INTERNATIONAL JOURNAL OF DEVELOPMENT MATHEMATICS

ISSN: 3026-8656 (Print) | 3026-8699 (Online)
journal homepage: https://ijdm.org.ng/index.php/Journals

Pourreza Transform Based Correction Functional Technique for the Solution of Nonlinear Fractional Order Differential Equations

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ARTICLE INFO

Article history:
Received 02 May 2025
Received in revised form 10 August 2025
Accepted 20 August 2025

Keywords:
Pourreza transform; Variational iteration method; Burger's equation; Newell Whitehead Segel equation, Caputo, Caputo Fabrizio.

MSC 2020 Subject classification:
35R11, 44A05, 65D25

ABSTRACT

This study presented the derivation of the Pourreza variational iteration method (PVIM) scheme and was used in determining the approximate numerical solutions of fractional Burger's equation as well as Newell-Whitehead-Segel equations in the area of Caputo and Caputo-Fabrizio derivatives. Also, the stability analysis of PVIM was conducted, and for diverse values of ϱ *, the numerical solution for the obtained proposed method was validated by comparison with the exact solution. The validity, efficiency, and accuracy of the method were illustrated by numerical and graphical results which shows that the methodology developed offers a robust mathematical tool for analysing nonlinear fractional order differential equation.*

1. Introduction

In recent times, the attention of scientists and engineers has been drawn to the exploration of fractional calculus due to its effective and efficient use in analyzing mathematical and physical problems applicable to real-life situations. It focuses on derivatives and integrals involving any real or complex arbitrary orders (Cherif and Ziane, 2018; Bastos Nuno, 2018; Hussein, 2024). The concept of differentiation can be extended to fractional orders in several approaches, which include Caputo and Caputo Fabrizio. One of the prominent advantages of Caputo fractional derivatives is the integer order initial conditions (Rida and Arafa, 2011). Caputo-Fabrizio pioneered the fractional differential operator, which has a nonsingular kernel (Bastos Nuno, 2018; Al-Refai and Pal, 2019; Martínez and Aguilar, 2019; Adel *et al*., 2023). The fact that a constant's derivative via Caputo-Fabrizio is zero is an additional advantage (Bouzenna *et al*., 2020). Because of the significance of this fractional operator, which has become the subject of numerous studies in the aspect of fractional calculus, making the interest of researchers to continually grow in finding approximate solutions to fractional differential equations containing this operator (Hussein, 2024).

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https://doi.org/10.62054/ijdm/0204.15

Notably, integral transform has been a major solution technique employed by researchers in recent years (Kumawat *et al*., 2024; Noor *et al*., 2024). Some of the integral transforms that have been used by researchers are the Laplace transform method (Adel *et al*., 2023), Khalouta transform (Kumawat *et al*., 2024), Aboodh transform (Noor *et al*.,2024), Sumudu transform method(Abedl-Rady *et al*., 2014), Yang transform method (Aruldoss and Jasmine, 2020), Elzaki transform (Ziane *et al*., 2018), Natural transform (Mistry *et al*., 2019; Igbal *et al*., 2022), Shehu transform (Bokhari, 2019), Alenezi transform (Ahmad, 2020), Mohand Transform (Khurshid and Shahnaz, 2024), Sawi transform (Liu *et al*., 2023), Mahgoub transform (Puhpam and Lydia, 2018), Pourreza transform (Ahmadi *et al*., 2019), and many others for solving problems involving differential equations (Hussein, 2022a; Hussein, 2022b; Akgül *et al*., 2023).

This work considered the fractional Newell-Whitehead-Segel equation and Burgers's equation. The Newell-Whitehead-Segel equation finds its applications in the numerical simulation of many fluid mechanics problems, including chemical reactions, Rayleigh-Benard convection, Faraday instability, biological systems, and nonlinear optics (Latif *et al*, 2020). Newell, Whitehead, and Segel modelled the equation given as;

$$u_t(x,t) = Iu_{xx}(x,t) + Ju(x,t) - Ku^{\eta}(x,t)\,, \tag{1}$$

with $i, j,$ and k taken to be real numbers, and n to be positive integers (Saadeh *et al*, 2019; Prakash and Verma, 2019).

Moreover, one of the numerous fundamental nonlinear partial differential equations in the area of fluid mechanics is the Burgers's equation. In many aspects of applied mathematics, including acoustic waves, heat conduction, traffic flow, and gas dynamics, Burger's equation model is often encountered (llhem *et al*., 2022). The time Burgers' equation of fractional order is given as;

$$D_t^{\varrho} u + Pu\frac{\partial u}{\partial x} - Q\frac{\partial^2 u}{\partial x^2} = 0 \qquad 0 < \varrho \le 1, \tag{2}$$

where P and Q are arbitrary constants (Uddin *et al*, 2021). Utilizing various numerical techniques, including the homotopy analysis method (Dehghan *et al*., 2010), the differential transform method

(Bildik and Konuralp, 2006), the Adomian decomposition method (Jafari and Daftardar-Gejji, 2006), the variational iteration method (Aljuhani *et al*., 2022), and the homotopy perturbation method (Sakar *et al*., 2016), among other methods that can be used to obtain the approximate analytical solution of differential equations.

Furthermore, researchers have introduced many methods that combine integral transforms with semi-analytical methods to determine the numerical solutions of nonlinear fractional-order differential equations like the Sawi homotopy perturbation method (Liu *et al*., 2023), the Kamal decomposition transform method (Owolabi and Oderinu, 2024), the multi-step homotopy analysis method (Martínez and Aguilar, 2019), the Elzaki decomposition method (Hussein, 2024), the homotopy perturbation natural transform (Mistry, 2019), the natural reduced differential transform method (Khalouta, 2021), the conformable fractional reduced differential transform with Adomian decomposition (Teppawar *et al.*, 2024), the natural variational iteration method (Hussein, 2022c), and the semi-analytical iterative method (Latif *et al*., 2020). The variational iteration method is a semi-analytical technique that can be used in solving both partial and ordinary differential equations. Implementing correction functional, which help in breaking down nonlinear functions into simpler ones, this technique is flexible and can solve nonlinear differential equations with ease (Adel *et al*., 2023).

Pourreza transform is one of the integral transforms and was developed by Ahmadi *et al*., (2019). It is an integral transform with a unique kernel, which will be used in this research. Using this transformation, the differential equations can be reduced to a set of algebraic ones for easy solution. Basically, most of these integral transforms cannot solve nonlinear fractional-order differential equations unless they are modified. As a result of that, this work is therefore aimed at solving nonlinear fractional order differential equations using the scheme Pourreza transform that is capable of solving nonlinear fractional-order differential equations for both Caputo and Caputo Fabrizio derivatives and applying it to solve some problems.

2. Preliminaries and Notations

2.1 Properties and definition of Pourreza transform

By obtaining the Pourreza transform over the set of functions as (Ahmadi *et al*, 2019; Akgül *et al*, 2023)

$$s=\left\{ f(t): z_1, z_2>0, |\, f(t)\,|< Me^{|t|/k}, t\in(-1)\times(0,\infty)\right\}, \tag{3}$$

where s denotes the set, M the constant with finite number, and z_1, z_2 finite or infinite constant. The operator $P(\cdot)$ defines the Pourreza transform by the integral equation given as:

$$P\{ f(t)\} = w\int_0^\infty e^{-w^2 t} f(t)dt = F(w) \qquad t>0\ , w>0. \tag{4}$$

2.2 Some definitions of Pourreza transforms and fractional calculus

Definition 1: For a function $f(t)$, the Pourreza transform in Caputo derivative is given as (Hussein, 2022a);

$$P\left\{ {}^{C}D_x^{\varrho} f(t)\right\} = w^{2\varrho}F(w)-\sum_{k=0}^{n-1} w^{2\varrho-2k-1} f^{k}\left(0\right). \tag{5}$$

Definition 2: For a function $f(t)$, the Pourreza transform in Caputo Fabrizio derivative is given as (Hussein, 2022b);

$$P\left\{ {}^{CF}D_x^{\varrho} f(t)\right\} = \frac{F(w)-\dfrac{f(0)}{w}}{1-\varrho+\varrho w^{-2}}. \tag{6}$$

Definition 3: The correction functional from the Variational iteration method is constructed as as (Abedl-Rady *et al*, 2014; Aljuhani *et al*, 2022; Adel *et al*, 2023;):

$$f_{\eta+1}(t) = f_\eta(t)+\int_0^t \psi(r)\left(D_x^{\varrho} f_\eta(r)+Lf_\eta(r)+Nf_\eta(r)-g(r)\right)d(r). \quad \eta=0,1,2,..., \tag{7}$$

and the general Lagrange multiplier is ψ, which can be optimally identified utilizing the variational theory. Subscript η is the nth approximation, and the restricted variation is f_η .

2.3 Some stability concepts

Definition 4: Let a metric space be $((\Omega),|.|)$ and A: $\Omega \to \Omega$ be a mapping which is said to be a contraction, if for all $\phi_1, \phi_2 \in \Omega$ and a real positive constant $\gamma < 1$ then, (Adel *et al*, 2023):

$$| A\phi_1 - A\phi_2 | \le \gamma | \phi_1 - \phi_2 |. \tag{8}$$

This means that any pair of points $\phi_1, \phi_2 \in \Omega$ have images closer than the points ϕ_1, ϕ_2 or in other words, the ratio in the equation below does not go beyond a positive constant γ, which is <1.,

$$\frac{| A\phi_1 - A\phi_2 |}{| \phi_1 - \phi_2 |}. \tag{9}$$

By uniqueness theorem and Picard's existence for differential equations, the initial value problem of first order to be considered is given as;

$$\dot{u}(t) = A(t;u), \quad u(t_0) = u_0 \tag{10}$$

Taking t_0 and u_0 to be two real numbers and A to be a continuous mapping on the rectangle, then

$$\bar{R} = \{(t;u) : | t - t_0 | \le a, | u - u_0 | \le b\} \tag{11}$$

Hence, A is bounded on $\bar{R}$. Then, for all $(t;u) \in \bar{R}$, $| A(t;u) | \le c$.

In its second argument, if A satisfies the Lipschitz condition on $\bar{R}$. Then, $\exists$ a Lipschitz constant γ, $\ni$ for all $(t;u),(t;v) \in \bar{R}$,

$$| A(t;u) - A(t,v) | \le \gamma | u - v |. \tag{12}$$

With the stated conditions, the initial value problem in (10) is with a unique solution in the interval $(t_0 - \psi, t_0 + \psi)$, where $\psi < \{a, \frac{b}{c}, \frac{1}{\gamma}\}$.

In order to apply Pourreza transform to solve differential equations some formula of Pourreza transform of derivatives needs to be established.

3. Methodology

The Pourreza transform of the first, second, and nth order derivative of a function is achieved by the definition of the Pourreza transform, the basic principle of integration, and the principle of mathematical induction. These results were employed in the derivation of the Pourreza transform scheme for solving nonlinear differential equations.

Theorem 1. Suppose $f(t)$ is a continuous function which is of exponential order and let $F(w)$ be a Pourreza transform of $f(t)$ where $P\{f(t)\} = F(w)$ then, the Pourreza transform of the first, second, and nth order derivatives is given by:

1. $P\{f^{'}(t)\} = w^2F(w) - wf(0)$,
2. $P\{f^{''}(t)\} = w^4F(w) - w^3f(0) - wf^{'}(0)$,
3. $P\{f^{n}(t)\} = w^{2n}P\{f(t)\} - \sum_{k=0}^{n-1} w^{2n-2k-1} f^{k}(0)$.

Proof:

Applying (4) on the first part of the theorem and using integrating by parts

$$P\{f^{'}(t)\} = w\int_0^\infty e^{-w^2t} f^{'}(t)dt = -wf(0) + w^3\int_0^\infty e^{-w^2t} f(t)dt, \tag{13}$$

therefore.

$$P\{f^{'}(t)\} = w^2F(w) - wf(0). \tag{14}$$

Similarly, applying (4) on the second theorem and using integrating by part

$$P\{f^{''}(t)\} = -wf^{'}(0) + w^3\int_0^\infty e^{-w^2t} f^{'}(t)dt, \tag{15}$$

then,

$$P\left\{f^{''}(t)\right\}=-wf^{'}(0)+w^{2}\left[-wf(0)+w^{2}F(w)\right], \tag{16}$$

therefore,

$$P\left\{f^{''}(t)\right\}=w^{4}F(w)-w^{3}f(0)-wf^{'}(0). \tag{17}$$

The last part of the theorem will be shown by using mathematical induction. If $n=1$ in the third theorem then (14) is gotten back, and it shows that $n=1$ is true. Let $n=\xi$ and assume it is true then,

$$P\left\{f^{\xi}(t)\right\}=w^{2\xi}P\left\{f(t)\right\}-\sum_{k=0}^{\xi-1}w^{2\xi-2k-1}f^{k}(0). \tag{18}$$

To prove that $n=\xi+1$ is true whenever $n=\xi$, then;

$$P\left\{f^{\xi+1}(t)\right\}=w^{2\xi+2}P\left\{f(t)\right\}-\sum_{k=0}^{\xi}w^{2\xi-2k+1}f^{k}(0), \tag{19}$$

then,

$$P\left\{f^{\xi+1}(t)\right\}=P\left\{\left(f^{\xi}(t)\right)\right\}=w^{2\xi}P\left\{f^{'}(t)\right\}-\sum_{k=0}^{\xi-1}w^{2\xi-2k-1}f^{k+1}(0), \tag{20}$$

substituting the value of $P\left\{f^{'}(t)\right\}$ gives;

$$P\left\{f^{\xi+1}(t)\right\}=w^{2\xi}\left[w^{2}F(w)-wf(0)\right]-\sum_{k=0}^{\xi-1}w^{2\xi-2k-1}f^{k+1}(0). \tag{21}$$

Let $y=k+1$ then,

$$P\left\{f^{\xi+1}(t)\right\}=w^{2\xi+2}F(w)-w^{2\xi+1}f(0)-\sum_{y=1}^{\xi}w^{2\xi-2y+1}f^{y}(0). \tag{22}$$

Simplifying (22) gives;

$$P\left\{f^{\xi+1}(t)\right\}=w^{2\xi+2}F(w)-\sum_{y=0}^{\xi}w^{2\xi-2y+1}f^{y}(0), \tag{23}$$

therefore,

$$P\left\{f^{\xi+1}(t)\right\}=w^{2\xi+2}F(w)-\sum_{k=0}^{\xi}w^{2\xi-2k+1}f^{k}(0). \tag{24}$$

Then, $n=\xi+1$ is true and so,

$$P\left\{f^{n}(t)\right\}=w^{2n}P\left\{f(t)\right\}-\sum_{k=0}^{n-1}w^{2n-2k-1}f^{k}(0). \tag{25}$$

3.1 Formulation of Pourreza transform of Caputo and Caputo Fabrizio derivative

New Pourreza transform schemes are obtained by incorporating the correction functional into the scheme of Pourreza transform.

Suppose the differential equation representing fractional order is written as,

$$^{C}D^{\varrho}f(t)+Lf(t)+Nf(t)=g(t), \tag{26}$$

where $^{C}D^{\varrho}f(t)$ is the fractional order derivative in Caputo sense, $Lf(t)$ represent the linear function, $Nf(t)$ denote the nonlinear function, and $g(t)$ denote the source function.

Applying Pourreza transform to (26), gives:

$$w^{2\varrho}F(w)-\sum_{k=0}^{\eta-1}w^{2\varrho-2k-1}f^{k}(0)=P\left[-Lf(t)-Nf(t)+g(t)\right], \tag{27}$$

simplifying,

$$F(w)=\sum_{k=0}^{\eta-1}w^{-2k-1}f^{k}(0)+\frac{1}{w^{2\varrho}}P\left[-Lf(t)-Nf(t)+g(t)\right]. \tag{28}$$

Applying inverse Pourreza transform to (28), gives;

$$f(t)=P^{-1}\left[\sum_{k=0}^{\eta-1}w^{-2k-1}f^{k}(0)+\frac{1}{w^{2\varrho}}P\left[-Lf(t)-Nf(t)+g(t)\right]\right], \tag{29}$$

then the initial approximation is,

$$f_0(t)=P^{-1}\left[\sum_{k=0}^{\eta-1}w^{-2k-1}f^{k}(0)\right]. \tag{30}$$

Hence,

$$f_0(t)=\sum_{k=0}^{\eta-1}\frac{t^k}{\Gamma(k+1)}f^{k}(0). \tag{31}$$

From the correction functional,

$$f_{\eta+1}(t)=f_\eta(t)+\int_0^t\psi(r)({}^{C}D_x^{\varrho}f_\eta(r)+Lf_\eta(r)+Nf_\eta(r)-g(r))d(r). \tag{32}$$

Taking Pourreza transform of (32) then,

$$P\left\{f_{\eta+1}(t)\right\}=P\left\{f_\eta(t)\right\}+\int_0^t\psi(r)({}^{C}D_x^{\varrho}f_\eta(r)+Lf_\eta(r)+Nf_\eta(r)-g(r))d(r), \tag{33}$$

simplifying,

$$\begin{aligned}P\left\{f_{\eta+1}(t)\right\}=P\left\{f_\eta(t)\right\}+\int_0^t\psi(r)(w^{2\varrho}F(w)-\sum_{k=0}^{\eta-1}w^{2\varrho-2k-1}f^{k}(0)\\+P[Lf_\eta(r)+Nf_\eta(r)-g(r)d(r)]),\end{aligned} \tag{34}$$

Regarding $Lf_\eta(r)+Nf_\eta(r)-g(r))d(r)$ as restricted term, differentiating W.R.T.

$P\left\{f_{\eta+1}(t)\right\}$ and setting

$\frac{P\left\{f_{\eta+1}(t)\right\}}{P\left\{f_\eta(t)\right\}}=0$ gives;

$$0 = 1 + \int_0^t \psi(r) w^{2\varrho}, \tag{35}$$

Which gives,

$$\psi(t) = \frac{-1}{w^{2\varrho}}. \tag{36}$$

Therefore (33) becomes,

$$P\left\{ f_{\eta+1}(t) \right\} = P\left\{ f_\eta(t) \right\} - \frac{1}{w^{2\varrho}} P[{}^C D_x^\varrho f_\eta(t) + Lf_\eta(t) + Nf_\eta(t) - g(t)], \tag{37}$$

Taking the inverse Pourreza transform, gives;

$$f_{\eta+1}(t) = f_\eta(t) - P^{-1}\left[\frac{1}{w^{2\varrho}} P[{}^C D_x^\varrho f_\eta(t) + Lf_\eta(t) + Nf_\eta(t) - g(t)] \right]. \tag{38}$$

Which is the Pourreza transform scheme of Caputo derivative. When $\eta = 0,1,2,3,...,$ the successive approximations for $u_1, u_2, ...u_\eta,$ can be obtained and converge to the exact solution.

Also, the general fractional order differential equation of Caputo Fabrizio derivative is stated as,

$${}^{CF} D^\varrho f(t) + Lf(t) + Nf(t) = g(t). \tag{39}$$

where ${}^{CF} D^\varrho f(t)$ define the fractional derivative in Caputo Fabrizio sense, $Lf(t)$ represent the linear function, $Nf(t)$ denote the nonlinear function, and $g(t)$ denote the source function.

In the same process from (26) and applying (6), gives;

$$\frac{F(w) - \dfrac{f(0)}{w}}{1 - \varrho + \varrho w^{-2}} = P[-Lf(t) - Nf(t) + g(t)]. \tag{40}$$

Simplifying,

$$F(w) = \frac{f(0)}{w} + (1 - \varrho + \varrho w^{-2}) P\left\{ -Lf(t) - Nf(t) + g(t) \right\}, \tag{41}$$

Applying inverse Pourreza transform to (41), we get;

$$f(t) = P^{-1}\left\{\frac{f(0)}{w} + (1-\varrho+\varrho w^{-2})P[-Lf(t) - Nf(t) + g(t)]\right\}, \tag{42}$$

then the initial approximation is,

$$f_0(t) = f(0). \tag{43}$$

From the correction functional,

$$f_{\eta+1}(t) = f_\eta(t) + \int_0^t \psi(r)({}^{CF}D_t^{\varrho} f_\eta(r) + Lf_\eta(r) + Nf_\eta(r) - g(r))d(r). \tag{44}$$

Taking Pourreza transform of (44) then,

$$P\left\{f_{\eta+1}(t)\right\} = P\left\{f_\eta(t) + \int_0^t \psi(r)({}^{CF}D_t^{\varrho} f_\eta(r) + Lf_\eta(r) + Nf_\eta(r) - g(r))d(r)\right\}, \tag{45}$$

simplifying,

$$P\left\{f_{\eta+1}(t)\right\} = P\left\{f_n(t)\right\} + \int_0^t \psi(r)\frac{F(w) - \frac{f(0)}{w}}{1-\varrho+\varrho w^{-2}} + P(Lf_\eta(r) + Nf_\eta(r) - g(r))d(r). \tag{46}$$

Regarding $Lf_\eta(r)+Nf_\eta(r)-g(r))d(r)$ as restricted term, differentiating W.R.T.

$P\left\{f_{\eta+1}(t)\right\}$ and setting

$\dfrac{P\left\{f_{\eta+1}(t)\right\}}{P\left\{f_\eta(t)\right\}} = 0$ gives;

$$0 = 1 + \int_0^t \frac{\psi(r)}{1-\varrho+\varrho w^{-2}}, \tag{47}$$

which gives,

$$\psi(t) = -(1-\varrho+\varrho w^{-2}). \tag{48}$$

Therefore (46) becomes,

$$P\left\{f_{\eta+1}(t)\right\}=P(f_\eta(t))-(1-\varrho+\varrho w^{-2})\left[\frac{F(w)-\dfrac{f(0)}{w}}{1-\varrho+\varrho w^{-2}}+P(Lf_\eta(t)+Nf_\eta(t)-g_\eta(t))\right]. \quad (49)$$

Using the inverse Pourreza transform, gives;

$$f_{\eta+1}(t)=f(0)-P^{-1}[(1-\varrho+\varrho w^{-2})P[Lf_\eta(t)+Nf_\eta(t)-g(t)]]. \quad (50)$$

which is the Pourreza transform scheme of Caputo Fabrizio derivative. Substituting $\eta=0,1,2,3,...,$ the successive approximations given as $u_1,u_2,...u_\eta$ can be obtained which converge to exact solution.

4. Stability Analysis of the PVIM Scheme

Theorem 2: Suppose $f(t)$ is a continous function of exponential order and γ be a positive real constant such that $\dfrac{|A(f_{\eta+1})-A(f_{\xi+1})|}{|f_\eta-f_\xi|}\le\gamma.$ Therefore, the scheme (38) is unconditionally stable.

Proof:

Considering the PVIM scheme (38) of Caputo derivative.

$$f_{\eta+1}(t)=f_\eta(t)-P^{-1}\left[\frac{1}{w^{2\varrho}}P\left[{}^{C}D_x^{\varrho}f_\eta(t)+Lf_\eta(t)+Nf_\eta(t)-g(t)\right]\right]. \quad (51)$$

Checking the stability of PVIM as follows:

$$f_{\eta+1}(t)=A\left\{f_{\eta+1}\right\}=f_\eta(t)-P^{-1}\left[\frac{1}{w^{2\varrho}}P[{}^{C}D_x^{\varrho}f_\eta(t)+Lf_\eta(t)+Nf_\eta(t)-g(t)]\right]. \quad (52)$$

Which later gives;

$$|A(f_{\eta+1}(t))-A(f_{\xi+1}(t))|\leq|f_\eta(t)-f_\xi(t)|-P^{-1}\left[\begin{array}{l}\frac{1}{w^{2\varrho}}P[{}^C D_x^{\varrho}(f_\eta(t)-f_\xi(t))+L(f_\eta(t)\\-f_\xi(t))+N(f_\eta(t)-f_\xi(t))]\end{array}\right], \quad (53)$$

implies,

$$\frac{|A(f_{\eta+1}(t))-A(f_{\xi+1}(t))|}{|f_\eta(t)-f_\xi(t)|}\leq 1-P^{-1}\left[\begin{array}{l}\frac{1}{w^{2\varrho}}P[{}^C D_x^{\varrho}(f_\eta(t)-f_\xi(t))+L(f_\eta(t)\\-f_\xi(t))+N(f_\eta(t)-f_\xi(t))]\end{array}\right]=\gamma \qquad (54)$$

Then,

$$\frac{|A(f_{\eta+1}(t))-A(f_{\xi+1}(t))|}{|f_\eta(t)-f_\xi(t)|}\leq\gamma, \qquad (55)$$

which implies that $|A(f_{\eta+1}(t))-A(f_{\xi+1}(t))|\leq\gamma|f_\eta(t)-f_\xi(t)|$. Hence, the proposed scheme (38) is unconditionally stable by (12)

Using the approach for proving the stability of PVIM for Caputo also holds for Caputo Fabrizio derivative.

5. Applications

This study aims at solving the fractional Burger's equations and Newell-Whitehead-Segel equation in Caputo and Caputo Fabrizio sense.

Example 1: Fractional Burger's equation in Caputo sense is given by (Llhem *et al*, 2022; Uddin *et al*, 2021).

$${}^C D_t^{\varrho}u+u\frac{\partial u}{\partial x}=\frac{\partial^2 u}{\partial x^2}, \qquad 0<\varrho\leq 1, \qquad (56)$$

with the initial condition

$$u(x,0)=x, \qquad (57)$$

The exact solution is given by

$$u(x,t)=\frac{x}{1+t}. \tag{58}$$

Following the PVIM for the Caputo derivative, the correction functional can be constructed as:

$$u_{\eta+1}=u_{\eta}-P^{-1}\left[\frac{1}{w^{2\varrho}}P[{}^{C}D_{t}^{\varrho}u_{\eta}+u_{\eta}(u_{\eta})_{x}-(u_{\eta})_{xx}]\right] \tag{59}$$

Starting with the initial approximations $u_0(t)=\sum_{\varrho=0}^{\eta-1}\frac{t^{\varrho}}{\Gamma(\varrho+1)}u^{\varrho}(0)$, then,

$$u_0=x.$$

From the correction functional substituting $\eta=0,1,2,...,$ then,

$$u_1=x-\frac{xt^{\varrho}}{\Gamma(\varrho+1)},$$

$$u_2=x-\frac{xt^{\varrho}}{\Gamma(\varrho+1)}+\frac{2x\Gamma(\varrho+1)t^{2\varrho}}{\Gamma(\varrho)\varrho\Gamma(2\varrho+1)}-\frac{x\Gamma(2\varrho+1)t^{3\varrho}}{\Gamma(\varrho)^{2}\varrho^{2}\Gamma(3\varrho+1)},...$$

Where u_3 is the solution to (56).

For Caputo Fabrizio derivative, with regard to (50), the correction functional can be constructed as:

$$u_{\eta+1}=u_{0}-P^{-1}\left[(1-\varrho+\varrho w^{-2})P[u_{\eta}(u_{\eta})_{x}-(u_{\eta})_{xx}]\right] \tag{60}$$

Starting with the initial approximations for the Caputo Fabrizio derivative $u_0(t)=u(0),$ then,

$$u_0=x.$$

From the correction functional substituting $\eta=0,1,2,...,$ then,

$$u_1=\varrho x-\varrho xt,$$

$$u_2 = x - \varrho^2 xt^2 + 2\varrho^2 xt - \varrho^2 x + 2\varrho^3 xt^2 - 3\varrho^3 xt + \varrho^3 x - \frac{1}{3}\varrho^3 xt^3,\ldots$$

$$u_3 = \varrho x + \varrho^4 x + \varrho^7 x - 4\varrho^3 x + 2\varrho^2 x + 3\varrho^5 x - 3\varrho^6 x - \varrho xt + \varrho^4 xt^2 + 2\varrho^2 xt^2 + 12\varrho^3 xt - 8\varrho^3 xt^2$$

$$+4/3\varrho^3 xt^3 - 4\varrho^2 xt - 5/6\varrho^4 xt^4 + 2\varrho^4 xt^3 + \frac{22\varrho^5 xt^4}{3} - \frac{62\varrho^5 xt^3}{3} + 26\varrho^5 xt^2 - \frac{13\varrho^5 xt^5}{15}$$

$$-4\varrho^4 xt - 15\varrho^5 xt - 15\varrho^6 xt^4 + \frac{100\varrho^6 xt^3}{3} - 36\varrho^6 xt^2 + \frac{46\varrho^6 xt^5}{15} + 18\varrho^6 xt - 2/9\varrho^6 xt^6$$

$$+16\varrho^7 xt^2 - 17\varrho^7 xt^3 - 7\varrho^7 xt + \frac{55\varrho^7 xt^4}{6} - \frac{38\varrho^7 xt^5}{15} + 1/3\varrho^7 xt^6 - \frac{\varrho^7 xt^7}{63},\ldots$$

Where u_3 is the solution to equation (56).

Example 2: Fractional Newell-Whitehead-Segel equation in Caputo sense (Saadeh *et al*, 2019).

$${}^{C}D_t^{\varrho} u = u_{xx} + 2u - 3u^2 \qquad 0 < \varrho \le 1, \tag{61}$$

with initial condition

$$u(x,0) = \psi. \tag{62}$$

The exact solution is given by,

$$u(x,t) = \frac{\frac{-2}{3}\psi e^{2t}}{\frac{-2}{3} + \psi - \psi e^{2t}}, \tag{63}$$

Following the PVIM for the Caputo derivative, the correction functional can be constructed as:

$$u_{\eta+1} = u_\eta - P^{-1}\left[\frac{1}{w^{2\varrho}} P\left[{}^{C}D_t^{\varrho} u_\eta + 3u_\eta^2 - 2u_\eta - (u_\eta)_{xx}\right]\right] \tag{64}$$

Starting with the initial approximations $u_0(t) = \sum_{\varrho=0}^{\eta-1} \frac{t^{\varrho}}{\Gamma(\varrho+1)} u^{\varrho}(0)$, then,

$$u_0 = \psi.$$

From the correction functional substituting $\eta = 0,1,2,...,$ then,

$$u_1 = \psi - \frac{3\psi^2 t^{\varrho}}{\Gamma(\varrho+1)} + \frac{2\psi t^{\varrho}}{\Gamma(\varrho+1)},$$

$$u_2 = \psi - \frac{3\psi^2 t^{\varrho}}{\Gamma(\varrho+1)} + \frac{2\psi t^{\varrho}}{\Gamma(\varrho+1)} + \frac{18\psi^3\Gamma(\varrho+1)t^{2\varrho}}{\Gamma(\varrho)\varrho\Gamma(2\varrho+1)} - \frac{18\psi^2\Gamma(\varrho+1)t^{2\varrho}}{\Gamma(\varrho)\varrho\Gamma(2\varrho+1)}$$

$$-\frac{27\psi^4\Gamma(2\varrho+1)t^{3\varrho}}{\Gamma(\varrho)^2\varrho^2\Gamma(3\varrho+1)} + \frac{36\psi^3\Gamma(2\varrho+1)t^{3\varrho}}{\Gamma(\varrho)^2\varrho^2\Gamma(3\varrho+1)} - \frac{12\psi^2\Gamma(2\varrho+1)t^{3\varrho}}{\Gamma(\varrho)^2\varrho^2\Gamma(3\varrho+1)} + \frac{4\psi\Gamma(\varrho+1)t^{2\varrho}}{\Gamma(\varrho)\varrho\Gamma(2\varrho+1)},...$$

Where u_3 is the solution to equation (61).

For Caputo Fabrizio derivative, with regard to (50), the correction functional can be constructed as:

$$u_{\eta+1} = u_0 - P^{-1}[(1-\varrho+\varrho w^{-2})P[3u_\eta^2 - 2u_\eta - (u_\eta)_{xx}]] \qquad (65)$$

Starting with the initial approximations for the Caputo Fabrizio derivative $u_0(t) = u(0),$ then,

$$u_0 = \psi.$$

From the correction functional substituting $\eta = 0,1,2,...,$ then,

$$u_1 = -3\varrho\psi^2 t + 3\varrho\psi^2 + 2\varrho\psi t - 2\varrho\psi - 3\psi^2 + 3\psi,$$

$$\begin{aligned} u_2 = {} & -9\varrho^3\psi^4t^3 + 54\varrho^3\psi^4t^2 + 12\varrho^3\psi^3t^3 - 81\varrho^3\psi^4t - 72\varrho^3\psi^3t^2 - 4\varrho^3\psi^2t^3 - 54\varrho^2\psi^4t^2 + 27\varrho^3\psi^4 + 108\varrho^3\psi^3t \\ & + 24\varrho^3\psi^2t^2 + 162\varrho^2\psi^4t + 81\varrho^2\psi^3t^2 - 36\varrho^3\psi^3 - 36\varrho^3\psi^2t - 81\varrho^2\psi^4 - 252\varrho^2\psi^3t - 33\varrho^2\psi^2t^2 - 81\varrho\psi^4t \\ & + 12\varrho^3\psi^2 + 126]\varrho^2\psi^3 + 108\varrho^2\psi^2t + 2\varrho^2\psi t^2 + 81\varrho\psi^4 + 144\varrho\psi^3t - 54\varrho^2\psi^2 - 8\varrho^2\psi t - 144\varrho\psi^3 \\ & - 75\varrho\psi^2t - 27\psi^4 + 4\varrho^2\psi + 75\varrho\psi^2 + 10\varrho\psi t + 54\psi^3 - 10\varrho\psi - 33\psi^2 + 7\psi,..., \end{aligned}$$

Where u_3 is the solution to (61)

6. Results and Discussion

A new numerical technique for the analysis of Burger's and Newell-Whitehead-Segel equations of fractional order was derived through the use of the fundamental principle of the Pourreza transform and the variational iteration method. With consideration to the fractional problems studied, the solution series converges to the exact solution at $\varrho = 1$ in their classical form, proving the reliability and validity of the used technique.

A comparative examination between the approximate solution derived using the suggested method and the exact solution is provided at the classical order by graphical analysis, which provides insight into the method's efficacy and accuracy. Three-dimensional charts illustrating the agreement between the exact and the approximate solutions of (56) in the Caputo and Caputo Fabrizio derivative, respectively, under the initial condition (57), are presented in figures 1 and 2. From the chart the velocity profile $u(x,t)$ evolves over space and time as governed by the viscous burger's equation. Furthermore, figures 3 and 4 present three-dimensional charts illustrating the agreement between the exact and the approximate solutions of (61) in the Caputo and Caputo Fabrizio senses, respectively which shows gradual increases in $u(x,t)$ over time under the initial condition (62), and was influenced by the reaction terms $-3u^2 + 2u,$ when $(\psi = 0.01)$.

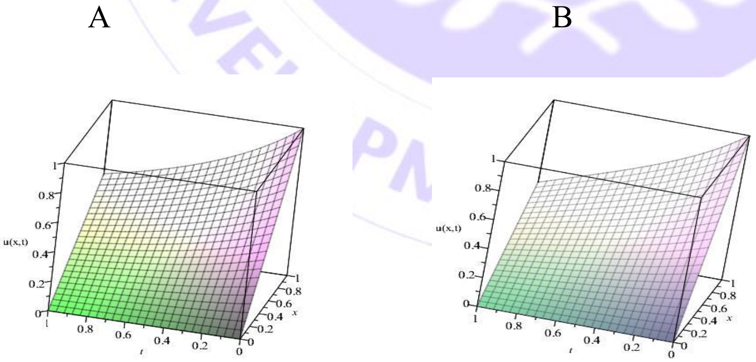

Figure 1: (A) is the exact solution $u(x,t)$ of (56) under initial condition (57), (B) is the numerical solution in Caputo derivative at $\varrho = 1.0$ under the same initial conditions.

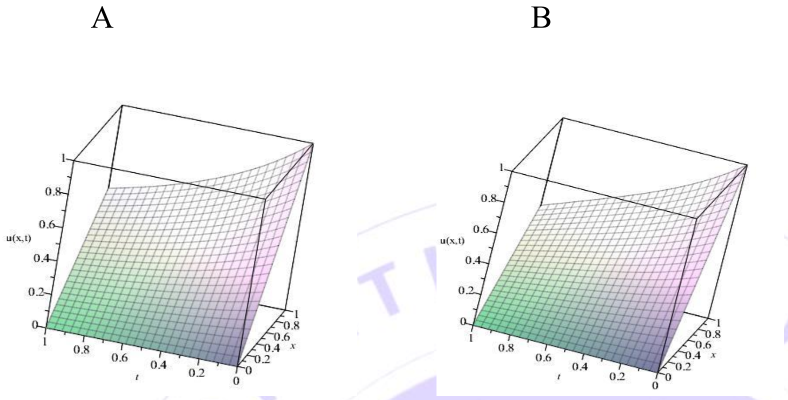

Figure 2: (A) is the exact solution $u(x,t)$ of (56) under initial conditions (57), (B) is the numerical solution in Caputo Fabrizio derivative at $\varrho = 1.0$ under the same initial conditions.

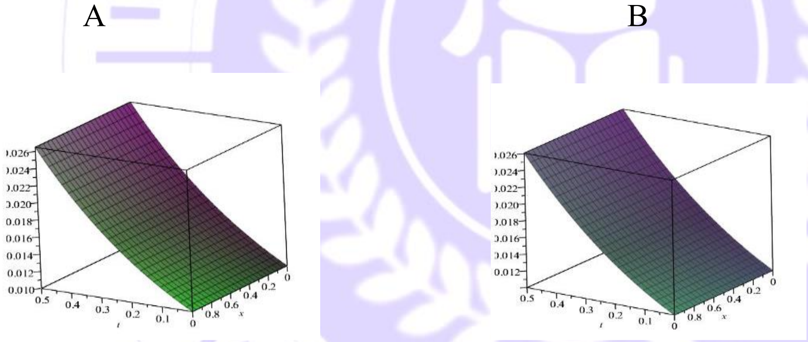

Figure 3: (A) is the exact solution $u(x,t)$ of (61) under the initial conditions (62) when $\psi = 0.01$, (B) is the numerical solution in Caputo sense at $\varrho = 1.0$ under the same initial conditions.

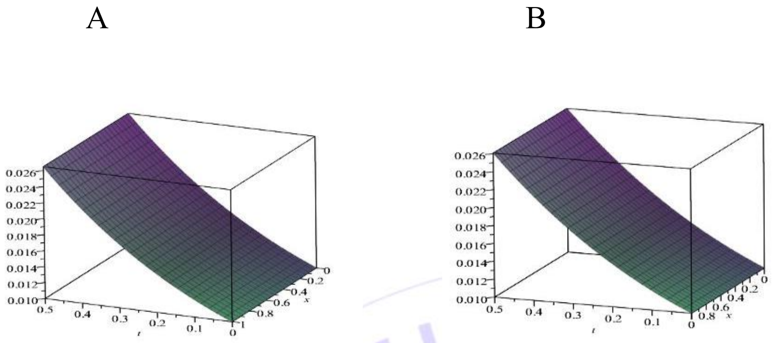

Figure 4: (A) is the exact solution $u(x,t)$ of (61) under initial conditions (62) when $\psi = 0.01$, (B) is the numerical solution in Caputo Fabrizio sense at $\varrho = 1.0$ under the same initial conditions.

In addition, with the comparison of the approximate solutions generated by PVIM in the Caputo and Caputo Fabrizio derivatives with the exact solutions, the resulting errors are displayed in Tables 1 and 4. In solving fractional order burger's differential equations, the efficacy of the technique is demonstrated by Table 1, which also compares the absolute errors between PVIM and the Elzaki Adomian decomposition method (EADM) by Hussein, (2024). The errors obtained by PVIM were found to be the least when compared with the errors in Hussein, (2024), where EADM was used. Tables 2, 3, 5, and 6 show the numerical solutions obtained from the PVIM when k and 1.00, which shows the memory and migratory effect in the values of $u(x,t)$ from fractional order to classical order. Together, these plots and tables offer a comprehensive overview of the accuracy and performance of the approach under different fractional orders and parameter settings, assisting in the assessment of the PVIM's effectiveness in solving nonlinear fractional-order differential equations.

Table 1: Results of illustration one when $\varrho = 1$ for both Caputo and Caputo Fabrizio derivatives

x	t	Error PVIM $_C$	Error PVIM $_{CF}$	Error [3] EADM $_{CF}$
0.2500	0.2500	$2.050490 \text{ x } 10^{-4}$	$2.050488 \text{ x } 10^{-4}$	$3.100 \text{ x } 10^{-3}$
0.7500	0.2500	$6.151472 \text{ x } 10^{-4}$	$6.151474 \text{ x } 10^{-4}$	$9.400 \text{ x } 10^{-3}$
0.2500	0.5000	$2.201141 \text{ x } 10^{-3}$	$2.201141 \text{ x } 10^{-3}$	$2.080 \text{ x } 10^{-2}$
0.5000	0.5000	$4.402281 \text{ x } 10^{-3}$	$4.402282 \text{ x } 10^{-3}$	$4.170 \text{ x } 10^{-2}$
0.7500	0.5000	$6.603422 \text{ x } 10^{-3}$	$6.603423 \text{ x } 10^{-3}$	$6.250 \text{ x } 10^{-2}$
0.2500	0.7500	$7.827758 \text{ x } 10^{-3}$	$7.827759 \text{ x } 10^{-3}$	$6.030 \text{ x } 10^{-2}$
0.5000	0.7500	$1.565551 \text{ x } 10^{-2}$	$1.565551 \text{ x } 10^{-2}$	$1.205 \text{ x } 10^{-2}$
0.7500	0.7500	$2.3483276 \text{ x } 10^{-2}$	$2.3483275 \text{ x } 10^{-2}$	$1.808 \text{ x } 10^{-1}$

Table 2: Fractional order results of illustration one for Caputo derivative

x	t	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1.0$
0.2500	0.2500	0.0857679108	0.1551553958	0.1826363049	0.1997949510
0.7500	0.2500	0.2573037322	0.4654661871	0.5479089141	0.5993848528
0.2500	0.5000	0.0390203530	0.1089719196	0.1477417519	0.1644655257
0.5000	0.5000	0.0780407060	0.2179438393	0.2954835039	0.3289310516
0.7500	0.5000	0.1170610585	0.3269157588	0.4432252559	0.4933965773
0.2500	0.7500	0.0090866602	0.0665027146	0.1144422346	0.1350293841
0.5000	0.7500	0.0181733212	0.1330054287	0.2288844690	0.2700587681
0.7500	0.7500	0.0272599848	0.1995081439	0.3433267030	0.4050881523

Table 3: Fractional order results of illustration one for Caputo Fabrizio derivative.

x	t	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1.0$
0.2500	0.2500	0.05888796439	0.1227050232	0.1723034450	0.1997949512
0.7500	0.2500	0.17666389340	0.3681150697	0.5169103359	0.5993848526
0.2500	0.5000	0.03915496743	0.0901534307	0.1390434368	0.1644655253
0.5000	0.5000	0.0781`q30993487	0.1803068616	0.2780868727	0.3289310507
0.7500	0.5000	0.11746490220	0.2704602924	0.4171303113	0.4933965770
0.2500	0.7500	0.02084979685	0.0579452139	0.1042423315	0.1350293835
0.5000	0.7500	0.04169959377	0.1158904281	0.2084846624	0.2700587684
0.7500	0.7500	0.06254939053	0.1738356420	0.3127269947	0.4050881528

Table 4: Results of illustration two when $\varrho = 1$ for both Caputo and Caputo Fabrizio derivative when $\psi = 0.01$.

t	Error PVIM $_C$	Error PVIM $_{CF}$
0.01	$6.0000 \text{ x } 10^{-11}$	$3.0000 \text{ x } 10^{-11}$
0.02	$9.5000 \text{ x } 10^{-10}$	$9.1000 \text{ x } 10^{-10}$
0.03	$4.9100 \text{ x } 10^{-9}$	$4.8300 \text{ x } 10^{-9}$
0.04	$1.5520 \text{ x } 10^{-8}$	$1.5450 \text{ x } 10^{-8}$
0.05	$3.8020 \text{ x } 10^{-8}$	$3.7960 \text{ x } 10^{-8}$

Table 5: Fractional order results of illustration two for Caputo derivative when $\psi = 0.01$.

t	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1.00$
0.01	0.02360246704	0.01265923812	0.01070747498	0.01019892286
0.02	0.02830272233	0.01406055105	0.01122553591	0.01040173924
0.03	0.03194531500	0.01527600807	0.01170507138	0.01060852064
0.04	0.03505099377	0.01640327891	0.01216734443	0.01081933847
0.05	0.03781637224	0.01747910689	0.01262114454	0.01103426385

Table 6: Fractional order results of illustration two for Caputo Fabrizio derivative when $\psi = 0.01$.

t	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1.00$
0.01	0.07441278252	0.03851636055	0.01881576807	0.01019892289
0.02	0.07487019108	0.03906106494	0.01921316980	0.01040173928
0.03	0.07532853102	0.03960895198	0.01961559524	0.01060852072
0.04	0.07578780216	0.04016002459	0.02002306692	0.01081933854
0.05	0.07624800428	0.04071428607	0.02043560699	0.01103426391

7. Conclusion

In this study, the PVIM scheme of Caputo and Caputo Fabrizio derivative was presented and used to derive the approximate solution of Burger's and Newell-Whitehead-Segel nonlinear fractional-order differential equations. The result was validated by comparing the approximate solutions to the exact solution and other existing solutions in the literature. From the plotted graph and the numerical results, it was observed that the obtained approximate solutions of fractional differential equations of Caputo and Caputo Fabrizio converge very fast to the exact solutions when $\varrho = 1.0$.

Therefore, PVIM is reliable, effective, and accurate and it can be seen as a potent mathematical tool for analyzing nonlinear fractional-order differential equations arising in the areas of science and engineering.

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