these control variables on all the compartments are observed and examined graphically by simulating the corruption model. Numerical simulation of the model indicates that sensitization program, prosecution and punishing of corrupt individuals can reduce the number of corrupted individuals which will further reduce the transmission of corruption, and even eradicate it to bearable minimum level. A significant and interesting direction for future research is to implement parameter estimation and cost-effectiveness analysis in the corruption dynamics model.

Data Availability

The data used to support the findings of the study are included within this article.

Conflict of Interest

The authors declare that they have no conflict of interest

Acknowledgements

The authors sincerely acknowledge the Department of Mathematics, Modibbo Adama University Yola and Nigerian Army University Biu for their support during the study.

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