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Optimal Inventory Policy for Deteriorating Item under Stock Level Dependent Demand, Controllable Deterioration, Trade Credit and Complete Backlogging

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ABSTRACT

This study examines the inventory management problem for deteriorating item with stock level dependent demand and controllable deterioration under trade credit and complete backlogging and derived the objective function which is the total average cost function and minimized it. We applied the optimization problem solver in MATLAB to solve the derived objective function. From the numerical illustrations and sensitivity analysis, it is observed that the optimal total average cost and the optimal order quantity are sensitive to the initial demand rate, stock level and the reduced deteriorating rate parameters.

1. Introduction

Physical commodities like fruits, vegetables, food items etc., when stored undergo depletion by direct spoilage. Also, some other commodities undergo deterioration through gradual loss of potential or utility as time passes. For instance, electronic goods, grains, chemicals, pharmaceutical products etc. Therefore, goods with these characteristics can be found in real life situation. Inventory theory can be seen as a set of mathematical models that describes the properties of a wide variety of inventory systems and as well as different methodologies studies that seek and analyze the best strategies that can be employed in inventories management (Luis *et al*., 2021). Rajan and Uthayakumar (2017) studied an economic order quantity model. In their work, they examined the optimal pricing and replenishment policies for instantaneous deteriorating items with backlogging, trade credit and inflation and obtained the optimal selling price, optimal order quantity and optimal replenishment time for profit maximization problem.

Luis *et al*. (2021) studied an inventory model where effects of time and selling price are component of the demand rate. In this work, the demand rate consists of two power functions. One depends on the selling price while the other depend on the time elapsed since the last inventory replenishment. They also allowed shortages and the shortages are fully backlogged. Furthermore, they determined the optimal inventory policies by optimizing the profit function and presented numerical examples to verify their results. The work of Amalesh *et al*. (2021) studied the impact of learning effect on rework policy, inflation and the time value of money and concluded that manufacturers can avoid loss due to presence of defective items, decrease the total cost of the manufacturing system and increase the rework rate of imperfect items by adopting rework policy.

Moreover, common practice demonstrates that if one lowers the selling price of a commodity, customers demand for that commodity increases. The work of Halim *et al.* (2021) explored this very idea. Their work considered stock dependent demand and nonlinear price dependent demand for production inventory model for deteriorating items.

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In practice, a firm might derive some advantages if customers can wait for certain period until the next replenishment to get their order. Hence, complete backlogging or partially backlogging of shortages is allowed. That is, any customer that come for transaction during the stock – out period is willing to wait until the next replenishment or some of them that come for transaction during the stock – out period are willing to wait until the next replenishment. Some of other work found in (Birbil *et al*., 2015; San – Jose *et al*., 2020; Mishra *et al*., 2015; San – Jose *et al*., 2019) also considered the concept of shortages and backlogging.

Offer of trade credit period is a widespread practice in business. It encourages patronage and new customers as well help in reducing on – hand inventory level since it might encourage customers to buy more. For instance, Banu *et al*. (2021) studied a supply chain model. The model considered demand rate that is stock dependent demand rate under trade credit financing. The work of Pervin *et al*. (2019) also studied an inventory models for deteriorating multi – items where the demand rate is stock dependent and price dependent under trade credit policy. Shaikh *et al*. (2020) examined a production model for deteriorating item under trade credit policy. Here, they studied how inflation and price dependent demand affect the model. Sushil and Rajput (2015) presented perishable item inventory model where demand depends on time varying stock under inflation. They assumed the supplier can offer trade credit to their customers and the length of the credit period depends on the quantity ordered. They derived the optimal policy through minimization of the total cost function and presented numerical examples to illustrate their results. One of their findings from their results is that if the credit period is large, then customers would want to order more items.

After the global financial crisis, large scale inflation has been experienced by many countries. As a result, inflation and time value of money has become an important component of inventory management. Shuai *et al*., (2013) considered an inventory model for perishable products with stock dependent demand under inflation. Here, they assumed that the supplier can offer trade credit but the quantity ordered determined the length of the trade credit. They also minimized the cost function to derive the optimal inventory policies and provided numerical examples to illustrate results.

Jie *et al*., (2010) also studied a model to illustrate real life business scenario by considering an inventory model under two level trade credit where the demand rate is stock dependent and the deterioration is also stock dependent. The kind of arrangement where both the vendor and the buyer can offer trade credit to their prospective customers can be referred to as two level trade credit. In the model, It is assumed that the supplier can offer a fixed credit period and the retailer in turn offers a fixed credit period to customers in order to promote market competition. Furthermore, they provided the necessary and sufficient conditions for the existence and uniqueness of the optimal solutions that maximized the average profit function.

Deterioration of physical goods is one of the important factors put into consideration in an inventory or production management. Hence, it has become very necessary to control and maintain inventories for decaying items. This work examined inventory management problem for deteriorating item with stock level dependent demand and controllable deterioration under trade credit and complete backlogging. Focusing on how to minimize the average cost function of the inventory.

2. Models Formulation

2.1 Constants and Variables

K The ordering cost per order

c the unit purchasing cost

h The holding cost per unit time (excluding interest charges)

s Unit selling price $(s > c)$

c_1 shortage cost per unit per order

c_2 Opportunity cost due to lost sales

I_e Interest earned per Naira per unit of time by the retailer

I_p Interest charges per Naira in stock per unit of time to the supplier

$I(t)$ The level of inventory at time t

I_m Maximum inventory level for each replenish cycle

I_b Maximum amount of demand backlogged per cycle

M Retailer's trade credit period offered by supplier per unit time

t_1 Time at which the inventory level falls to zero

T Inventory cycle length

Q Retailer's order quantity

ω Maximum capital constraint

ξ Preservation technology cost per unit time for reducing deterioration rate in order to preserve the products $(0 \leq \xi \leq \omega)$

2.2 Assumptions and Models

i. The replenishment rate is infinite

ii. Lead time is zero

iii. Planning horizon of the inventory system is infinite

iv. There is no repair or replacement of deteriorated items during the period under consideration

v. The inventory model deals with single item

vi. The reduced deterioration rate $N(\xi)$ is an increasing function of the preservation technology cost ξ where $\lim_{\xi \to \infty} N(\xi) = \theta$

vii. The demand rate function $D(t)$ is deterministic and a function of instantaneous stock level $I(t)$. When inventory is positive, $D(t)$ is given by

$$D(t) = \alpha + \beta I(t), \qquad 0 \leq t \leq t_1$$

And when inventory is negative, $D(t)$ is given by

$$D(t) = \alpha, \qquad 0 \leq t \leq T$$

$\alpha > 0$ and $0 < \beta < 1$, are demand rate and a fixed fraction of stock level parameters respectively

viii. Shortages are allowed. That is, the demands that are not satisfied are backlogged and the fraction of shortages backordered is $b(t) = e^{-\delta t}\ \delta > 0, t$ is the time of waiting for the next replenishment and $0 \le b(t) \le 1,\ b(0) = 1$. Note that if $b(x) = 1$ (or 0) for all t, then the shortages are completely backlogged (or lost). We assumed that the shortages are completely backlogged.

ix. If the trade credit period M is offered, the retailer would settle the account at $t = M$ and pay for the interest charges on the items in stock with rate I_p over the interval $[M, t_1]$ if $t_1 \ge M$ and if the retailer settles the account at $t = M$, the retailer pays no interest charge on items in stock during the whole cycle if $t_1 \le M$

x. The retailer can accumulate revenue and earn interest from the beginning of the inventory cycle until the end of the trade credit period offered by the supplier. i.e., the revenue of the retailer can accumulate and the retailer can also earn interest during the period from $t = 0\ to\ t = M$ with rate I_e under the trade credit arrangement.

Considering the assumptions above, the model for the inventory level at any time is given by

$$\frac{dI(t)}{dt} = \begin{cases} -\alpha - \beta I(t) - [\theta - N(\xi)]I(t) & 0 \le t \le t_1 \\ -\alpha b(t) & t_1 \le t \le T \end{cases} \tag{1}$$

With boundary condition $I(t_1) = 0$

During the interval $[0, t_1]$, The decreases in the inventory level is due to combined effects of deterioration and demand, inventory drops to zero during the time interval $[0, t_1]$. During the interval $[t_1, T]$, shortages occurred which are completely backlogged. From equation (1), the rate of change of inventory level at any time t can be represented by the following differential equation:

$$\frac{dI(t)}{dt} = -\alpha - \beta I(t) - [\theta - N(\xi)]I(t) \quad 0 \le t \le t_1$$

$$\frac{dI_1}{dt} = -\alpha b(t) \qquad t_1 \le t \le T$$

With the boundary conditions $I(0) = I_M, I_1(t_1) = 0$, now

$$\frac{dI(t)}{dt} = -\alpha - \beta I(t) - [\theta - N(\xi)]I(t) \quad 0 \le t \le t_1$$

$$\frac{dI(t)}{dt} = -\alpha - [\beta + (\theta - N(\xi))]I(t)$$

$$= -(\alpha + [\beta + \theta - N(\xi)]I(t))$$

$$\int_t^{t_1} \frac{dI(t)}{\alpha + [\beta + \theta - N(\xi)]I(t)} = \int_t^{t_1} -dt$$

$$\int_t^{t_1} \frac{dI(t)}{\alpha + [\beta + \theta - N(\xi)]I(t)} = -\int_t^{t_1} dt$$

$$\ln\left[\frac{\alpha + [\beta + \theta - N(\xi)]I(t_1)}{\alpha + [\beta + \theta - N(\xi)]I(t)}\right] = -(\beta + \theta - N(\xi))(t_1 - t)$$

$$\frac{\alpha + [\beta + \theta - N(\xi)]I(t_1)}{\alpha + [\beta + \theta - N(\xi)]I(t)} = e^{-(\beta + \theta - N(\xi))(t_1 - t)}$$

Since $I(t_1) = 0$, we Have

$$\frac{\alpha}{\alpha + [\beta + \theta - N(\xi)]I(t)} = e^{-(\beta+\theta-N(\xi))(t_1-t)}$$

$$\alpha + [\beta + \theta - N(\xi)]I(t) = \alpha e^{(\beta+\theta-N(\xi))(t_1-t)}$$

$$[\beta + \theta - N(\xi)]I(t) = \alpha e^{(\beta+\theta-N(\xi))(t_1-t)} - \alpha$$

$$I(t) = \frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta + \theta - N(\xi)}$$

$$I_m = I(0) = \frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} \quad (2)$$

3. The Objective Function Components

Ordering cost per cycle $= K$ (3)

Holding Cost $= h\int_0^{t_1} I(t)\,dt$

$$= h\int_0^{t_1} \frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta + \theta - N(\xi)}dt$$

$$= \frac{\alpha h}{\beta + \theta - N(\xi)}\int_0^{t_1}[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]\,dt$$

$$= \frac{\alpha h}{\beta + \theta - N(\xi)}\left|\frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{-(\beta + \theta - N(\xi))} - t\right|_0^{t_1}$$

$$= \frac{\alpha h}{\beta + \theta - N(\xi)}\left(\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{-(\beta + \theta - N(\xi))} - t_1\right] - \left[\frac{e^{(\beta+\theta-N(\xi))(t_1-0)}}{-(\beta + \theta - N(\xi))} - 0\right]\right)$$

$$= \frac{\alpha h}{\beta + \theta - N(\xi)}\left(-\frac{1}{(\beta + \theta - N(\xi))} - t_1 + \frac{e^{(\beta+\theta-N(\xi))t_1}}{(\beta + \theta - N(\xi))}\right)$$

$$= \frac{\alpha h}{\beta+\theta-N(\xi)}\left(\frac{[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - t_1\right) \quad (4)$$

For Unsatisfied demand completely backlogged, we have

$$\frac{dI_1}{dt} = -\alpha b(t) \qquad t_1 \le t \le T$$

$b(t) = 1$, therefore,

$$\frac{dI_1}{dt} = -\alpha$$

$$I_1(t) = -\alpha\int_{t_1}^{t} dt = -\alpha(t - t_1) = \alpha(t_1 - t) \quad (5)$$

Maximum backordered quantity

$$I_1(T) = I_{1m} = \alpha(t_1 - T) \tag{6}$$

Here, no lost sales or opportunity cost

Purchase cost (case of complete backlogging)

Maximum quantity $= Q = I_m - I_{1m}$

$$Q = I_m - I_{1m} = \frac{\alpha[e^{(\beta+\theta-N(\xi))t_1 - 1}]}{\beta+\theta-N(\xi)} + \alpha(T - t_1) \tag{7}$$

$$\text{Purchase cost} = cQ = c\left(\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} + \alpha(T - t_1)\right)$$

$$= \frac{\alpha c[e^{(\beta+\theta-N(\xi))t_1 - 1}]}{\beta+\theta-N(\xi)} + \alpha c(T - t_1) \tag{8}$$

Shortage cost (case of complete backlogging)

$$\text{Shortage cost} = -c_1 \int_{t_1}^{T} I_1(t)\, dt = -c_1\alpha \int_{t_1}^{T} (t_1 - t)\, dt$$

$$= -c_1\alpha \left| t_1 t - \frac{t^2}{2} \right|_{t_1}^{T}$$

$$= -c_1\alpha \left[\left(t_1 T - \frac{T^2}{2}\right) - \left(t_1^2 - \frac{t_1^2}{2}\right)\right]$$

$$= -c_1\alpha \left[t_1 T - \frac{T^2}{2} - t_1^2 + \frac{t_1^2}{2}\right]$$

$$= c_1\alpha \left[-t_1 T + \frac{T^2}{2} + t_1^2 - \frac{t_1^2}{2}\right]$$

$$= c_1\alpha \left[\frac{T^2}{2} - \frac{t_1^2}{2} + t_1^2 - t_1 T\right] \tag{9}$$

Preservation technology cost $= \xi T$ (10)

$$\text{Sales revenue} = s\left(\int_0^{t_1} D(t)\, dt - I_1(T)\right)$$

$$= s\int_0^{t_1} (\alpha + \beta I(t))\, dt - s\alpha(t_1 - T)$$

$$= s\int_0^{t_1} \left(\alpha + \beta\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta+\theta-N(\xi)}\right) dt + s\alpha(T - t_1)$$

$$= s\int_0^{t_1} \left(\alpha + \beta\alpha\frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{\beta+\theta-N(\xi)} - \frac{\beta\alpha}{\beta+\theta-N(\xi)}\right) dt + s\alpha(T - t_1)$$

$$= s\left(\alpha t - \frac{\beta\alpha}{\beta+\theta-N(\xi)} \times \frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{\beta+\theta-N(\xi)} - \frac{\beta\alpha t}{\beta+\theta-N(\xi)}\right)\Big|_0^{t_1} + s\alpha(T-t_1)$$

$$= s\left(\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha t}{\beta+\theta-N(\xi)}\right)\Big|_0^{t_1} + s\alpha(T-t_1)$$

$$= s\left[\left(\alpha t_1 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}\right) - \left(0 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta+\theta-N(\xi)]^2} - \frac{0}{\beta+\theta-N(\xi)}\right)\right]$$

$$+s\alpha(T-t_1)$$

$$= s\left[\alpha t_1 - \frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha t_1}{\beta+\theta-N(\xi)} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right] + s\alpha(T-t_1)$$

$$= s\left[\alpha t_1 - \frac{\beta\alpha t_1}{\beta+\theta-N(\xi)} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}\right] + s\alpha(T-t_1)$$

$$= s\left[\frac{\alpha t_1[\beta+\theta-N(\xi)] - \beta\alpha t_1}{\beta+\theta-N(\xi)} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1} - \beta\alpha}{[\beta+\theta-N(\xi)]^2}\right] + s\alpha(T-t_1)$$

$$= s\left[\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)} + \frac{\beta\alpha(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta+\theta-N(\xi)]^2}\right] + s\alpha(T-t_1)$$

$$= s\alpha\left[\frac{t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)} + \frac{\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta+\theta-N(\xi)]^2}\right] + s\alpha(T-t_1) \tag{11}$$

$$\text{Deteriorating cost} = d_c\left[I_m - \int_0^{t_1} D(t)\,dt\right] = d_c\left[I_m - \int_0^{t_1}(\alpha+\beta I(t))\,dt\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - \int_0^{t_1}\left(\alpha + \beta\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta+\theta-N(\xi)}\right)dt\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - \left(\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)} + \frac{\alpha\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta+\theta-N(\xi)]^2}\right)\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - \frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)} - \frac{\alpha\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - \frac{\alpha\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta+\theta-N(\xi)]^2} - \frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\right]$$

$$= d_c\left[\frac{\alpha[\beta+\theta-N(\xi)][e^{(\beta+\theta-N(\xi))t_1} - 1] - \alpha\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta+\theta-N(\xi)]^2} - \frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\right]$$

$$= d_c\left[\frac{\alpha[\theta-N(\xi)][e^{(\beta+\theta-N(\xi))t_1} - 1]}{[\beta+\theta-N(\xi)]^2} - \frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\right]$$

$$= \frac{d_c\alpha[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\left[\frac{[e^{(\beta+\theta-N(\xi))t_1 - 1]}}{\beta+\theta-N(\xi)} - t_1\right] \tag{12}$$

Interest payable and earned

If the end of the credit period is shorter than or equal to the length of period in which the inventory is positive $(M \leq t_1)$, payment for goods is settled and the retailer starts paying the capital opportunity cost for the items in stock with rate I_p. We also assumed that while the account is yet to be settled, the retailer can sell the goods and continue to accumulate sales revenue and earn interest with the rate I_e. Hence, interest earned and payable per cycle for different cases are given below:

Case I: $M \leq t_1$

Interest payable $= cI_p \int_M^{t_1} I(t)dt$

$$= cI_p \int_m^{t_1} \left(\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta + \theta - N(\xi)} \right) dt$$

$$= \frac{cI_p\alpha}{\beta + \theta - N(\xi)} \int_M^{t_1} \left(e^{(\beta+\theta-N(\xi))(t_1-t)} - 1\right) dt$$

$$= \frac{cI_p\alpha}{\beta + \theta - N(\xi)} \left(\frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{-(\beta + \theta - N(\xi))} - t \right) \Big|_M^{t_1}$$

$$= \frac{cI_p\alpha}{\beta + \theta - N(\xi)} \left[\left(\frac{e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{-(\beta + \theta - N(\xi))} - t_1 \right) - \left(\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}}{-(\beta + \theta - N(\xi))} - M \right) \right]$$

$$= \frac{cI_p\alpha}{\beta + \theta - N(\xi)} \left[\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}}{(\beta + \theta - N(\xi))} + M - \frac{1}{\beta + \theta - N(\xi)} - t_1 \right]$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)} \left[\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}-1}{(\beta+\theta-N(\xi))} + M - t_1 \right] \quad (13)$$

Interest earned $= sI_e \int_0^m \int_0^t D(s)ds\, dt$

$$= sI_e \int_0^m \left[\int_0^t D(s)ds \right] dt$$

$$= sI_e \int_0^m \left[\int_0^t (\alpha + \beta I(s))ds \right] dt$$

$$= sI_e \int_0^m \left[\int_0^t \left(\alpha + \frac{\beta\alpha[e^{(\beta+\theta-N(\xi))(t_1-s)} - 1]}{\beta + \theta - N(\xi)} \right) ds \right] dt$$

$$= sI_e \int_0^M \left[\int_0^t \left(\alpha + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{\beta + \theta - N(\xi)} - \frac{\beta\alpha}{\beta + \theta - N(\xi)} \right) ds \right] dt$$

$$= sI_e \int_0^M \left[\left(\alpha s - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha s}{\beta + \theta - N(\xi)} \right) \Big|_0^t \right] dt$$

$$= sI_e \int_0^M \left[\left(\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha t}{\beta+\theta-N(\xi)} \right) - \left(0 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta+\theta-N(\xi)]^2} - 0 \right) \right] dt$$

$$= sI_e \int_0^M \left[\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha t}{\beta+\theta-N(\xi)} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2} \right] dt$$

$$= sI_e \left(\frac{\alpha t^2}{2} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^3} - \frac{\beta\alpha t^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha t e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2} \right) \Big|_0^M$$

$$= sI_e \left[\left(\frac{\alpha M^2}{2} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-M)}}{[\beta+\theta-N(\xi)]^3} - \frac{\beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2} \right) \right.$$
$$\left. - \left(0 + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3} - 0 + 0 \right) \right]$$

$$= sI_e \left[\frac{\alpha M^2}{2} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-M)}}{[\beta+\theta-N(\xi)]^3} - \frac{\beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2} \right.$$
$$\left. - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3} \right]$$

$$= sI_e \left[\frac{\alpha M^2}{2} - \frac{\beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-M)}}{[\beta+\theta-N(\xi)]^3} - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3} \right.$$
$$\left. + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2} \right]$$

$$= sI_e \left[\frac{\alpha M^2(\beta+\theta-N(\xi)) - \beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}(e^{-(\beta+\theta-N(\xi))M} - 1)}{[\beta+\theta-N(\xi)]^3} \right.$$
$$\left. + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2} \right]$$

$$= sI_e \left[\frac{\alpha M^2(\theta-N(\xi))}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}(e^{-(\beta+\theta-N(\xi))M} - 1)}{[\beta+\theta-N(\xi)]^3} \right.$$
$$\left. + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2} \right] \quad (14)$$

Case II: $t_1 \leq M$

Here, the cycle time t, is less than or equal to the credit period M, so no interest will be paid. Hence, the retailer pays no interest at the end of the inventory cycle. Thus,

$$\text{Interest payable} = 0$$

However, from time 0 to t_1, the retailer can sell the goods and continue to accumulate sales revenue to earn interest $sI_e \int_0^{t_1} \int_0^t D(s)\, ds\, dt$. Also, from time t_1 to M, the retailer uses the sales revenue generated in $[0, t_1]$ to earn interest

$sI_e \int_0^{t_1} D(s)ds\,(M - t_1)$. Thus, the interest earned in this period per cycle can be described by:

$$\text{Interest earned} = sI_e\left[\int_0^{t_1}\int_0^{t} D(s)\,ds\,dt + \int_0^{t_1} D(t)dt\,(M - t_1)\right]$$

$$= sI_e\int_0^{t_1}\int_0^{t} D(s)\,ds\,dt + sI_e\int_0^{t_1} D(t)dt\,(M - t_1)$$

$$= sI_e\int_0^{t_1}\int_0^{t} D(s)\,ds\,dt + sI_e\int_0^{t_1} D(t)dt\,(M - t_1)$$

$$= sI_e\int_0^{t_1}\left[\int_0^{t} D(s)ds\right]dt + sI_e\int_0^{t_1} D(t)dt\,(M - t_1)$$

$$= sI_e\int_0^{t_1}\left[\int_0^{t}(\alpha + \beta I(s))ds\right]dt + sI_e(M - t_1)\int_0^{t_1}(\alpha + \beta I(t))dt$$

$$= sI_e\int_0^{t_1}\left[\int_0^{t}\left(\alpha + \frac{\beta\alpha[e^{(\beta+\theta-N(\xi))(t_1-s)} - 1]}{\beta + \theta - N(\xi)}\right)ds\right]dt$$

$$+sI_e(M - t_1)\int_0^{t_1}\left(\alpha + \frac{\beta\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta + \theta - N(\xi)}\right)dt$$

$$= sI_e\int_0^{t_1}\left[\int_0^{t}\left(\alpha + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{\beta + \theta - N(\xi)} - \frac{\beta\alpha}{\beta + \theta - N(\xi)}\right)ds\right]dt$$

$$+sI_e(M - t_1)\int_0^{t_1}\left(\alpha + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{\beta + \theta - N(\xi)} - \frac{\beta\alpha}{\beta + \theta - N(\xi)}\right)dt$$

$$= sI_e\int_0^{t_1}\left[\left(\alpha s - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha s}{\beta + \theta - N(\xi)}\right)\Bigg|_0^t\right]dt$$

$$+sI_e(M - t_1)\left(\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t}{\beta + \theta - N(\xi)}\right)\Bigg|_0^{t_1}$$

$$= sI_e\int_0^{t_1}\left[\left(\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t}{\beta + \theta - N(\xi)}\right) - \left(0 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta + \theta - N(\xi)]^2} - 0\right)\right]dt$$

$$+sI_e(M - t_1)\left[\left(\alpha t_1 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t_1}{\beta + \theta - N(\xi)}\right) - \left(0 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta + \theta - N(\xi)]^2} - 0\right)\right]$$

$$= sI_e\int_0^{t_1}\left[\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t}{\beta + \theta - N(\xi)} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta + \theta - N(\xi)]^2}\right]dt$$

$$+sI_e(M - t_1)\left[\alpha t_1 - \frac{\beta\alpha}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t_1}{\beta + \theta - N(\xi)} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta + \theta - N(\xi)]^2}\right]$$

$$= sI_e\left(\frac{\alpha t}{2}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^3}-\frac{\beta\alpha t}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right)\Big|_0^{t_1}$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= sI_e\left[\left(\frac{\alpha t_1^2}{2}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{[\beta+\theta-N(\xi)]^3}-\frac{\beta\alpha t_1^2}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right)\right.$$

$$\left.-\left(0+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta+\theta-N(\xi)]^3}-0+0\right)\right]$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= sI_e\left[\frac{\alpha t_1^2}{2}+\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^3}-\frac{\beta\alpha t_1^2}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right.$$

$$\left.-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}\right]$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= sI_e\left[\frac{\alpha t_1^2}{2}-\frac{\beta\alpha t_1^2}{2(\beta+\theta-N(\xi))}-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}+\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^3}\right.$$

$$\left.+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= sI_e\left[\frac{\alpha t_1^2(\beta+\theta-N(\xi))-\beta\alpha t_1^2}{2(\beta+\theta-N(\xi))}-\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^3}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$+sI_e(M-t_1)\left[\frac{\alpha t_1[\beta+\theta-N(\xi)]-\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= sI_e\left[\frac{\alpha t_1^2(\theta-N(\xi))}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^3}\right]$$

$$+sI_e(M-t_1)\left[\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^2}\right] \quad (15)$$

4. Cost Minimization Problem for the Case of Complete Backlogging

Our problem here is to determine the optimal value of t which minimizes $X(t)$. The necessary condition for minimization of the Total cost function $X(t)$ are:

$$\frac{d}{dt}X(t) = 0 \quad (16)$$

Equations (16) can be solved for t to obtain the optimal value of t (say t^*). The sufficient condition for $X(t)$ to be a minimum is that the

$$\frac{d^2}{dt^2}X(t) > 0 \quad (17)$$

If we put the above components into consideration, then the objective function can be described by:

$$X(t) = \frac{1}{T}\{\text{odering cost} + \text{holding cost} + \text{purchasing cost} + \text{deteriorating cost} + \text{shortage cost} + \text{opportunity cost} + \text{preservation cost} + \text{interest payable}\}$$

Therefore, for

Case I: $M \leq t_1$

Here, opportunity cost $= 0$, therefore, the total average cost is:

$$X_1(t_1) = \frac{1}{T}\Bigg\{K + \frac{\alpha h}{\beta+\theta-N(\xi)}\left(\frac{[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - t_1\right) + \left(\frac{\alpha c[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} + \alpha c(T - t_1)\right) + \frac{d_c\alpha[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\left[\frac{[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - t_1\right] + c_1\alpha\left[\frac{T^2}{2} - \frac{t_1^2}{2} + t_1^2 - t_1T\right] + \xi T + \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-M)} - 1}{(\beta+\theta-N(\xi))} + M - t_1\right]\Bigg\} \quad (18)$$

Case II: $t_1 \leq M$

$$X(t) = \frac{1}{T}\left\{\begin{matrix}\text{odering cost} + \text{holding cost} + \text{purchasing cost} + \text{deteriorating cost} + \text{shortage cost} \\ +\text{opportunity cost} + \text{preservation cost} + \text{interest payable}\end{matrix}\right\}$$

interest payable $= 0$

opportunity cost $= 0$

Therefore, the total average cost is:

$$X_2(t_1) = \frac{1}{T}\Bigg\{K + \frac{\alpha h}{\beta+\theta-N(\xi)}\left(\frac{[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - t_1\right) + \left(\frac{\alpha c[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} + \alpha c(T - t_1)\right) + \frac{d_c\alpha[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\left[\frac{[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - t_1\right] + c_1\alpha\left[\frac{T^2}{2} - \frac{t_1^2}{2} + t_1^2 - t_1T\right] + \xi T\Bigg\} \quad (19)$$

5. Numerical and Sensitivity Analysis

Case I: $M \leq t_1$

For numerical examples, we solved Equ. (18)., an inventory system with the following parameter set. α=40units,

β=0.04, T=40days, M=30days, c=1Naira/unit, h=1Naira/unit time, c_1=1Naira/unit/order, θ=0.7, K=30, $N(\xi)$=0.02, ξ=0.3, d_c=0.3, I_p=0.14.

The optimal time, $t_1^* = 34.6870$

Optimal Total average cost, $X_1^* = 226.0181$

Optimal order quantity, $Q^* = 4116$

Table 1 The effect of the parameter θ on X_1^* and Q^*

θ	X_1^*	Q^*
0.2	676.9940	7889.4
0.3	496.2063	6058.6
0.4	371.2616	5145.1
0.5	298.2561	4623.6
0.6	253.9602	4308
0.7	226.0181	4116
0.8	208.0314	4005.5

α=40units, β=0.04, T=40days, M=30days, c=1Naira/unit, h=1Naira/unit time, c_1=1Naira/unit/order, K=30, $N(\xi)$=0.02, ξ=0.3, d_c=0.3, I_p=0.14.

Table 2 The effect of the parameter α on X_1^* and Q^*

α	X_1^*	Q^*
20	104.5255	2023.3
30	156.2784	3034.9
40	208.0314	4005.5
50	259.7844	5058.2

β=0.04, T=40days, M=30days, c=1Naira/unit, h=1Naira/unit time, c_1=1Naira/unit/order, θ=0.8, K=30, $N(\xi)$=0.02, ξ=0.3, d_c=0.3, I_p=0.14.

Table 3 The effect of the parameter β on X_1^* and Q^*

β	X_1^*	Q^*
0.04	208.0314	4046.6
0.05	206.4093	4042.5
0.06	204.8543	4038.8
0.07	203.3635	4035.5
0.08	201.9346	4032.6

α=40units, T=40days, M=30days, c=1Naira/unit,

h=1Naira/unit time, c_1=1Naira/unit/order, θ=0.8, K=30,

$N(\xi)$=0.02, ξ=0.3, d_c=0.3, I_p=0.14.

Case II: $t_1 \leq M$

For numerical examples, we solved Equ. (19)., an inventory system with the following parameter set. α=40units, β=0.04, T=40days, c=1Naira/unit, h=1Naira/unit time, c_1=1Naira/unit/order, θ=0.7, K=30, $N(\xi)$=0.02, ξ=0.3, d_c=0.3, I_p=0.14.

The optimal time, $t_1^* = 35.0474$

Optimal Total average cost, $X_2^* = 224.5341$

Optimal order quantity, $Q^* = 4542.7$

Table 4 The effect of the parameter θ on X_1^* and Q^*

θ	X_2^*	Q^*
0.3	505.1133	4845.4
0.4	372.0728	4697
0.5	297.2408	4459.4
0.6	252.5230	4270.5
0.7	224.5341	4142.7
0.8	206.6033	4067.9

α=40units, β=0.04, T=40days, c=1Naira/unit, h=1Naira/unit time,

c_1=1Naira/unit/order, K=30, $N(\xi)$=0.02, ξ=0.3, d_c=0.3.

Table 5 The effect of the parameter α on X_1^* and Q^*

α	X_2^*	Q^*
20	112.7747	2071.3
30	168.6544	3107
40	224.5341	4142.7
50	280.4138	5178.4

β=0.04, T=40days, c=1Naira/unit, h=1Naira/unit time,

c_1=1Naira/unit/order,θ=0.7, K=30, $N(\xi)$=0.02, ξ=0.3, d_c=0.3.

Table 6 The effect of the parameter β on X_1^* and Q^*

β	X_2^*	Q^*
0.04	224.5341	4142.7
0.05	222.1042	4133.5
0.06	219.7769	4124.9
0.07	217.5475	4116.7
0.08	215.4117	4109

α=40units, T=40days, c=1Naira/unit, h=1Naira/unit time,

c_1=1Naira/unit/order, θ=0.7, K=30, $N(\xi)$=0.02, ξ=0.3, d_c=0.3.

From Tables (1) and (4), we can see that the reduced deterioration rate parameter θ is sensitive to the optimal total average cost and the optimal ordered quantity. As the parameter θ increases, the optimal decision is to reduce the optimal quantity and as the result, the average cost also reduces as in the Tables. Similarly, from Tables (2) and (5), we observed that the initial demand rate parameter α is also sensitive to the optimal total average cost and the optimal ordered quantity. That is as the parameter α increases, the optimal decision is to increase the optimal quantity and hence the increase in the average cost. Furthermore, the stock level parameter β is sensitive to the optimal total average cost and the optimal ordered quantity as shown in Tables (3) and (6). That is, as the parameter β increases, optimal quantity reduces with the average cost.

6. Conclusion

We have considered an inventory management problem for deteriorating item with stock level dependent demand and controllable deterioration under trade credit and complete backlogging and derived the objective function which is the total average cost and minimizes the objective function to obtained the optimal inventory management strategies for the inventory management problem as shown in the numerical and sensitivity analysis.

References

Amalesh, K. M., Leopoldo, E. C., Barun, D., Ali, A. S., Armando, C. & Gerardo, T. (2021). An imperfect Production Model for Breakable Multi – item with Dynamic Demand and Learning Effect on Rework Over Random Planning Horizon. Journal of Risk and Financial Management. 14, 574.

http://dx.doi.org/10.3390/jrfm14120574

Banu, A., Amalesh, K. M. & Shyaamal, K. M. (2021). Adjustment of Credit Period and Stock – Dependent Demand in a Supply Chain Model With Variable Imperfectness. RAIRO – Operations Research. 55: 291 – 324.

Birbil, S. L., Bulbul, K., Frenk, J. B. G. & Mulder, H. M. (2015). On EOQ Cost Models With Arbitrary Purchase and Transportation Costs. Journal of Industrial and Management Optimization. 11: 1211 – 1245.

Halim, M. A., Paul, A., Mona, M., Alshahrani, B., Atheelah, Y. M. A. & Gamal, M. I. (2021). An Overtime Production Inventory Model for Deteriorating Items With Nonlinear Price and Stock Dependent Demand. Alexandria Engineering Journal. 60: 79 – 86.

Jie, M., Yong – Wu, Z. & Ju, Z. (2010). An Inventory Model for Deteriorating Items Under Stock – Dependent Demand and Two – level Trade Credit. Applied Mathematical modelling. 34, 3273 – 3285. doi:10.016/j.apm.2010.02.019

Luis, A. S., Joaquin, S., Manuel, G. & Jaime, F. (2021). Optimal Price and Lot Size for an EOQ model With Full Backordering Under Power Price and Time Dependent Demand. Mathematics. 9: 1848. http://doi.org/10.3390/math9161848

Mishra, S. S., Gupta, S., Yadav, S. K. & Rawat, S. (2015). Optimization of Fuzzified Economic Order Quantity Model Allowing Shortage and Deterioration With Full Backlogging. American Journal of Operations Research. 5: 103 – 110.

Pervin, M., Sankar, K. R. & Gerhard, W. W. (2019). Multi – Item Deteriorating Two – echelon Inventory Model With Price – and Stock – Dependent Demand: A Trade – Credit Policy. Journal of Industrial and Management Optimization. 15: 45 – 73.

Rajan, R. S. & Uthayakumar, R. (2017). Optimal Pricing and Replenishment Policies for Instantaneous Deteriorating Items with Backlogging and Trade Credit Under Inflation. Journal of Industrial Engineering International, Springer, Heidelberg, 13(4), 427 – 443. http://dx.doi.org/10.1007/s40092-017-0200-3

Shaikh, A. A., Leopoldo, E. C., Amalesh, K. M. & Armando, C. (2020). An Economic Production Quantity (EPQ) Model for a Deteriorating Item with Partial Trade Credit Policy for Price Dependent Demand Under Inflation and Reliability. Yugoslav Journal of Operations Research. 31: 39 – 51.

San – Jose, L. A., Sicilia, J., Gonzalez – De – La – Rosa, M. & Febles – Acosta, J. (2019). Analysis of an Inventory System with Discrete Scheduling Period, Time Dependent Demand and Backlogged Shortages. Computational Operations Research. 109: 200 – 208. http://dx.doi.org/10.1016/j.cor.2019.05.003

San – Jose, L. A., Sicilia, J., Gonzalez – De – La – Rosa, M. & Febles – Acosta, J. (2020). Best Pricing and Optimal Policy for an Inventory System Under Time – and Price – Dependent Demand and Backordering. Ann. Oper. Res. 286: 351 – 369. http://dx.doi.org/10.1007/s10479-2953-5

Sushi, K. & Rajput, U. S. (2015). An Inventory Model for Perishable Items With Time Varying Stock dependent Demand and Trade Credit Under Inflation. American Journal of Operations Research. 5: 435 – 449. http://dx.doi.org/10.4236/ajor.2015.55036

Shaui, Y., Chulung, L. & Anming, Z. (2013). An inventory Model for Perishable Products With Stock – Dependent Demand and Trade Credit Under Inflation. Mathematical Problems in Engineering. Vol. 2013, Article ID 702939, 8 pages. http://dx.doi.org/10.1155/2013/702939