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Measuring Uncertainty in PPP Aggregation: A Study of GEKS and Weighted CPD Approaches Using African Data

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ABSTRACT

This study investigates the estimation of Purchasing Power Parities (PPPs) above the basic heading (BH) level conducted across 47 African countries with Nigeria as the numeraire. Using price and expenditure data aggregated from detailed item-level structures, the study compares five prominent PPP aggregation methods: GEKS-Fisher, GEKS-Tornqvist, Weighted GEKS-Fisher, Weighted GEKS-Tornqvist, and Weighted Country-Product-Dummy (WCPD) with an emphasis on the computation of their associated standard errors. The study reveals consistency among the methods with marginal differences in country-level PPP estimates. Notably, the standard errors of the estimates are also similar but Weighted GEKS-Tornqvist has the smallest Standard error value of 1.1676. This comparative evaluation provides empirical insight into the trade-offs and robustness of PPP aggregation approaches and guides methodological choices for regional and global ICP applications.

1. Introduction

Purchasing Power Parities (PPPs) serve as indispensable tools in international economics, allowing for the comparison of currency values across countries by adjusting for variations in price levels. This adjustment is crucial for accurate analyses of economic indicators such as income, consumption, and productivity across national borders (World Bank, 2013). At the forefront of global PPP estimation is the International Comparison Program (ICP), a large-scale statistical initiative coordinated by the World Bank under the direction of the United Nations Statistical Commission. The ICP operates through a network of regional partners including the Organisation for Economic Co-operation and Development (OECD) and Eurostat, ensuring consistent methodologies and extensive geographic coverage (Majumder, *et al.,* 2017 and Weinard, 2022).

A standard definition of PPP, as articulated in the ICP documentation, states that it represents "the number of currency units required to purchase the amount of goods and services which can be bought with one currency unit of the base or numeraire country" (World Bank, 2013 and Atamanov *et al*, 2020). This concept enables users to make meaningful cross-country comparisons that reflect real purchasing power,

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rather than nominal exchange rates. For instance, during the 2011 round of the ICP, which included 199 countries, the PPP for Nigeria was calculated as 74.378 naira per U.S. dollar, substantially different from the nominal exchange rate of 153.903 naira, highlighting the extent of deviation due to local price levels (World Bank, 2015).

The ICP follows a hierarchical structure in the estimation of PPPs, differentiating between calculations made below and above the basic heading (BH) level (Rui *et al.*, 2023). BHs represent the most disaggregated level of expenditure categories used in the comparison process (Abe and Rao, 2022; and Clement *et al.,* 2022). Foundational documentation of this structure can be traced back to United Nations in 1992, with a comprehensive methodological exposition provided by World Bank (2013), particularly in chapters by Rao and Diewert. The methodology integrates both economic theory and statistical index number approaches, evolving over time from axiomatic and economic-theoretic models toward a renewed focus on stochastic methods.

Historically, stochastic approaches to index number theory trace back to early work by Jevons, Edgeworth, Bowley, and Fisher. More recent contributions have revitalized interest in stochastic frameworks, including studies by Clements and Izan (1981), Selvanathan and Rao (1994), Hill (2004), Selvanathan and Selvanathan (2009), Diewert (2010, 2020, 2024), Hajargasht (2022) and others. These efforts have culminated in the adoption and refinement of models like the Country-Product-Dummy (CPD) model, which supports the flexible construction of multilateral price comparisons and has been enhanced with hedonic regression techniques (Dikhanov *et al.*, 2011; Blade *et al.*,2020; Rao, 2005; Silver and Heravi, 2003; Hajargasht, 2022; Hill and Syed, 2015; Ray and Singh, 2021, Weinard, 2022).

At the BH level, PPPs are typically estimated using methods like the unweighted CPD model, particularly in cases of missing price data. These estimates are then aggregated to produce PPPs for broader economic aggregates such as Gross domestic product (GDP) and actual individual consumption (AIC) (Heston and Rao, 2021, and Li and Liu, 2024). Above the BH level, several aggregation methods are employed, including the Geary-Khamis (GK), Eltetö-Köves-Szulc (GEKS), weighted GEKS (WGEKS), Iklé, and weighted CPD (Rao and Hajargasht, 2016 and Hajargasht, 2022).

Although PPPs are widely used and considered robust tools in both policy and research, a key limitation of the current ICP methodology is its lack of published measures of statistical uncertainty, such as standard errors. This gap limits the empirical credibility of comparisons and policy decisions based on PPPs (Rao and Hajargasht, 2016). Therefore, a central aim of this study is to calculate the standard errors associated with different PPP estimates, while the main focus of this study is to explore and evaluate different methods for estimating PPPs above the basic heading (BH) level, building on below-BH level prices with a primary focus on the African region. The rest of the paper is organized as follows. The PPPs methodology for GEKS, WGEKS, and WCPD are described in section 2. Section 3 comprises of Results and Discussion, while section 4 gives the conclusion.

2. PPPs methodology

The estimation of Purchasing Power Parities (PPPs) begins at the Basic Heading (BH) level. At this stage, the Country-Product-Dummy (CPD) regression method is employed to compute average price relatives across countries for each BH. Specifically, the CPD model regresses the logarithm of observed item prices on country and item indicators, capturing systematic price differences while controlling for item-specific effects (World Bank, 2015).

Once BH-level PPPs are established, they are aggregated using expenditure data from national accounts. The aggregation process involves several steps. First, bilateral Laspeyres-type PPPs are calculated for each country pair using expenditure weights that reflect the base country's consumption patterns. Next, Paasche-type bilateral PPPs are computed using the partner country's expenditure weights. These two indices are then combined by taking their geometric mean, resulting in Fisher-type bilateral PPPs, which balance the perspectives of both countries in the comparison (World Bank, 2015).

To ensure consistency and comparability, the matrix of Fisher-type bilateral PPPs is further processed using the Gini-Éltető-Köves-Szulc (GEKS) method. This method adjusts the bilateral Fisher indices to enforce transitivity and base-country invariance, producing a set of multilateral PPPs that are suitable for international comparisons (Diewert, *et al*. 2018).

2.1. Gini–Eltetö–Köves–Szulc (GEKS) Framework

The GEKS method is a widely adopted multilateral index number approach used in the ICP to aggregate BH level PPPs to higher-level aggregates such as GDP or household consumption. The GEKS method ensures transitivity and base-country invariance by deriving multilateral indices from a set of bilateral comparisons (Hill, 2004; Silver, 2010 and Majumder and Ray, 2020).

i. GEKS-Fisher Index

The GEKS-Fisher index is constructed by taking the geometric mean of bilateral Fisher indexes between all countries in a comparison set (Li and Liu, 2024; Majumder and Ray,2020). For countries i and j, the GEKS-Fisher index is calculated as:

$$P_{ij}^{GEK-F} = \left(\prod_{k=1}^{N} \frac{P_{ik}^{F}}{P_{jk}^{F}}\right)^{1/N} \quad (1)$$

where:

- P_{ik}^{F} is the bilateral Fisher index between countries i and k,

- P_{jk}^{F} is the bilateral Fisher index between countries j and k,

-N is the number of countries.

The Fisher index is the geometric mean of the Laspeyres and Paasche indices:

$$P_{ij}^{F} = \sqrt{P_{ij}^{L} * P_{ij}^{P}} \quad (2)$$

ii. GEKS-Törnqvist Index

Alternatively, GEKS can be implemented using the Törnqvist index, which is a superlative index rooted in economic theory (Hajargasht, 2022). The bilateral Törnqvist index between countries i and j is defined as:

$$lnP_{ij}^{T} = \sum_{h=1}^{H} \bar{S}_{ijh} * ln\left(\frac{p_{ih}}{p_{jh}}\right) \quad (3)$$

where:

- $\bar{S}_{ijh} = \frac{1}{2}\left(S_{ih} + S_{jh}\right)$ is the average expenditure share for item h across countries i and j,
- p_{ih} and p_{jh} are prices of item h in countries i and j,
- H is the number of items.

The GEKS-Törnqvist multilateral index is:

$$P_{ij}^{GEKS-T} = exp\left(\frac{1}{N}\sum_{k=1}^{N} ln\left(\frac{P_{ik}^{T}}{P_{jk}^{T}}\right)\right) \quad (4)$$

2.2. Weighted GEKS (WGEKS)

The Weighted GEKS (WGEKS) framework extends GEKS by incorporating expenditure weights to better reflect the relative economic importance of items or basic headings in aggregate comparisons.

i. WGEKS-Fisher

In the WGEKS-Fisher approach, the bilateral Fisher indexes are weighted using aggregate expenditure shares:

$$P_{ij}^{WGEK-F} = exp\left(\sum_{k=1}^{N} w_k \, ln\left(\frac{P_{ik}^{F}}{P_{jk}^{F}}\right)\right) \tag{5}$$

where w_k is the normalized expenditure share of country k.

ii. WGEKS-Törnqvist

The WGEKS-Törnqvist index weights the logarithmic price relatives by expenditure shares:

$$P_{ij}^{WGEK-T} = exp\left(\sum_{k=1}^{N} w_k \, ln\left(\frac{P_{ik}^{T}}{P_{jk}^{T}}\right)\right) \tag{6}$$

2.3. Weighted Country-Product-Dummy (Weighted CPD)

It is possible to generalize the CPD method discussed in the context of aggregation below the BH level. Rao in 1995 has generalized the CPD method by making use of quantity and value data directly into the CPD method. The basic idea behind this generalization comes from the fact that the standard CPD regression model attempts to track the logarithm of the observed prices using an unweighted residual sum of squares. Weighted-CPD is achieved by minimizing a weighted residual sum of squares, with each observation weighted according to the expenditure share of the commodity in a given country (Rao and Hajargasht, 2016). Thus, the generalized CPD method of Summers (1973) in the equation below:

$$\ln P_{ij} = \pi_1 D_1 + \pi_2 D_2 + \ldots + \pi_M D_M + \mu_1 D_1^{*} + \mu_2 D_2^{*} + \ldots + \mu_N D_N^{*} + \varepsilon_{ij} \tag{7}$$

is conducted after weighting each observation according to its value share. This is equivalent to the application of ordinary least squares after transforming the equation pre-multiplied by $\sqrt{w_{ij}}$

$$\sqrt{w_{ij}} \ln P_{ij} = \pi_1 \sqrt{w_{ij}} D_1 + \pi_2 \sqrt{w_{ij}} D_2 + \ldots + \pi_M \sqrt{w_{ij}} D_M + \mu_1 \sqrt{w_{ij}} D_1^{*} + \mu_2 \sqrt{w_{ij}} D_2^{*} + \ldots + \mu_N \sqrt{w_{ij}} D_N^{*} + \varepsilon_{ij} \tag{8}$$

Where $w_{ij} = \frac{p_{ij} q_{ij}}{\sum_{i=1}^{N} p_{ij} q_{ij}}$ is the value share of $i-th$ basic heading in $j-th$ country $\sum w_{ij} = 1$ the sum of budget shares in country j.

Rao (2005) has shown that PPPs resulting from the least squares estimation of the above weighted CPD equation (8) are equivalent to a system of expenditure-share weighted log-change system.

The equivalence of purchasing power parities and international prices derived from the application of the weighted-CPD method with those arising out of the Rao-system for multilateral comparisons implies that the weighted-CPD method is a natural method of aggregation at all levels of aggregation within the context of international comparisons. Thus, the weighted CPD method may be considered as a bridge between the GK approach to International Comparisons and the standard CPD stochastic approach to index numbers (Rao, and Hajargasht, 2016 and Hajargasht, 2022).

3. RESULTS AND DISCUSSION

The ICP dataset 2011 which consists of twelve (12) BHs PPPs out of which seven (7) BHs PPPs with their various expenditure data were used in this study. This heading includes: Food and Non-alcoholic beverages (BH1), Alcoholic beverages, narcotic, and tobacco (BH2), Clothing and footwear (BH3), Household goods and services (BH5), Transport (BH7), Post and Communication (BH8), and Catering and Accommodation services (BH11) respectively.

This study compares PPP estimates for African countries using five multilateral index number methods: GEKS-Fisher, GEKS-Tornqvist, Weighted GEKS-Fisher, Weighted GEKS-Tornqvist and Weighted CPD (WCPD). All PPPs are normalized with Nigeria (NGA) as the numeraire currency (PPP = 1). The computations were performed using R statistical software (version 4.4.2).

Table 1: Estimates of PPPs

Country	GEKS-Fisher	Tornvist GEKS	WGEKS Fisher	WGEKS Tornqvist	WCPD
AGO (Angola)	0.9021	0.9011	0.9002	0.8992	0.9065
BDI (Burundi)	8.0145	7.9995	7.9934	7.9798	8.1970
BEN (Benin)	2.8587	2.8564	2.8569	2.8550	2.8959
BFA (Burkina Faso)	3.0335	3.0314	3.0271	3.0251	3.0774
BWA (Botswana)	0.0517	0.0518	0.0516	0.0517	0.0524
CAF (Central Afr. Rep.)	3.8609	3.8591	3.8569	3.8544	3.9096
CIV (Cote d'Ivoire)	3.3801	3.3748	3.3698	3.3650	3.4143
CMR (Cameroon)	3.3004	3.2984	3.2960	3.2941	3.3451
COD (Congo, Dem. Rep.)	8.0757	8.0619	8.0666	8.0528	8.2316
COG (Congo)	4.0438	4.0451	4.0371	4.0382	4.0786
COM (Comoros)	3.1362	3.1166	3.1292	3.1105	3.1052
CPV (Cabo Verde)	0.5599	0.5594	0.5593	0.5586	0.5603
DJI (Djibouti)	1.2977	1.2958	1.2940	1.2920	1.3005
DZA (Algeria)	0.4057	0.4039	0.4045	0.4028	0.4031
ETH (Ethiopia)	0.0728	0.0727	0.0727	0.0727	0.0733
GAB (Gabon)	4.7589	4.7771	4.7521	4.7700	4.7805
GHA (Ghana)	0.0109	0.0109	0.0109	0.0109	0.0108
GIN (Guinea)	39.1918	39.1422	39.1218	39.0639	39.6495
GMB (Gambia)	0.1444	0.1443	0.1444	0.1443	0.1463
GNB (Guinea-Bissau)	3.4430	3.4339	3.4341	3.4257	3.4629
GNQ (Equatorial Guinea)	4.2612	4.2565	4.2527	4.2484	4.2985
KEN (Kenya)	0.4657	0.4654	0.4646	0.4644	0.4734
LBR (Liberia)	0.0070	0.0070	0.0070	0.0070	0.0071
LSO (Lesotho)	0.0515	0.0514	0.0514	0.0514	0.0525
MAR (Morocco)	0.0555	0.0554	0.0554	0.0552	0.0560
MDG (Madagascar)	9.1244	9.1295	9.0992	9.1055	9.2307
MLI (Mali)	3.0477	3.0418	3.0374	3.0319	3.0921
MOZ (Mozambique)	0.2148	0.2142	0.2140	0.2136	0.2156
MRT (Mauritania)	1.4208	1.4120	1.4175	1.4089	1.4077
MUS (Mauritius)	0.2236	0.2232	0.2229	0.2226	0.2291
MWI (Malawi)	1.0750	1.0726	1.0717	1.0694	1.0846
NAM (Namibia)	0.0618	0.0619	0.0617	0.0618	0.0628
NER (Niger)	3.0702	3.0707	3.0669	3.0673	3.0985
NGA (Nigeria)	1	1	1	1	1
RWA (Rwanda)	3.7780	3.7676	3.7679	3.7589	3.8043
SDN (Sudan)	0.0171	0.0170	0.0171	0.0169	0.0169
SEN (Senegal)	3.1568	3.1479	3.1475	3.1394	3.1548
SLE (Sierra Leone)	24.2543	24.3066	24.2245	24.2686	24.3320
SYC (Seychelles)	0.1001	0.0987	0.0998	0.0985	0.0975
TCD (Chad)	3.4707	3.4620	3.4595	3.4512	3.4928
TGO (Togo)	3.1571	3.1558	3.1514	3.1502	3.1863
TUN (Tunisia)	0.0091	0.0091	0.0091	0.0091	0.0094
TZA (Tanzania)	7.4940	7.4665	7.4801	7.4543	7.5660
UGA (Uganda)	12.9975	12.9861	12.9687	12.9583	13.2320
ZAF (South Africa)	0.0593	0.0592	0.0591	0.0590	0.0609
ZMB (Zambia)	33.3098	33.1569	33.2019	33.0630	32.9861
ZWE (Zimbabwe)	0.0069	0.0068	0.0069	0.0068	0.0068

Table 1 presents a comparative assessment of PPP estimates derived from five different computational methods across a range of countries. Overall, the results reveal a high degree of consistency among the methods, with only minor variations observed, indicating computational efficiency and methodological coherence.

For the unweighted GEKS framework, the Fisher and Tornqvist indices produce remarkably similar results, differing only marginally—typically at the fourth decimal place. This alignment underscores the superlative properties of both indices. For instance, in the case of Angola, the GEKS-Fisher estimate is 0.9021, compared to 0.9011 for GEKS-Tornqvist. Similarly, for Burundi, the respective estimates are 8.0145 and 7.9995, while for Guinea, the figures are 39.1918 and 39.1421. These close correspondences confirm that GEKS-Fisher and GEKS-Tornqvist are not only robust but also highly correlated, as theoretically expected.

When incorporating expenditure weights under the weighted GEKS (WGEKS) method, the estimates continue to track closely with their unweighted counterparts. However, slight downward adjustments are evident, reflecting the influence of national expenditure patterns. For example, Mozambique's GEKS-Fisher estimate is 0.21475, which declines slightly to 0.21405 under WGEKS-Fisher. Similarly, Guinea's value shifts from 39.1918 (GEKS-Fisher) to 39.1218 (WGEKS-Fisher). These adjustments illustrate the WGEKS method's capacity to address asymmetries in consumption structures more effectively.

The weighted CPD (WCPD) method, while still broadly consistent with GEKS-based estimates, tends to exhibit slightly greater deviations—particularly for countries with extreme PPP values. For instance, Burundi's PPP estimate increases from 39.1918 under GEKS-Fisher to 39.6495 using WCPD. A similar trend is observed for Sierra Leone, where the estimate rises from 24.2543 (GEKS-Fisher) to 24.3320 (WCPD). These discrepancies suggest that WCPD may be more sensitive to outlier consumption or price patterns.

Among the 47 countries analyzed, 27 display PPP values exceeding that of the designated numeraire country. The highest PPP estimates are observed in Zambia, Guinea, and Sierra Leone. In contrast, 19 countries exhibit lower PPP values relative to the numeraire, with the lowest estimates found in Liberia, Tunisia, and Zimbabwe.

Table 2: The standard errors (SEs) provide a measure of spread across the methods:

Methods	Standard error
GEKS-Fisher	1.1722
GEKS-Tornqvist	1.1702
WGEKS-Fisher	1.1697
WGEKS-Tornqvist	**1.1676 (lowest)**
WCPD	*1.1875 (highest)*

Table 2 provides a comparative evaluation of the stability of PPP estimation methods, using standard error as a criterion for assessing methodological performance. Among the methods examined, the Weighted GEKS-Tornqvist (WGEKS-Tornqvist) approach yields the lowest standard error values across countries, indicating it provides the most stable and consistent estimates. This suggests that, from a statistical precision standpoint, WGEKS-Tornqvist is the most reliable among the methods analyzed.

In contrast, the Weighted CPD (WCPD) method exhibits comparatively higher standard errors, reflecting a greater degree of dispersion in its estimates. This implies that WCPD may be more sensitive to underlying price or expenditure variability, particularly in contexts with more heterogeneous data.

Additionally, the results reinforce earlier findings that both Fisher and Tornqvist indices produce closely aligned PPP estimates, regardless of whether weighted or unweighted versions are employed. This consistency further affirms the methodological robustness and comparability of these two superlative index approaches.

3. Conclusion

This study has examined the performance of five different PPP estimation methods across 47 countries, highlighting both their accuracy and statistical stability. The results show that GEKS-Fisher and GEKS-Tornqvist yield highly consistent estimates, differing only at minimal decimal levels. Incorporating expenditure weights through the WGEKS framework introduced slight yet meaningful adjustments, enabling the estimates to better capture national consumption patterns. Among all methods, WGEKS-

Tornqvist proved to be the most statistically stable, as evidenced by its lowest standard error. In contrast, the Weighted CPD method exhibited greater variability, particularly in countries with extreme PPP values, indicating reduced robustness. Furthermore, the strong alignment between Fisher and Tornqvist indices in both weighted and unweighted forms reinforces their reliability and theoretical soundness. These findings suggest that weighted superlative methods, especially WGEKS-Tornqvist, offer the most balanced approach to international PPP estimation, combining precision, representativity, and methodological rigor. As such, future applications in global economic comparisons should prioritize these methods to enhance the credibility and relevance of PPP-based analyses.

References

Abe, N., & Rao, D. P. (2022). *Transitivity, substitution bias and the fixed basket multilateral Walsh index for international price and real expenditure comparisons*. Research Center for Economic and Social Risks.

Atamanov, A., Lakner, C., Mahler, D. G., Tetteh Baah, S. K., & Yang, J. (2020). *The effect of new PPP estimates on global poverty: A first look*. World Bank.

Blades, D., Chen, S., Coonan, J., & Deaton, A. S. (2020). Measuring the Real Size of the World Economy: The Framework, Methodology, and Research of the International Comparison Program (ICP).

Clements, K. W., & Izan, H. Y. (1981). A note on estimating divisia index numbers. *International Economic Review*, 745-747.

Clements, K. W., Lan, Y., Liu, H., and Vo, L. (2022). The ICP, PPP, and household expenditure patterns. *Social Science Research Network*. https://doi.org/10.2139/ssrn.4253930

Diewert, W. E. (2010). On the stochastic approach to index numbers. *Price and productivity measurement*, *6*, 235-62.

Diewert, W. E. (2020). Stochastic approaches to index number theory. *Consumer price index theory*.

Diewert, W. E. (2024, May). The Ottawa Group After 30 Years. In *18th Meeting of the Ottawa Group*.

Diewert, W. E., & Fox, K. J. (2018). Substitution bias in multilateral methods for CPI construction using scanner data. *UNSW Business School Research Paper*, (2018-13).

Dikhanov, Y., Palanyandy, C., & Capilit, E. (2011). *Updating 2005 Purchasing Power Parities to 2009 in the Asia and Pacific Region: Methodology and Empirical Results* (No. 246). ADB Economics Working Paper Series.

Hajargasht, G. (2022). *Reliability of Ideal Indexes*. arXiv preprint arXiv:2210.13684. https://doi.org/10.48550/arXiv.2210.13684

Heston, A. W., & Prasada Rao, D. S. (2021). Price Levels, Size, Distribution and Growth of the World Economy: Insights from recent International Comparisons of Prices and Real Product.

Hill, R. J. (2004). Constructing price indexes across space and time: the case of the European Union. *American Economic Review*, *94*(5), 1379-1410.

Hill, R. J., & Syed, I. A. (2015). Improving International Comparisons of Prices at Basic Heading Level: An Application to the Asia-Pacific Region. *Review of Income and Wealth*, *61*(3), 515-539.

Li, L., and Liu, B. (2024). *Regional price differences of medical services: Evidence from China*. BMC Public Health, 24(1) 2353. https://doi.org/10.1186/s12889-024-19719-9

Majumder, A., Ray, R., & Santra, S. (2017). Sensitivity of purchasing power parity estimates to estimation procedures and their effect on living standards comparisons. *Journal of Globalization and Development*, *8*(1), 20170006.

Majumder, A., & Ray, R. (2020). National and subnational purchasing power parity: a review. *Decision*, *47*(2), 103-124.

Rao, D. P. (2005). On the equivalence of Weighted Country-Product-Dummy (CPD) method and the Rao-system for multilateral price comparisons. *Review of Income and Wealth*, *51*(4), 571-580.

Rao, D. P., & Hajargasht, G. (2016). Stochastic approach to computation of purchasing power parities in the International Comparison Program (ICP). *Journal of Econometrics*, *191*(2), 414-425.

Ray, R., & Singh, P. (2021). Regionally disaggregated estimates of global income inequality with evidence on sensitivity to purchasing power parity. *Journal of the Asia Pacific Economy*, *26*(2), 252-270.

Rui, M., Chang, X., and Jian, W. (2023). Substitution bias in purchasing power parity aggregation methods. National Social Science Fund of China.

Selvanathan, E. A., & Rao, D. P. (1994). *Index numbers: a stochastic approach*. University of Michigan Press.

Selvanathan, S., & Selvanathan, S. (2009). Recent developments in the stochastic approach to index numbers. In *International Conference on Frontiers of Interface between Statistics and Sciences*. CR Rao AIMSCS.

Silver, M. (2010). The wrongs and rights of unit value indices. *Review of Income and Wealth*, *56*, S206-S223.

Silver, M., & Heravi, S. (2003). The measurement of quality-adjusted price changes. In *Scanner data and price indexes* (pp. 277-316). University of Chicago Press.

Summers, R. (1973). International price comparisons based upon incomplete data. *Review of Income and Wealth*, *19*(1), 1-16.

United Nations. Statistical Office. (1992). *Handbook of the international comparison programme* (No. 62). New York: United Nations.

Weinand, S. (2022). Measuring spatial price differentials at the basic heading level: A comparison of stochastic index number methods. *AStA Advances in Statistical Analysis, 106*(1), 117–143. https://doi.org/10.1007/s10182-021-00409-5

World Bank. (2013). Measuring the real size of the world economy: The framework, methodology, and results of the International Comparison Program (ICP). Washington, DC: World Bank.

World Bank. (2015). Purchasing power parities and the real size of world economies: A comprehensive report of the 2011 International Comparison Program. Washington, DC: World Bank.