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The Isomorphism of Some Eulerian Graph Models Due to the Algebraic Properties of the (132)-Avoiding Class of Aunu Permutation Patterns

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ABSTRACT

In this article, the three Eulerian Graph models obtained from the adjacency tables (matrix) of the subsets C_1, C_2 *and* C_3 *of* A^5*, the (132)- avoiding class of AUNU (Audu and Aminu, for short) permutation patterns are examined as regard to whether they are Isomorphic or not. Two Graphs* G_1 *and* G_2 *are said to be Isomorphic to each other if there exist a one to one correspondence between their vertices sets say* V_1 *to* V_2 *and also between their edges sets say* E_1 *to* E_2 *such that incidence relationship is preserved and written as* $G_1 \cong G_2$*. We first disregard the order of these graph models and consider them as undirected graphs. Next, the four basic requirements (conditions) for Isomorphism are now verified on these graph models to ascertain whether the said graph models are Isomorphic or otherwise. Finally, it is shown that graphs* B *and* C *are Isomorphic to each other while graph* A *is not isomorphic to either of them.*

1. Introduction

The concept of Isomorphism has been of tremendous help in the study of rare (complicated) mathematical structures using existing (simpler) ones.

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. A function $f: V_1 \to V_2$ is called a graphs isomorphism if

(*i*) *f* is one-to-one and onto.

(*ii*) for all $a, b \in V_1$, $\{a, b\} \in E_1$ if and only if $\{f(a), f(b)\} \in E_2$ when such a function exists, G_1 and G_2 are called isomorphic graphs and is written as $G_1 \cong G_2$. According to Leslie (2007), isomorphic graphs are cospectral. That is whenever, the said graphs have the same spectrum (i.e if the multi set of the Eigen values of their adjacency matrices are the same) see Chun (2018) and Leslie (2007). As noted by Columbia University (28, feb.,2021), the two graphs only differ by names of edges and vertices

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but are structurally equivalent. Also, William (2023) puts it that, because isomorphism preserves some structural aspect of set or mathematical group, it is often used to map a complicated set onto a simpler or better known set in order to establish the original set's properties. Abdulsamad *et al.* (2009) reported that a Eulerian graph named after its inventor Euler is a graph (circuit) that starts and ends at the same vertex. Also, Kaptcianos (2008) and Pevzner *et al.* (2001) has reported that Eulerian circuits have been used in a proposed approach to the problem of DNA fragment assembly.

The special classes of (123)-avoiding and (132)-avoiding permutation patterns were first reported in by Ibrahim and Audu (2005) in connection with some group theoretic applications (properties). The non associative property of these structures was ascertained in Ibrahim and Abubakar (2016). The special (132)-avoiding class was also used in the construction of Latin squares as demonstrated in Ibrahim and Ibrahim, (2011). The suitability of these structures in coding theory was extensively demonstrated see Chun (2018), Chun *et al.* (2016a) and Chun *et al.* (2016b). In this article, the three Eulerian Graph models obtained from the adjacency tables (matrix) of the subsets $C_1, C_2, and\ C_3$ of A^5, the (132)- avoiding class of AUNU (Audu and Aminu, for short) permutation patterns as in Ibrahim *et al.* (2012) are examined as regard to whether they are Isomorphic or otherwise.

2. Some Basic Preliminary Concepts

Here, we examine the following basic concepts with examples where applicable for the ease of understanding of this paper;

2.1 Graphs

A graph G consists of a set of objects $\mathrm{V} = \{v_1, v_2, v_3, \ldots\}$ called vertices (also called points or nodes) and another set $\mathrm{E} = \{e_1, e_2, e_3, \ldots\}$ whose elements are called edges (also called lines or arcs). The set V(G) is called the vertex set of G and E(G) is the edge set. Usually the graph is denoted as G = (V, E). Let G be a graph and $\{u, v\}$ an edge of G Vasudev (2007). Since $\{u, v\}$ is 2-element set, we may write $\{v, u\}$ instead of $\{u, v\}$. It is often more convenient to represent this edge by *uv* or *vu.* If *e* = *uv* is an edge of a graph G, then we say that *u* and *v* are adjacent in G and that *e* joins *u* and *v*. (We may also say that each that of *u* and *v* is adjacent to or with the other). For example, a graph G is defined by the sets V(G) = $\{u, v, w, x, y, z\}$ and E(G) = $\{uv, uw, wx, xy, xz\}$. Now we have the following graph by considering these sets.

Figure 2.1.1 A Simple Graph

Every graph has a diagram associated with it. The vertex *u* and an edge *e* are incident with each other as are *v* and *e*. If two distinct edges say *e* and *f* are incident with a common vertex, then they are adjacent edges (see, Vasudev 2007).

2.2 Directed and Undirected Graphs

2.2.1 Directed Graph

A directed graph or diagraph G consist of a set V of vertices and a set E of edges such that $e \in E$ is associated with an orderd pair of vertices. In other words if each edge of a graph G has a direction, then the graph is called a directed graph. In the diagram of a directed graph, each edge $e = (u\, v)$ is represented by an arrow or a curved from an initial point u of e to a terminal point v. Figure 2.2.1 (a) is an example of a directed graph.

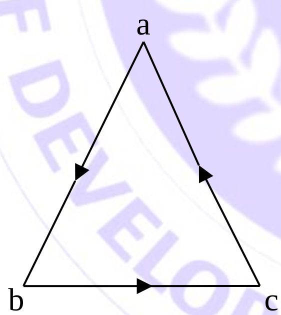


Fig. 2.2.1(*a*). A Directed graph.

Suppose $e = (u, v)$ is a directed edge in a digraph, then

i. *u* is called the **initial vertex** of *e* and *v* is the terminal vertex of *e*

ii. *e* is said to be **incident** from *u* and to be incident to *v*.

iii. *u* is adjacent to *v*, and *v* is adjacent from *u.* (Vasudev, 2007)

2.2.2. Undirected graph

An un-directed graph G consists of set V of vertices and a set E of edges such that each edge $e \in E$ is associated with an unordered pair of vertices. In other words, if each edge of the graph G has no direction then the graph is called un-directed graph. Figure 2.2.2 (b) below is an example of an undirected graph. We can refer to an edge joining the vertex pair *i* and *j* as either (*i, j*) or (*j, i*) (Vasudev, 2007).

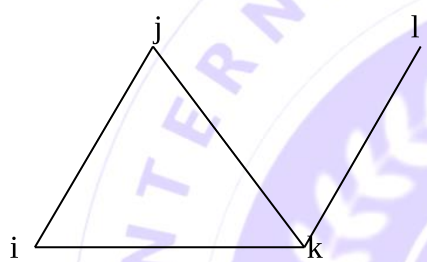


Figure 2.2.2. (a). An Undirected graph.

2.3 Graph Isomorphisms and Examples

(a) Definition 2.3.1

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. A function $f: V_1 \to V_2$ is called a graphs isomorphism if

i. *f* is one-to-one and onto.

ii. for all a, b $\in V_1$, $\{a, b\} \in E_1$ if and only if $\{f(a), f(b)\} \in E_2$ when such a

function exists, G_1 and G_2 are called isomorphic graphs and is written as $G_1 \cong G_2$.

In other words, two graphs G_1 and G_2 are said to be isomorphic to each other if there is a oneto- one correspondence between their vertices and between edges such that incidence relationship is preserve. The necessary conditions for two graphs to be isomorphic are

1. Both must have the same number of vertices

2. Both must have the same number of edges

3. Both must have equal number of vertices with the same degree.

4. They must have the same degree sequences and same cycle vector $(c_1, c_2, \ldots c_n)$, where c_i is the number of cycles of length i.(Vasudev, 2007).

Example 2.4

Verify if the following graphs (a) and (b) are Isomorphic or not

(a) (b)

Solution:

We start by checking the above four conditions:

1. Are the number of vertices in both graphs the same? Yes, both graphs has four vertices
2. Are the number of edges in both graphs the same? Yes, both graphs has four edges
3. Are the degree sequence in both graphs the same? Yes, each vertex is of degree two (2).
4. If the vertices in one graph can form a cycle of length k, can we find the same cycle length in the other graph? Yes, both graphs has a cycle of length four (4).

Since the above four necessary conditions are all satisfied, we now conclude that the given graphs (a) and (b) are isomorphic or that an Isomorphism exist between them. (Vasudev, 2007)

3. Methodology

3.1. Adjacency Matrices/Tables

We first consider the adjacency matrices constructed as in Ibrahim *et al.* (2012)

using subsets C_1, C_2 and C_3, of π_A of $(132) \times \Omega_5$ as follows;

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Adjacency matrix for subset C_1 of $\pi_{(132)} \times \Omega_5$

$$B = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

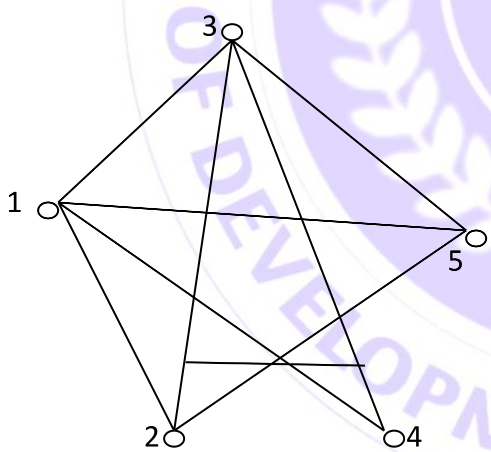
Adjacency matrix for subset C_2 of $\pi_{(132)} \times \Omega_5$

$$C=\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

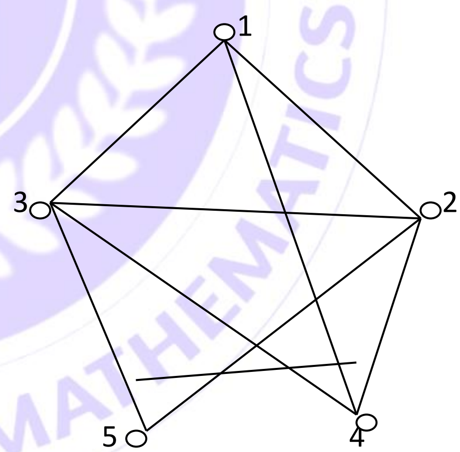
Adjacency matrix for subset C_3 of $\pi_{(132)}\times\Omega_5$

3.2 Eulerian Graphs Due to Aunu Permutation Patterns

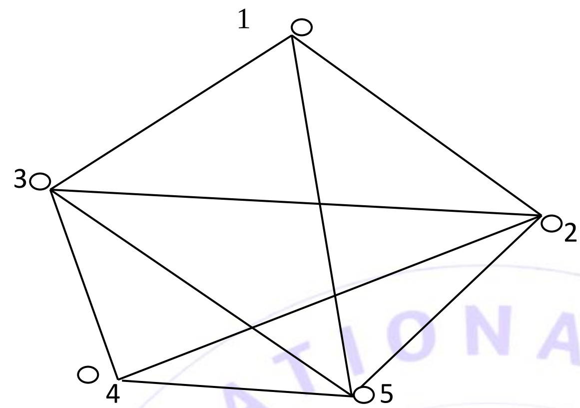
Next, we consider the following three Eulerian, graphs generated by the subsets of the AUNU numbers C_1, C_2 and C_3 as in Ibrahim *et al*. (2012) but disregarding their order. Thus, we have the following corresponding undirected graphs A, B and C.



(A)



(B)



(C)

Next, we examine our four necessary conditions for Isomorphism as outlined in definition 2.3.1 and demonstrated in example 2.4

Now,

Condition 1: Number of Vertices: All the three graphs A, B and C has the same number of Vertices i.e five vertices each.

Condition 2: Number of Edges: All the three graphs A, B and C has the same number if edges i.e five edges each.

Condition 3: Degree sequences: All the three graphs A, B and C has the same degree sequence as {3, 3, 4, 4, 4}.

Condition 4: Same number of cycles lengths in both or all graphs:

i. In graph A, we have {No cycle of length 5, two cycles of length 4 and six cycles of length 3}

ii. In graph B, we have {one cycle of length 5, three cycles of length 4 and seven cycles of length 3}

iii. In graph B, we have {one cycle of length 5, three cycles of length 4 and seven cycles of length 3}.

4. Results

From the foregoing in our methodology, it is clear that the seemingly isomorphic graphs A, B and C are not all Isomorphic. It is evident that graphs B and C are Isomorphic to each other while graph A is not isomorphic to either of them. This is simply for the fact that, graph A does not have a cycle of length 5 as it is the case with graphs B and C despite satisfying all the other three conditions along side B and C. .Moreover, the number of cycles of lengths 4 and 3 in graph A also differ from those of graphs B and C..

5. Conclusion

This research paper has exposed the true status of the seemingly isomorphic Eulerian graphs due to the (132) - avoiding class of the known AUNU permutation Patterns. The cospectoral analysis or determination of the isomorphism of these graphs models and other graph properties has been uncovered. The Eulerian graph models exrayed here could also find some applications interconnection network analysis and related fields.

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