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Evaluating Technical Efficiency of Selected Healthcare Centres in Yola North using Data Envelopment Analysis

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ABSTRACT

Healthcare facilities enjoy particular significance and sensitivity in health economics since they are among the largest organizations offering health and hygiene services. A novel approach to healthcare efficiency evaluation is Data Envelopment Analysis (DEA), a multi-criteria approach to hospital performance evaluation and decision-making. In order to evaluate a few chosen hospitals in Yola North, Adamawa State, Nigeria, the study applied DEA. Nine DMUs were specifically chosen for the case study, and the number of outpatients was regarded as the output. Two inputs were taken into consideration: the number of clinical staff members and beds utilized in the hospital. The Charnes, Cooper and Rhodes (CCR) model was deployed to evaluate the technical efficiency of the DMU's. The results of the efficiency evaluation for the period of (2024) shows that specialists' hospital, Peace hospital and New Bosheng hospital were technically efficient, with efficiency score of 100% indicating the inputs and outputs used in the hospital was satisfactorily used. The inefficient DMU'S were DMU B with efficiency of 83%, DMU C with efficiency score of 41%, DMU E with efficiency score of 59%, DMU F with efficiency score of 43%, DMU H with efficiency score of 39% and DMU I with efficiency score of 94%. The overall efficiency of the hospitals was not satisfactory; this indicates that these hospitals need to improve their performance, managers and policymakers of the health system in these private hospitals should provide an appropriate context to change the existing situation to improve the efficiency of the hospital service.

1. Introduction

In the field of health services, the word "performance" has gained widespread recognition. The level of achievement of predetermined goals is measured by performance. A tool for balancing quality, efficiency, and effectiveness is the idea of performance in the context of health services. As a result, the concept of performance is multifaceted and encompasses a number of different elements, including evidence-based practice (EBP), continuity and integration in healthcare services, health promotion, and patient-centeredness (Minvielle *et al*., 2008).

Regulations inspection, public opinion polls, third-party evaluations, and statistical indicators are frequently used to measure how well major hospitals are performing. Often referred to as performance indicators, or PIs, such an evaluation is typically based on a list of precise requirements. The number of beds occupied, average length of stay in the hospital, use of medical equipment, treatment error rate, patient mortality rate, etc. are some of the most crucial healthcare Key Performance Indicators (KPIs) and metrics that healthcare managers and experts use to assess the quality of healthcare services (Barliba , Andrei & Silviu-Mihail 2012).

Understanding the performance of the hospital simply entails being aware of how it carries out its mission because

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performance genuinely relates to efficacy, efficiency, and quality. Knowing how well the hospital is performing is even more crucial when you take into account the fact that it must constantly adapt to a changing external environment in order to carry out its job in all circumstances (Akazili *et al*., 2008).

In order to pursue the objective of enhancing the population's health, health systems in developing nations, like Nigeria, must overcome severe resource limitations. The ineffectiveness of the healthcare systems, particularly within public health institutions, worsens the already restricted capacity to appropriately satisfy healthcare requirements. Within the majority of health systems, efficiency has emerged as the primary goal of policymakers. This is especially true in Africa, where severe inefficiencies, particularly in the hospital sector, make it difficult to provide adequate healthcare. Since 2000, 14 African countries have conducted studies on the efficiency of their health facilities in an effort to help them develop interventions that will help them use their limited resources more efficiently (WHO, 2013).

The demand for healthcare is increasing even as there are fewer resources available to meet it due to a variety of variables, including newly emerging and reemerging health issues. The perceived widespread inefficiency in the healthcare systems, particularly inside the health centers, exacerbates the inadequate capacity to adequately satisfy healthcare requirements.

Medical errors are apparently blamed for an alarming high number of deaths in thousands of people per year in Nigerian hospitals. Nigeria's healthcare system, like those of many other Sub-Saharan African nations, struggles to meet the needs of the population due to a lack of resources. Poor economic performance, rapid growing populations, and a drop in government expenditures may all be contributing factors to the shortage of healthcare resources. In addition, communicable, material, nutritional, and newborn diseases continue to be prevalent in sub-Saharan Africa, placing further strain on the regions already limited healthcare resources (World Bank, 2013).

Although there is possibility to increase efficiency in our hospital, a stronger emphasis on transparency and guaranteeing value for money is essential. Healthcare spending is rising along with rising demand in many countries. A fundamental framework for assessing the resource management of healthcare systems and comparing their productivity to other systems is provided by measuring their relative efficacy. Also, improved utilization of healthcare resources could result in the delivery of better fundamental health services (Abbasi, Tavakoli, & Moslehi, 2012).

Hospitals use a variety of health system inputs to generate a variety of health services through a production process. Medical and surgical treatment combines labor and capital as inputs to produce outputs. The marginal change in health status, which is the ultimate goal of healthcare, is difficult to quantify in most data sets, therefore intermediate outputs, or episodes of care (such as the number of procedures and outpatient visits), typically take precedence as the main findings of studies. This production process does not occur in isolation; a number of environmental factors, both inside and outside the hospital, may have an impact on how effectively the production process occurs (Coelli, *et al*., 2005).

In recent years, a wide range of Data Envelopment Analysis (DEA) applications have been used to assess the performances of various types of entities engaging in various contexts in various nations. DEA for hospital evaluation has been the subject of numerous practical researches, which highlights the value of this method (Jacob, 2013).

DEA is a relatively recent "data-oriented" method for assessing the effectiveness of a group of peer entities known as Decision Making Units (DMUs), which transform a variety of inputs into a variety of outputs. A DMU can be defined in a variety of ways. A single efficiency measure is provided by the linear programming method used by DEA. It is founded on the idea that an organization can be seen as efficient when it is able to generate the maximum amount of commodities, products, or practicable services with a specific combination of employed resources as input, or alternatively when it does so with the least amount of input. (Garcia *et al*., 2002).

A number of sub-Saharan African nations have employed DEA to examine the technical effectiveness of their hospitals. In order to quantify efficiency, several studies have employed various inputs and outputs. Studies carried out in Eritrea (WHO, 2010), and Botswana (Asbu *et al*., 2010),for instance, are just a few examples. The majority of studies concentrated on assessing the first stage of efficiency analysis; a small number of studies carried out the second stage analysis to look at the factors that influence technical (in) efficiency using panel data.

Little investigations into the effectiveness of hospitals in Ethiopia have been made. The technical effectiveness of Ethiopian health facilities was the subject of a study by Tamiu (2002), while Amare, Yitayal and Amare (2020) modeled the technical efficiency of Maternal health services provision in chosen public hospitals in Ethiopia, which used a two-stage data envelopment analysis which identified that health expenditure was on the high side while also the efficiency in the provision of maternal health services was high.

The effectiveness of hospital subunits, including surgical treatment, medical care, and quality was assessed by Khushalani and Ozcan (2017) using the Dynamic Network DEA. Their findings indicated that efficiency of quality of hospitals input and overall hospital efficiency became more effective during the time they were studying.

2. Methodology

Definition 1.1 (Extended Pareto-Koopmans Definition of Efficiency): A DMU achieves full (100%) efficiency if and only if none of its inputs or outputs can be enhanced without making some of its other inputs or outputs poorer.

The levels of efficiency that could theoretically be achieved won't be known in the majority of management or social science applications. As a result, the previous definition is replaced by the one that emphasizes only the information that is empirically available while emphasizing its uses: A DMU is to be rated as fully (100%) efficient on the basis of the information available such that only when the performances of other DMUs do not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs.

Definition 1.2 (Relative Efficiency): A DMU is to be rated as fully (100%) efficient on the basis of the available evidence.

You will note that this definition does not rely on prices or other weighting assumptions that are meant to indicate the relative significance of the various inputs or outputs. Also, it eliminates the requirement for explicitly defining the formal relationships that are anticipated to exist between inputs and outputs. This fundamental kind of efficiency, known as "technical efficiency" in economics, can, however, be expanded to additional forms of efficiency provided data like prices, unit costs, etc. are accessible for use in DEA.

Technological efficiency is accomplished when a health decision-making unit (DMU) generates a specific level of health service outputs with the least amount of health system inputs, such as human resources in the health sector, pharmaceutical and non-pharmaceutical supplies, and capital inputs (buildings, beds, equipment, vehicles, etc.).

A DMU is to be judged as entirely (100%) efficient on the basis of the evidence available when the performances of other DMUs do not reveal that some of its inputs or outputs can be improved without deteriorating some of its other inputs or outputs. Efficiency ratings are capped at zero (totally inefficient).

2.1 Data collection

The data was gathered using a questionnaire and a standardized form for secondary data. The input and output data were gathered using the structured form. The numbers of clinical staff and beds or cots were the input data. The number of outpatients was one of the output data. Nine hospitals that served as decision-making units had their technical and allocation efficiencies estimated using the inputs and outputs that had been gathered for the year 2024.

Efficiency measuring

In this study, the technical efficiency of the nine chosen hospitals in Yola North, was assessed using data envelopment analysis by maximizing production factors with the use of cross-sectional data. The technical effectiveness of the hospitals was assessed using the Charnes, Cooper, and Rhodes (CCR) model.

As a maximum ratio of the total sum of weighted outputs to the total sum of weighted inputs, the efficiency can be represented as

Efficiency = $\frac{\textit{weighted sum of hospitals outputs}}{\textit{weighted sum of hospital inputs}}$

Estimating Efficiency of inefficient DMU

Relative Efficiency = $\frac{\textit{Best possible performance}}{\textit{Actual performance}}$

The general output maximization of CCR model can be represented as follows

$$Max\ TE\ (U,V) = \sum_{r=1}^{s} U_r Y_{rj} \tag{1}$$

Subject to:

$$\sum_{i=1}^{m} = V_i X_{ij} = 1 \tag{2}$$

$$\sum_{r=1}^{s} U_r Y_{rj} - \sum_{i=1}^{m} V_i X_{ij} \leq 0 \ \ j = 1,2, \dots n$$

$U_r \geq \in$, r = 1, 2… s

$V_i \geq \in$, i = 1, 2…m

Where $\in$ is an infinitesimal non-Archimedean quantity greater than zero

$TE = Teachnical\ efficiency$

$X_{ij} = input\ i\ used\ by\ the\ j\ hospital$

$Y_{rj} = output\ r\ produce\ by\ j\ hospital$

$U_r =$ weight given to health system output r

$V_i = weight\ given\ to\ health\ system\ input\ i$

The data were entered into excel software then the necessary steps to filter the data was followed in other to determine the technical efficiency of the hospitals.

For Specialists hospital (DMU A)

$$\text{MAX } Z = 351V_{B,A} + 251V_{C,A} \tag{3}$$

Subject to:

$52{,}146U_{O,A} = 1$

$52{,}146U_{O,A} - (351V_{B,A} + 251V_{C,A}) \leq 0$

$16{,}920U_{O,A}-(13V_{B,A} + 26V_{C,A}) \leq 0$

$11{,}016U_{O,A} - (10V_{B,A} + 14V_{C,A}) \leq 0$

$21{,}018U_{O,A} - (14V_{B,A} + 18V_{C,A}) \leq 0$

$9{,}530U_{O,A} - (16V_{B,A} + 12V_{C,A}) \leq 0$

$1{,}400U_{O,A} - (13V_{B,A} + 10V_{C,A}) \leq 0$

$30{,}100U_{O,A} - (30V_{B.A} + 19V_{C,A}) \leq 0$

$3{,}090U_{O,A} - (12V_{B,A} + 11V_{C,A}) \leq 0$

$1{,}800U_{O,A} - (10V_{B,A} + 11V_{C,A}) \leq 0$

$U_{O,A,}, V_{B,A}, , V_{C,A,} \geq 0$ and integer

For Da'ama hospital (DMU B)

$$\text{MAX } Z = 13V_{B,A} + 26V_{C,A} \tag{4}$$

Subject to:

$16{,}920U_{O,A} = 1$

$16{,}920U_{O,A} - (13V_{B,A} + 26V_{C,A}) \leq 0$

$52{,}146U_{O,A} - (351V_{B,A} + 251V_{C,A}) \leq 0$

$11{,}016U_{O,A} - (10V_{B,A} + 14V_{C,A}) \leq 0$

$21{,}018U_{O,A} - (14V_{B,A} + 18V_{C,A}) \leq 0$

$9{,}530U_{O,A} - (16V_{B,A} + 12V_{C,A}) \leq 0$

$1{,}400U_{O,A} - (13V_{B,A} + 10V_{C,A}) \leq 0$

$30{,}100U_{O,A} - (30V_{B,A} + 19V_{C,A}) \leq 0$

$3{,}090U_{O,A} - (12V_{B,A} + 11V_{C,A}) \leq 0$

$1{,}800U_{O,A} - (10V_{B,A} + 11V_{C,A}) \leq 0$

$U_{O,A}, V_{B,A}, V_{C,A} \geq 0$ and integer

For Valada hospital (DMU C)

$$\text{MAX } Z = 10V_{B,A} + 14V_{C,A} \tag{5}$$

Subject to:

$11{,}016U_{O,A} = 1$

$11{,}016U_{O,A} - (10V_{B,A} + 14V_{C,A}) \leq 0$

$52{,}146U_{O,A} - (351V_{B,A} + 251V_{C,A}) \leq 0$

$16{,}920U_{O,A} - (13V_{B,A} + 26V_{C,A}) \leq 0$

$21{,}018U_{O,A} - (14V_{B,A} + 18V_{C,A}) \leq 0$

$9{,}530U_{O,A} - (16V_{B,A} + 12V_{C,A}) \leq 0$

$1{,}400U_{O,A} - (13V_{B,A} + 10V_{C,A}) \leq 0$

$30{,}100U_{O,A} - (30V_{B,A} + 19V_{C,A}) \leq 0$

$3{,}090U_{O,A} - (12V_{B,A} + 11V_{C,A}) \leq 0$

$1{,}800U_{O,A} - (10V_{B,A} + 11V_{C,A}) \leq 0$

$U_{O,A}, V_{B,A}, V_{C,A} \geq 0$ and integer

For Peace hospital (DMU D)

$$MAXZ = 14V_{B,A} + 18V_{C,A} \tag{6}$$

Subject to:

$21{,}018U_{O,A} = 1$

$21{,}018U_{O,A} - (14V_{B,A} + 18V_{C,A}) \leq 0$

$52{,}416U_{O,A} - (351V_{B,A} + 251V_{C,A}) \leq 0$

$16{,}920U_{O,A} - (13V_{B,A} + 26V_{C,A}) \leq 0$

$11{,}016 - (10V_{B,A} + 14V_{C,A}) \leq 0$

$9{,}539U_{O,A} - (16V_{B,A} + 12_{C,A}) \leq 0$

$1{,}400U_{O,A} - (13V_{B,A} + 10V_{C,A}) \leq 0$

$30{,}100U_{O,A} - (30V_{B,A} + 19V_{C,A}) \leq 0$

$3{,}090U_{O,A} - (12V_{B,A} + 11V_{C,A}) \leq 0$

$1{,}800U_{O,A} - (10V_{B,A} + 11_{C,A}) \leq 0$

$U_{O,A,}, V_{B,A,} V_{C,A} \geq 0$ and integer

For Galbose (DMU E)

$$MAX\ Z = 16V_{B,A} + 12V_{C,A} \tag{7}$$

Subject to:

$9{,}350 = 1$

$9{,}350U_{O,A} - (16V_{B,A} + 12V_{C,A}) \leq 0$

$52{,}416U_{O,A} - (351V_{B,A} + 251V_{C,A}) \leq 0$

$16{,}920U_{O,A} - (13V_{B,A} + 26V_{C,A}) \leq 0$

$11{,}016U_{O,A} - (10V_{BA} + 14V_{C,A}) \leq 0$

$1{,}400U_{O,A} - (13V_{B,A} + 10V_{C,A}) \leq 0$

$30{,}100U_{O,A} - (30V_{B,A} + 19V_{C,A}) \leq 0$

$3{,}090U_{O,A} - (12V_{B,A} + 11V_{C,A}) \leq 0$

$1{,}800U_{O,A} - (10V_{B,A} + 11V_{C,A}) \leq 0$

$21{,}018U_{O,A} - (14V_{B,A} + 18V_{C,A}) \leq 0$

$U_{O,A,}, V_{B,A}, V_{B,C.} \geq 0$ and integer

For Freedom-Poly Hospital (DMU F)

$$\text{MAX } Z = 13V_{B,A} + 10V_{C,A} \tag{8}$$

Subject to:

$1{,}400U_{O,A} = 1$

$1{,}400U_{O,A} - (13V_{B,A} + 10V_{C,A}) \leq 0$

$52{,}416U_{O,A} - (351V_{B,A} + 251V_{C,A}) \leq 0$

$16{,}920U_{O,A} - (13V_{B,A} + 26V_{C,A}) \leq 0$

$11{,}016U_{O,A} - (10V_{B,A} + 14V_{C,A}) \leq 0$

$9{,}530U_{O,A} - (16V_{B,A} + 12V_{C,A}) \leq 0$

$21{,}018U_{O,A} - (14V_{O,A} + 18V_{C,A}) \leq 0$

$30{,}100U_{O,A} - (30V_{B,A} + 19V_{C,A}) \leq 0$

$3{,}090U_{O,A} - (12V_{B,A} + 11V_{C,A}) \leq 0$

$1{,}800U_{O,A} - (10V_{B,A} + 11V_{C,A}) \leq 0$

$U_{O,A}, V_{B,A,}, V_{C,A.} \geq 0$ and integer

For New Bosheng (DMU G)

$$\text{MAX } Z = 30V_{B,A} + 19V_{C,A} \tag{9}$$

Subject:

$30{,}100U_{O,A} = 1$

$30{,}100U_{O,A} - (30V_{B,A} + 19V_{C,A}) \leq 0$

$52{,}416U_{O,A} - (351V_{B,A} + 251V_{C,A}) \leq 0$

$16{,}920U_{O,A} - (13V_{B,A} + 26V_{C,A}) \leq 0$

$11{,}016U_{O,A} - (10V_{B,A} + 14V_{C,A}) \leq 0$

$21{,}018U_{O,A} - (14V_{B,A} + 18V_{C,A}) \leq 0$

$9{,}530U_{O,A} - (16V_{B,A} + 12V_{C,A}) \leq 0$

$1{,}400U_{O,A} - (13V_{B,A} + 10V_{C,A}) \leq 0$

$3{,}090U_{O,A} - (12V_{B,A} + 11V_{C,A}) \leq 0$

$1{,}800U_{O,A} - (10V_{B,A} + 11V_{C,A}) \leq 0$

$U_{O,A}, V_{B,A}, V_{C,A} \geq 0$ and integer

For Munagu hospital (DMU H)

$$\text{MAX } Z = 12V_{B,A} + 11V_{C,A} \quad (10)$$

Subject to:

$3{,}090U_{O,A} = 1$

$3{,}090U_{O,A} - (12V_{B,A} + 11V_{C,A}) \leq 0$

$52{,}416U_{O,A} - (351V_{B,A} + 251V_{C,A}) \leq 0$

$16{,}920U_{O,A} - (13V_{B,A} + 26V_{C,A}) \leq 0$

$11{,}016U_{O,A} - (10V_{B,A} + 14V_{C,A}) \leq 0$

$21{,}018U_{O,A} - (14V_{B,A} + 18V_{C,A}) \leq 0$

$9{,}530U_{O,A} - (16V_{B,A} + 12V_{C,A}) \leq 0$

$1{,}400U_{O,A} - (13V_{B,A} + 10V_{C,A}) \leq 0$

$30{,}100U_{O,A} - (30V_{B,A} + 19V_{C,A}) \leq 0$

$1{,}800U_{O,A} - (10V_{B,A} + 11V_{C,A}) \leq 0$

$U_{O,A}, V_{B,A}, V_{C,A} \geq 0$ and integer

For Bamaiyi (DMU I)

$$\text{MAX } Z = 10V_{B,A} + 11V_{C,A} \quad (11)$$

Subject to:

$1{,}800U_{O,A} = 1$

$1{,}800U_{O,A} - (10V_{B,A} + 11V_{C,A}) \leq 0$

$52{,}416U_{O,A}-(351V_{B,A}+251V_{C,A}) \leq 0$

$16{,}920U_{O,A}-(13V_{B,A}+26V_{C,A}) \leq 0$

$11{,}016U_{O,A}-(10V_{B,A}+14V_{C,A}) \leq 0$

$21{,}018U_{O,A}-(14V_{B,A}+18V_{C,A}) \leq 0$

$9{,}530U_{O,A}-(16V_{B,A}+12V_{C,A}) \leq 0$

$1{,}400U_{O,A}-(13V_{B,A}+10V_{C,A}) \leq 0$

$30{,}100U_{O,A}-(30V_{B,A}+19V_{C,A}) \leq 0$

$3{,}090U_{O,A}-(10V_{B,A}+11V_{C,A}) \leq 0$

$U_{O,A}, V_{B,A}, V_{C,A} \geq 0$ and integer

Where U is the weight of output used and V weight of input used

3. Results and Discussion

TABLE 3.1 Technical efficiency outputs of DMU'S (Maximization)

DMU'S	Efficiency	Reference set	Relative value (V1)	Relative value (V2)
A	1	N/A	0.002849003	0
B	0.83003952	DMU A (0.037037037)	0.076923077	0
C	0.412844037	DMU A (0.028490028)	0.1	0
D	1	N/A	0.083333333	0
E	0.59870455	DMU D (0.166306821) DMU G (0.371418568)	0.028768162	0.069557099
F	0.433628319	DMU D (0.098551891) DMU G (0.088696702)	0.085545723	0.050147493
G	1	N/A	0.032115172	0.018826135
H	0.39931740	DMU D (0.072603165) DMU G (0.07863428)	0.098976109	0.05820478
I	0.943643512	DMU G (0.0550458716)	0.142857143	0

The efficiency score of DMU A, DMU D, and DMU G is 100%, meaning that for the year 2024, these DMUs were totally efficient. As a result, the inputs and outputs used in these hospitals were used satisfactorily, which is what makes them entirely efficient. The remaining DMUs (DMU B, DMU C, DMU E, DMU F, and DMU I) were operating inefficiently when 2024 was taken into account.

DMU C also has an efficiency score of 41%, implying that it is inefficient. The inefficiency could be attributed to the hospital's bed shortage and the excess amount of clinical staff that is available. DMU B has an efficiency score of 83%, implying that it is inefficient. This inefficiency could be likened to the hospital's bed shortage and the excessive amount of clinical staff. DMU B and DMU C should therefore replicate DMU A by using it as their benchmark for efficiency.

They ought to use DMU D as a model for comparison because DMU E, which also has an efficiency score of 59%, DMU F, which has a score of 43%, and DMU H, which also has a score of 39%, all exhibit to be inefficiency which could be attributed to the hospital's excess of beds and medical personnel.

DMU I have a 94% efficiency rating. They should use DMU-G as a benchmark for efficiency because it is inefficient and the inefficiency may be linked to the hospital's overabundance of beds and clinical staff. DMUs B, C, E, F, H, and I are among the ineffective DMUs.

With an efficiency rating of (0.83003952), Da'ama Specialist Hospital was determined to be inefficient. The inefficiency may be attributed to the hospital's lack of beds and surplus of clinical staff. Specialists Hospital, a successful hospital, serves as the model for Da'ama Hospital, which it emulates with a reference set of (0.037037037) and a relative value (V1) of (0.076923077).

The efficiency rating of Da'ama Hospital needs to go up by (1-0.83003952) = 0.1699648 in order for it to be considered effective.

To accomplish this,

1. Reducing input 1 by $\frac{\boldsymbol{Increase\ in\ Efficiency\ Rating}}{\boldsymbol{v1}} = \frac{\mathbf{0.16996048}}{\mathbf{0.076923077}} = 2.209486238$

With an efficiency rating of 0.412844037, Valada Hospital was determined to be inefficient. The lack of beds and excess clinical staff could be blamed for the inefficiency, and the Valada hospital's reference set of an efficient hospital, which has a reference set of (0.028490028) and a relative value (V1) of (0.1), is intended to be emulated.

Valada Hospital needs raise its efficiency rating by (1-0.412844037) = 0.587155963 in order to be effective.

This can be achieved by

1. Reducing input 1 by $\frac{\boldsymbol{Increase\ in\ Efficiency\ Rating}}{\boldsymbol{V1}} = \frac{\mathbf{0.587155963}}{\mathbf{0.1}} = 5.87155963$

Galbose Hospital's efficiency rating of (0.59870455) indicates that it is inefficient. The hospital's inefficiency may be attributed to the excess clinical staff and beds available, where DUM D (Peace Hospital) and DMU G (New Bosheng Hospital) serve as the benchmarks for Galbose Hospital to imitate, with D (0.166306821) and G (0.371418568) as their respective reference sets.

Galbose Hospital needs to raise its efficiency score by (1 - 0.59870455) = 0.40129545 in order to be considered efficient.

This can be achieved by

1. Reducing input 1 by $\frac{\boldsymbol{Increase\ in\ Efficiency\ Rating}}{\boldsymbol{V1}} = \frac{\mathbf{0.40129545}}{\mathbf{0.028768162}} = 13.94929054$
2. Reducing input 2 by $\frac{\boldsymbol{Increase\ in\ Efficiency\ Rating}}{\boldsymbol{V2}} = \frac{\mathbf{0.40129545}}{\mathbf{0.06557099}} = 5.769295381$

Freedom Poly Hospital's efficiency rating of (0.0433628319) indicates that it is inefficient. The overabundance of beds and clinical staff available for the hospital where DMU D and DMU G form the reference set for Freedom Poly Hospital to replicate with a reference set of D (0.098551891) and G (0.08869702) could be the cause of the inefficiency.

Freedom Poly Hospital must raise its efficiency rating by (1 - 0.0433628319) = 0.566371681 in order to be considered effective.

This can be achieved by

1. Reducing input 1 by $\frac{\textbf{\textit{Increase in Efficiency Rating}}}{\textbf{\textit{V1}}} = \frac{\mathbf{0.566371681}}{\mathbf{0.08554723}} = 6.620572998$

2. Reducing input 2 by $\frac{\textbf{\textit{Increase in Efficiency Rating}}}{\textbf{\textit{V2}}} = \frac{\mathbf{0.0566371681}}{\mathbf{0.050147493}} = 11.2941175$

An efficiency score of (0.039931740) indicated that Munagu Hospital was ineffective. The overabundance of beds and clinical staff available for the hospital could be blamed for the inefficiency. Munagu Hospital should replicate DMU D and DMU G using a reference set of D (0.072603165) and G (0.07863428).

In order for Munagu Hospital to be effective, it must raise its efficiency score by (1 - 0.39931740) = 0.6006826.

1. Reducing input 1 b$y\frac{\textbf{\textit{Increase in Efficiency rating}}}{\textbf{\textit{V1}}} = \frac{\mathbf{0.6006826}}{\mathbf{0.098976109}} = 6.068965592$

2. Reducing input 2 by $\frac{\textbf{\textit{Increase in Efficiency rating}}}{\textbf{\textit{V2}}} = \frac{\mathbf{0.6006826}}{\mathbf{0.058020478}} = 10.35294125$

With an efficiency rating of 0.943643512, Bamaiyi Hospital was determined to be inefficient. The inefficiency may be ascribed to the excess of beds and clinical staff available for the hospital, where DMU G serves as the benchmark for Bamaiyi Hospital to imitate with a benchmark of G (0.0550458716).

The efficiency rating of Bamaiyi Hospital must rise by (1 - 0.943643512) = 0.05635488 in order for it to be considered effective.

This can be achieved by

1. Reducing input 1 by $\frac{\textbf{\textit{Increase in Efficiency Rating}}}{\textbf{\textit{V1}}} = \frac{\mathbf{0.05635488}}{\mathbf{0.142857143}} = 0.3944841596$

In order to evaluate a few chosen hospitals in Yola, North, this study applied data envelope analysis. The purpose of the study was to examine the technical effectiveness of the hospitals and determine which ones were effective and which ones were ineffective. Evaluation of the technical effectiveness of a few chosen Yola hospitals (both training and non-training hospitals) was the goal.

Estimate the level of output growth and/or input reductions needed to make some private hospitals in Yola, North, more efficient. This includes estimating the relative technical efficiency of specialty hospitals. Nine DMUs were specifically chosen for this study's case study, which took into account two inputs: the number of beds used and the number of clinical personnel working in the hospital, as well as one output: the number of outpatients.

The technical effectiveness of the DMUs was assessed using the CCR model. The specialists' hospital, Peace hospital, and New Bosheng hospital all received a 100% efficiency score in the efficiency evaluation for the period of 2024, indicating that they were technically efficient. As a result, the inputs and outputs used in the hospital were used satisfactorily, which makes it efficient.

4. Conclusion

The use of data envelopment analysis with the possibility of comparison, ranking, and modeling can be an important step in the continuous improvement of hospital performance, especially in the country's health sector, given the high importance of hospitals in providing healthcare services as well as their high impact on the effectiveness of the health management system.

Technical efficiency of nine chosen hospitals in Yola North, was assessed using data envelopment analysis by maximizing production factors with the use of cross-sectional data. Since the identified hospitals' overall effectiveness was unsatisfactory, it is clear that they must raise their game.

The least effective DMUs were DMU I, which had an efficiency score of 94%, DMU B, which had an efficiency score of 83%, DMU E, which had a score of 59%, DMU F, which had a score of 43%, DMU C, which had a score of 41%, and DMU H, which had a score of 39%.

DMU I, DMU B, DMU E, DMU F, DMU C, and DMU H hospitals should increase the size of their hospitals into manageable size so as to have more space for beds and for outpatient department for an effective service delivery.

Both the inefficient and efficient DMU's should monitor their services delivery efficiency and need to identify inputs that are underutilized and outputs that fall shortage of targets. This may help hospitals identify input/output services needed to increase or decrease so that they might be able to deliver better services. Therefore the allocation of adequate resources and appropriate use of resources in this sector is of great importance.

Among the tools that can improve hospital performance are organizational training that focuses on improving staff abilities, staff and physician evaluations, and reviews of new methods and technology in the healthcare industry. Therefore, managers and policymakers of the health system in these private hospitals should offer a suitable framework to alter the current circumstance and enhance the effectiveness of the hospital service.

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