International Journal of Development Mathematics Vol 2 Issue 3 Page No. 055 - 083

INTERNATIONAL JOURNAL OF DEVELOPMENT MATHEMATICS

ISSN: 3026-8656 (Print) | 3026-8699 (Online)

journal homepage: https://ijdm.org.ng/index.php/Journals

Modelling and Sensitivity Analysis of Insecticide Susceptibility Status of Anopheles-gambiae Mosquitoes and Community Awareness of Malaria

Adamu Gambo[a*], Ibrahim I. Adamu [a], Holy-Heavy M. Balami [a], Ahmed. K. Dotia [a], Alhaji M. Usman [b], Abdulmumini Husseini [a] and Amidu A. Oyebanjo [a]

[a]Department of Mathematics, Nigerian Army University Biu, Borno State, Nigeria

[b]Department of Environmental Health Sciences, Federal University of Health Science Azare, Bauchi State, Nigeria

ARTICLE INFO

Article history:
Received 30 June 2025
Received in revised form 25August 2025
Accepted 15 September 2025

Keywords:
Differential equation, Modelling, Sensitivity, Differential and Mosquitoes Dynamics.

MSC 2020 Subject classification:
93A30, 92D30, 49J15.

ABSTRACT

This study investigates the susceptibility of Anopheles-gambiae mosquitoes to various insecticides and assesses community awareness of malaria. Using modelling and sensitivity analysis, the paper evaluates the effectiveness of different insecticide treatments on mosquito populations. The study also evaluated community awareness and knowledge regarding malaria prevention and control measures. The findings aim to inform public health strategies and improve malaria management in the region. Also, the results highlight critical insights into the resistance patterns of mosquitoes and suggest strategies for enhancing community engagement in malaria reduction efforts. Result of the sensitivity analysis revealed that the expansion of mosquitoes in the community was significantly influenced by the parameters with positive index. As their values increasing, the burden on mosquitoes in the community is reduced by parameters with negative index, which were significant factors affecting insecticide susceptibility in Anopheles-gambiae mosquitoes. Variations in these environmental conditions led to fluctuations in the mosquito's response to the insecticides. Additionally, based on the findings, it is recommended to implement targeted insecticide resistance management strategies to maintain the efficacy of current control measures. Finally, enhancing community awareness programs on malaria prevention can further aid in reducing transmission rates. Collaboration between local health authorities and researchers is crucial to adapt interventions to the specific needs of the area.

1. Introduction

The Malaria remains a significant public health challenge in many parts of the world, particularly in sub-Saharan Africa. Anopheles-gambiae mosquitoes, the primary vectors of malaria, have shown varying levels of susceptibility to different insecticides, which complicates control efforts. Malaria continues to cause intolerable rates of disease and death, according to the World Health Organization, (2023) World Malaria Report. As of April 18–20, 2023, there were an estimated 249 million cases and 608,000 fatalities globally, according to the most recent study. The world should prioritise reducing the burden of illness and mortality while pursuing the long-term objective of eradicating malaria because it is preventable and treatable. Female Anopheles mosquitoes transmit malaria, which is caused by Plasmodium parasites. The four-malaria species that infect people are P. falciparum, P. vivax, P. malariae, and P. ovale. Of them, P. falciparum is the most dangerous. P. falciparum and P. vivax are the most prevalent of them. The zoonotic plasmodium P. knowlesi is also found to infect humans. Despite these advancements, endemic malaria

* Corresponding author. Tel.: +2347039020817
E-mail address: adamu.gambo@naub.edu.ng (Adamu Gambo)
https://doi.org/10.62054/ijdm/0203.04

remains prevalent in all six WHO regions, with the African Region carrying the heaviest burden of the disease, accounting for an estimated 90% of all malaria deaths. Nigeria and the Democratic Republic of the Congo account for almost 40% of the estimated global malarial mortality rate. The majority of malaria cases and fatalities go unreported, and millions of people worldwide still lack access to prevention and treatment (World Health Organisation, 2023). More people will live in countries where malaria is a risk as the world's population grows by 2030, placing additional strain on national health systems and malaria program funding (World Health Organisation, 2024). In addition, 44 nations and one territory were declared malaria-free as of December 2024. However, there were 11 million more cases of malaria worldwide than the year before, and the highest increases in malaria death since 2015 occurred in Ethiopia and Nigeria. 95% of the 597,000 recorded deaths in 2023 still took place in African nations, with 76% of those deaths involving children under the age of five. Feachem *et al.,* (2019) estimate that Anopheles gambiae is the malaria vector that causes around 214 million cases of malaria globally, with 438,000 deaths annually as a result. The burden of mosquito-borne diseases continues despite global efforts to avoid mosquito-borne infections through vector control programs and preventative therapy because resistance vectors are emerging to insecticides used for vector control. In order to prevent the transmission of disease, health organisations most commonly employ adult and larval chemical operations as vector control measures. When mosquitoes like Anopheles and Culex develop from larvae to adults, they are able to proliferate because these chemical insecticides (adulticides and lecticides) do not adequately target breeding habitats and residential areas (Fagbohun *et al.,* 2020). Additionally, parasites that feed on blood supply and guarantee hatchability are present in female Anopheles gambiae and Culex quinquefasciatus mosquitoes. Additionally, Ukpai and Ekedo (2019) claim that because vectors are becoming increasingly resistant to common insecticides, disease is disseminated by them. Numerous studies have been carried out worldwide to determine the extent to which Anopheline and Culicine mosquitoes are vulnerable to organophosphate insecticides. Furthermore, studies on community host awareness and pesticide sensitivity status have been conducted in several parts of the world (Kura *et al.,* 2022). Three pesticides that are frequently used for basic control techniques were studied by Shehu *et al.,* (2023) for their impact on the pesticide sensitivity status of two mosquito vectors that are crucial to medicine, Anopheles gambiae and Culex quinquefasciatus. According to Busari *et al.,* (2023) Osogbo also has adult mosquitoes that were susceptible to multiple pesticide classes. Before being examined, the mosquito larvae were collected during the rainy season and grown to adulthood in a laboratory setting. In dengue and malaria hotspots along the Thai-Myanmar border, Pusawang *et al.,* (2022) also assess the pesticide susceptibility of dengue and malaria vectors. Lees *et al.,* (2023) employed insecticide susceptibility status to control mosquitoes and enhance techniques for validating results to back up evidence-based judgements. Additionally, Duguay *et al.,* (2023) used a descriptive successive approach procedure to assess risk factors for schistosomiasis, malaria, and co-infection in school-age children.

Then, in order to determine which fundamental characteristic most strongly affects the rise or fall in malaria incidence, Haile *et al.,* (2025) developed a mathematical model for malaria transmission and used sensitivity analysis. In order to inform efficient control measures, Opaginnian and Durojaye, (2025) created a compartmental SEIIR-SEI model to assess the effects of early and late treatment interventions on malaria transmission among children under the age of five. Additionally, in order to investigate the spread of malaria in areas affected by war, Alhaj and Nyabadza, (2025) create a compartmental mathematical model that considers both the consequences of armed conflicts and malaria management measures. The findings have implications for malaria control recommendations in conflict-affected areas, highlighting the necessity of focused interventions and more robust control methods in spite of the conflicts. Helikumi *et al.,* (2024) developed a fractional order model for the dynamics of malaria transmission that includes two control measures: pesticide use and health education initiatives. The results showed a significant decrease in the population's exposure to malaria. To study the dynamics of malaria transmission in the presence of parasites and mosquitoes resistant to antimalarial medications (Mwanga, 2025: Wako *et al.,* 2025) created a mathematical model for studying the dynamics of malaria transmission. It considers repercussions including severe anaemia and organ failure, as well as how sickness outcomes and healthcare systems are affected. The findings emphasise the need for improved vector management and complication control in order for malaria treatments to be successful. Additionally, Ayalew *et al.,* (2024) presented and investigated a non-linear, deterministic mathematical model of malaria transmission dynamics. The findings of this study also provide methods for examining the extent of Anopheles gambiae and Culex mosquito susceptibility to the chemical components of insecticides, including malathion, propoxur, and permethrin. Therefore, the aim of this paper is to investigating the susceptibility status of Anopheles-gambiae mosquitoes to various insecticides and to assess the level of community awareness of malaria, using mathematical modelling and sensitivity analysis. In conclusion, none of the researchers developed a mathematical model or offered a case study for the sensitivity analysis of insecticide susceptibility status on An. gambiae mosquitoes and the local knowledge of malaria, despite the previously cited pertinent literature. As a result, this research paper must be carried out.

The paper is organized into several sections. The introduction provides background information on malaria and the significance of understanding insecticide susceptibility. The methodology details the modeling techniques and sensitivity analysis employed, followed by the results section which presents the findings on mosquito susceptibility and community awareness. Finally, the discussion interprets the results and suggests implications for malaria control strategies in the region.

2. Formulation of Model

To model the mosquito population dynamics, it begins by defining key parameters such as the mosquito birth rate, natural death rate, and the rate of insecticide-induced mortality. The equations incorporate these factors to simulate changes in mosquito populations over time. The Anopheles mosquitoes transmit malaria, which is caused by Plasmodium parasites. Subsequently, this study introduces a novel approach by integrating both insecticide susceptibility modeling and community awareness analysis, which has not been extensively explored in past studies. The links between the several parameters influencing Anopheles gambiae mosquitoes' susceptibility to insecticides are depicted in the model diagram. It contains portions on environmental factors that affect populations of susceptible individuals, aware individuals, aquatic mosquitoes that are healthy and infected, and the frequency of insecticide application. Arrows also show the interactions and possible feedback loops between the parameters and compartments, providing a thorough transmission dynamic of the system. Therefore, the diagram can be deduced in figure 1, as follow;

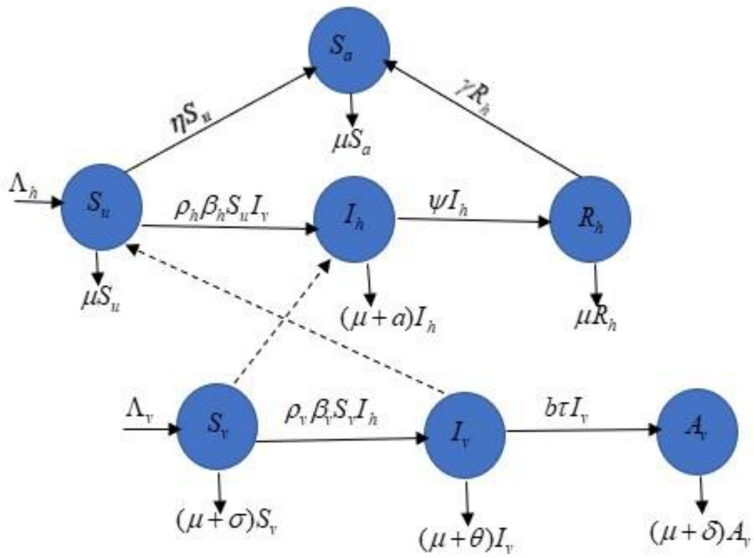

Figure 1: Showing the interaction flow diagram of Anopheles-gambiae Mosquitoes.

The non-linear differential equations of insecticide susceptibility status of anopheles-gambiae mosquitoes transition dynamics and community awareness of malaria, is given as;

$$\left.\begin{aligned}
\frac{dS_u}{dt} &= \Lambda_h - \rho_h\beta_h S_u I_v - (\mu+\eta)S_u,\\
\frac{dS_a}{dt} &= \eta S_u + \gamma R_h - \mu S_a,\\
\frac{dI_h}{dt} &= \rho_h\beta_h S_u I_v - (\psi+\mu+a)I_h,\\
\frac{dR_h}{dt} &= \psi I_h - (\mu+\gamma)R_h,\\
\frac{dS_v}{dt} &= \Lambda_v - \rho_v\beta_v S_v I_h - (\mu+\sigma)S_v,\\
\frac{dI_v}{dt} &= \rho_v\beta_v S_v I_h - (b\tau+\mu+\theta)I_v,\\
\frac{dA_v}{dt} &= b\tau I_v - (\mu+\delta)A_v.
\end{aligned}\right\} \tag{1}$$

with initial conditions given as;

$S_u(0)\geq 0, S_a(0)\geq 0, I_h(0)\geq 0, R_h(0)\geq 0, S_v(0)\geq 0, I_v(0)\geq 0, A_v(0)\geq 0$.

Table 1. Description of Variables

Variables	Description
$S_u(t)$	Density of susceptible unaware human at time
$S_a(t)$	Density of susceptible aware human at time
$I_h(t)$	Density of infectious human at time
$R_h(t)$	Density of recovered human at time
$S_v(t)$	Density of healthy susceptible Anopheles mosquitoes at time
$I_v(t)$	Density of infectious Anopheles mosquitoes at time
$A_v(t)$	Density of Aquatic mosquitoes at time

Table2. Description of Parameters

Parameters	Description
Λ_h	Recruitment rate of humans per-capita
η	Probability rate of unaware become aware humans due to indoor insecticide spread
γ	Progression rate of recovered humans join susceptibility aware humans
μ	Natural death rates of both human and Anopheles mosquito population respectively
ρ_v	Transmission rate from infected Anopheles mosquitoes to humans
ρ_h	Transmission rate from infected humans to Anopheles mosquitoes
β_h	Per-capita contact rate of humans become infected by infected Anopheles mosquitoes
β_v	Per-capita contact rate of Anopheles mosquitoes become infected by infectious humans
σ	Mortality rate of healthy Anopheles mosquitoes due to human activities
θ	Mortality rate of infected Anopheles mosquitoes due to human activities
δ	Mortality rate of aquatic mosquitoes due to insecticide
ψ	Recovery rate of infectious humans
b	Rate of infected Anopheles mosquitoes laid eggs per-oviposition
Λ_v	Rate at which Anopheles mosquitoes recruit per-capita
a	Disease induced-death rate of infectious humans
τ	Oviposition of Anopheles rate

2.1 Assumption of the model

The model assumes that the population of Anopheles gambiae mosquitoes has a consistent exposure to insecticides used in the area. It also presumes that community awareness campaigns about malaria are uniformly distributed and have a measurable impact on mosquito control efforts. Additionally, both infected and healthy aquatic insects in this study are aquatic mosquitoes. Also, it assumes that the Anopheles gambiae mosquito population is relatively stable and that their insecticide susceptibility can be adequately measured through standardised tests. It also presumes that community awareness of female Anopheles mosquitoes caused by malaria is a variable that can be systematically assessed through surveys and educational programmes. Moreover, the model assumes that the Anopheles gambiae mosquito population, is representative of the broader region. It also presumes that community awareness efforts are constant over the study period, influencing local malaria incidence rates. Additionally, the model considers environmental factors, such as temperature and humidity, to remain stable throughout the analysis. Lastly, the model considers that environmental factors and human behaviours affecting mosquito exposure remain consistent throughout the study period. Finally, the parameter estimation is based on mortality rates of both humans and mosquitoes, while some are hypothetical data from the published work review in this paper. Also, due to an insufficient database, the parameter values are limited on malaria.

2.2 Description of model

The insecticide susceptibility status of Anopheles-gambiae mosquitoes, indicates varying degrees of resistance to commonly used insecticides, complicating efforts to control malaria transmission. Community awareness of malaria is crucial in this context, as it influences preventive measures and the effectiveness of control strategies.

Density of susceptible unaware human $S_u(t)$: Risk of malaria transmission is a significant concern. When a high density of people in a community lack awareness about malaria prevention and control measures, the likelihood of mosquito bites and subsequent malaria infection increases, and educating communities to reduce their vulnerability to the disease, which gives;

$$\frac{dS_u}{dt} = \Lambda_h - \rho_h \beta_h S_u I_v - (\mu + \eta) S_u . \tag{2}$$

Density of susceptible aware human at $S_a(t)$: The density of susceptible, aware humans at time (t) can be represented by the variable $S_a(t)$. This variable accounts for individuals within the community who have knowledge about malaria prevention but are still vulnerable to infection, this gives;

$$\frac{dS_a}{dt} = \eta S_u + \gamma R_h - \mu S_a . \tag{3}$$

Density of infectious human at $I_h(t)$: The density of infectious humans in an area can significantly impact the transmission dynamics of malaria. In regions with a high density of infected individuals, the likelihood of mosquitoes acquiring and subsequently spreading the disease increases, gives;

$$\frac{dI_h}{dt} = \rho_h \beta_h S_u I_v - (\psi + \mu + a) I_h . \tag{4}$$

Density of recovered human at $R_h(t)$: The density of recovered humans at a given time can have a significant impact on the transmission dynamics of malaria. If the density is high, it may indicate effective treatment and recovery efforts, potentially reducing the mosquito population's access to infected hosts and gives;

$$\frac{dR_h}{dt} = \psi I_h - (\mu + \gamma) R_h \quad (5)$$

Density of healthy susceptible mosquitoes at $S_v(t)$: The density of susceptible mosquitoes at various locations in Biu, Nigeria, was found to vary significantly. This variation was likely influenced by environmental factors and the level of insecticide resistance in the mosquito population, give;

$$\frac{dS_v}{dt} = \Lambda_v - \rho_v \beta_v S_v I_h - (\mu + \sigma) S_v \quad (6)$$

Density of infectious mosquitoes at $I_v(t)$: The density of infectious mosquitoes in the area plays a critical role in the transmission dynamics of malaria. High mosquito density increases the likelihood of human-mosquito contact, thereby elevating the risk of malaria infections within the community, as;

$$\frac{dI_v}{dt} = \rho_v \beta_v S_v I_h - (b\tau + \mu + \theta) I_v \quad (7)$$

Density of Aquatic mosquitoes at $A_v(t)$: The transmission dynamics of malaria densities can increase the likelihood of human-mosquito interactions, thereby elevating the risk of malaria outbreaks. Effective control measures must therefore focus on reducing these densities to manage and prevent the spread of the disease, which gives;

$$\frac{dA_v}{dt} = b\tau I_v - (\mu + \delta) A_v \quad (8)$$

Therefore, the total population of both humans and vectors respectively are, given as;

$$\frac{dN}{dt} = \frac{dS_u}{dt} + \frac{dS_a}{dt} + \frac{dI_h}{dt} + \frac{dR_h}{dt} + \frac{dS_v}{dt} + \frac{dI_v}{dt} + \frac{dA_v}{dt} \quad (9)$$

2.3 Basic properties of the model

The model incorporates variables such as mosquito population density, insecticide resistance levels, and environmental factors influencing mosquito habitats. It uses differential equations to simulate the dynamics of mosquito populations under various insecticide interventions. Sensitivity analysis is employed to identify critical parameters that significantly affect the model's outcomes, helping to inform effective malaria control strategies.

2.3.1 Positivity of the solutions

The subsection highlights the effectiveness of targeted interventions in reducing mosquito populations and improving community health. Since the system (1) monitors the temporal dynamics of the populations of humans and mosquitoes, all its associated parameters and state variables are non-negative for all $t \geq 0$.

Therefore, using the idea of Haile *et al.,* (2025) on system (1) and divide the compartment into two for examination, which are both human and mosquito group, respectively, and the following theorem as;

Theorem 1.1 *All solutions of the system (1) with non-negative initial condition remains positive for all time* $t \geq 0$.

Proof: Let $t_1 = \sup\left\{t > 0 : S_u > 0, S_a > 0, I_h \geq 0, R_h \geq 0, S_v > 0, I_v \geq 0, A_v \geq 0 \in [0,t]\right\}$ Thus, it follows from the first equation of system (1) that: $\frac{dS_u}{dt} = \Lambda_h - (\lambda + \mu + \eta)S_u$.

Which can be written as

$$\frac{d}{dt}\left[S_u(t)\exp\left\{\int_0^t \lambda(u)\,du + (\eta+\mu)t\right\}\right] \geq \Lambda_h \exp\left\{\int_0^t \lambda(u)\,du + (\eta+\mu)t\right\}. \tag{10}$$

Hence;

$$S_u(t_1)\exp\left\{\int_0^t \lambda(u)\,du + (\eta+\mu)t\right\} - S_h(0),$$

$$\geq \int_0^{t_1} \Lambda_h \exp\left\{\int_0^x \lambda(u)\,du + (\eta+\mu)x\right\} dx. \tag{11}$$

Therefore,

$$S_u(t_1) \geq \exp\left\{-\int_0^{t_1} \lambda(u)\,du - (\eta+\mu)t_1\right\} \times$$

$$\left[S_u(0) - \int_0^{t_1} \Lambda_h \exp\left\{\int_0^x \lambda(u)\,du + (\eta+\mu)x\right\} dx\right] > 0. \tag{12}$$

This implies that $S_u(t_1) > 0$ for all $t_1 > 0$,hence, $S_u(t) > 0$ for $t > 0$. Likewise, it can be demonstrated that all other state variables of model (1) remain non-negative for all $t \geq 0$**,** provided the initial conditions are non-negative. As a result, any solution to system (1) that begins with initial values that are not negative remains so in all subsequent times.

Theorem 1.2: *The solution of the system (1) is bounded in the closed set*

$$\Omega = \Omega_h \cup \Omega_v \subset \mathbb{R}_+^4 \times \mathbb{R}_+^3$$

where;

$$\Omega_h = \left\{ \left(S_u, S_a, I_h, R_h \right) \in \mathbb{R}_+^4 : N_h(t) \le \frac{\Lambda_h}{\mu} \right\},$$

$$\Omega_v = \left\{ \left(S_v, I_v, A_v \right) \in \mathbb{R}_+^3 : N_v(t) \le \frac{\Lambda_v}{\mu} \right\}.$$

Furthermore, the set Ω *is positively invariant and attracting with respect to system (1).*

Proof: For the human population, we sum up the first four equations of system (1) to get

$$\frac{dN_h}{dt} \le \Lambda_h - \mu N_h$$

It follows that

$$\frac{dN_h}{dt} \ge -\mu N_h, \quad \frac{dN_h}{dt} \le \Lambda_h - \mu N_h. \tag{13}$$

Now, by using comparison theorem, it can be established that

$$N_h(t) \ge N_h(0)e^{-\mu t}, \qquad N_h(t) \le N_h(0)e^{-\mu t} + \frac{\Lambda_h}{\mu}\left[1 - e^{-\mu t}\right]$$

$$N_h(0)e^{-\mu t} \le N_h(t) \le N_h(t) \le N_h(0)e^{-\mu t} + \frac{\Lambda_h}{\mu}\left[1 - e^{-\mu t}\right]. \tag{14}$$

Thus, we have

$$0 \le \lim_{t\to\infty} \inf N_h(t) \le \lim_{t\to\infty} \sup N_h(t) \le \frac{\Lambda_h}{\mu}. \tag{15}$$

For the mosquito population, this consider the sum of the last three equation of system (1)

$$N_v(t) \le \Lambda_v - \mu N_v.$$

It follows that

$$N_v(t) \ge -\mu N_v, \quad N_v(t) \le \Lambda_v - \mu N_v. \tag{16}$$

Now, by using comparison theorem, it can be established that

$$N_v(t) \geq N_v(0)e^{-\mu t}, \quad N_v(t) \leq N_v(0)e^{-\mu t} + \frac{\Lambda_v}{\mu}\left[1-e^{-\mu t}\right]$$

$$N_v(0)e^{-\mu t} \leq N_v(t) \leq N_v(t) \leq N_v(0)e^{-\mu t} + \frac{\Lambda_v}{\mu}\left[1-e^{-\mu t}\right]. \tag{17}$$

Thus, we have

$$0 \leq \lim_{t\to\infty} \inf N_v(t) \leq \lim_{t\to\infty} \sup N_v(t) \leq \frac{\Lambda_v}{\mu}. \tag{18}$$

In particular, from equations (4) and (5), if $N_h(0) \leq \frac{\Lambda_h}{\mu}$ and $N_v(0) \leq \frac{\Lambda_v}{\mu}$ then

$N_h(t) \leq \frac{\Omega_h}{\mu}$ and $N_v(t) \leq \frac{\Omega_v}{\mu}$, respectively.

Therefore, the set Ω is positively invariant, which means that any trajectory of model system (1) starting from an initial state in Ω will remain in Ω. In addition, if then either the solution enters the region Ω in finite time or $N_h(t) \to \frac{\Omega_h}{\mu}$ asymptotically and $N_v(t) \to \frac{\Omega_v}{\mu}$ asymptotically. Hence, this conclude that Ω is an attracting set in $\mathbb{R}^7_+$.

2.4 Malaria Disease Free-Equilibrium

The Malaria Disease Free Equilibrium refers to a state in which there is no transmission of malaria within a population. The equilibrium points are found by setting the right-hand side of system (1) to zero as;

$$\frac{dS_u}{dt} = \frac{dS_a}{dt} = \frac{dI_h}{dt} = \frac{dR_h}{dt} = \frac{dS_v}{dt} = \frac{dI_V}{dt} = \frac{dA_v}{dt} = 0 \tag{19}$$

Also, in absence of malaria in a population, meaning I_h, R_h, I_v and $A_v = 0$. Therefore, disease free-equilibrium is calculated by the used of Maple23 mathematical tools software and gives; $E_* = \left(\frac{\Lambda_h}{\mu+\eta}, \frac{\Lambda_h \eta}{(\mu+\eta)\mu}, 0, 0, \frac{\Lambda_v}{\mu+\sigma}, 0, 0\right)$. The mosquito free-equilibrium point of the system (1), is given by E_*, which implies $E_* = S_u^*, S_s^*, I_h^*, R_h^*, S_v^*, I_v^*, A_v^*$ and when malaria exist in a community, $I_h, I_v \neq 0$ in this study.

2.5 Endemic Equilibrium Point of Malaria Disease

The study found the existence of a malaria equilibrium, where the transmission rate and recovery rate reached a steady state. The endemic disease in a population is obtained from the model system (1) with the help of Maple23 mathematical tools software, and obtained;

$$\left.\begin{aligned}
S_u^* &= \frac{b_5(\Lambda_h\rho_v\beta_v + b_2b_4)}{\rho_v\beta_v(\Lambda_v\rho_h\beta_h + b_1b_5)},\\
S_v^* &= \frac{b_2(\Lambda_v\rho_h\beta_h + b_1b_5)}{\rho_h\beta_h(\Lambda_h\rho_v\beta_v + b_2b_4)},\\
I_h^* &= \frac{\Lambda_h\Lambda_v\rho_h\rho_v\beta_h\beta_v - b_1b_2b_4b_5}{\rho_v\beta_v(\Lambda_v\rho_h\beta_h + b_1b_5)b_2},\\
R_h^* &= \frac{\psi(\Lambda_h\Lambda_v\rho_h\rho_v\beta_h\beta_v - b_1b_2b_4b_5)}{\rho_v\beta_v(\Lambda_v\rho_h\beta_h + b_1b_5)b_2b_3},\\
A_v^* &= \frac{b\tau(\Lambda_h\Lambda_v\rho_h\rho_v\beta_h\beta_v - b_1b_2b_4b_5)}{\rho_h\beta_h(\Lambda_h\rho_v\beta_v + b_2b_4)b_5b_6},\\
I_v^* &= \frac{\Lambda_h\Lambda_v\rho_h\rho_v\beta_h\beta_v - b_1b_2b_4b_5}{\rho_h\beta_h(\Lambda_h\rho_v\beta_v + b_2b_4)b_5},\\
S_a^* &= \frac{\eta b_2^2b_3b_4b_5 + b_5(\eta b_3\Lambda_h\rho_v\beta_v - \psi\gamma b_1b_4)b_2 + \psi\gamma\Lambda_h\Lambda_v\rho_h\rho_v\beta_h\beta_v}{\rho_v\beta_v(\Lambda_v\rho_h\beta_h + b_1b_5)b_2\mu b_3}.
\end{aligned}\right\} \tag{20}$$

where;

$$\begin{aligned}
b_1 &= (\mu + \eta)\\
b_2 &= (\psi + \mu + a)\\
b_3 &= (\mu + \gamma)\\
b_4 &= (\mu + \sigma)\\
b_5 &= (b\tau + \mu + \theta)\\
b_6 &= (\mu + \delta)
\end{aligned}$$

2.6 Basic Reproduction Number R_0

The basic reproduction number, often denoted as R_0, is a crucial metric in understanding the transmission dynamics of malaria. It represents the average number of secondary cases generated by one infected individual in a completely susceptible population. In this paper, the potential spread of malaria and the effectiveness of control measures were analysed using the idea of (Van den Driessche, 2017). Therefore, using the method in (Diekmann, (2010); Brauer, 2016), the result follows as;

$$F = \begin{bmatrix} \rho_h \beta_h S_u I_v \\ 0 \\ \rho_v \beta_v S_v I_h \\ 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} +(\psi + \mu + a) I_h \\ -\psi I_h + (\mu + \gamma) R_h \\ +(b\tau + \mu + \theta) I_v \\ -b\tau I_v + (\mu + \delta) A_v \end{bmatrix}.$$

Therefore, putting the compartments with infectious transmission into Jacobian matrix of F and V, then differentiate with respect to I_h, R_h, I_v and A_v at disease free equilibrium E_*, gives;

$$F = \begin{pmatrix} 0 & 0 & \dfrac{\rho_h \beta_h \Lambda_h}{\mu + \eta} & 0 \\ 0 & 0 & 0 & 0 \\ \dfrac{\rho_v \beta_v \Lambda_v}{\mu + \sigma} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$-V = \begin{pmatrix} (\psi + \mu + a) & 0 & 0 & 0 \\ -\psi & (\mu + \gamma) & 0 & 0 \\ 0 & 0 & (b\tau + \mu + \theta) & 0 \\ 0 & 0 & -b\tau & (\mu + \delta) \end{pmatrix}.$$

Subsequently, by further simplification, gives;

$$F = \begin{pmatrix} 0 & \dfrac{\rho_h \beta_h \Lambda_h}{\mu + \eta} \\ \dfrac{\rho_v \beta_v \Lambda_v}{\mu + \sigma} & 0 \end{pmatrix}, \text{ and } V = \begin{pmatrix} (\psi + \mu + a) & 0 \\ 0 & (b\tau + \mu + \theta) \end{pmatrix}.$$

Thus, interchanging columns two to one and one to column on F, gives;

$$F = \begin{pmatrix} \dfrac{\rho_h \beta_h \Lambda_h}{\mu + \eta} & 0 \\ 0 & \dfrac{\rho_v \beta_v \Lambda_v}{\mu + \sigma} \end{pmatrix}, \quad V = \begin{pmatrix} (\psi + \mu + a) & 0 \\ 0 & (b\tau + \mu + \theta) \end{pmatrix}.$$

Then;

$$FV^{-1} = \begin{pmatrix} \frac{1}{\psi+\mu+a} & 0 \\ 0 & \frac{1}{b\tau+\mu+\theta} \end{pmatrix}.$$

Therefore, by finding the eigenvalues of FV^{-1} at the disease-free equilibrium points, which gives;

$$FV^{-1} = \begin{pmatrix} \frac{\rho_h \beta_h \Lambda_h}{(\psi+\mu+a)(\mu+\eta)} - \lambda & 0 \\ 0 & \frac{\rho_v \beta_v \Lambda_v}{(b\tau+\mu+\theta)(\mu+\sigma)} - \lambda \end{pmatrix}.$$

The following results are given by the use of Maple2023 software and therefore, the basic reproduction number of system (1) denoted by $R_0 = \rho FV^{-1}$, where ρ is the spectral radius of the product FV^{-1}, which gives;

$$R_h = \frac{\rho_h \beta_h \Lambda_h}{(\psi+\mu+a)(\mu+\eta)}. \tag{21}$$

$$R_v = \frac{\rho_v \beta_v \Lambda_v}{(b\tau+\mu+\theta)(\mu+\sigma)}. \tag{22}$$

where; R_h and R_v which are both Basic reproduction number of human and vector respectively, and denoted as R_0. It follows from Van den Driessche and Watmough, (2002) that the effective control can also lead to the reduction of the basic reproduction number R_0 below unity, which in turn reduces the burden of malaria transmission in the community. The above following R_0 result is established by standard technique in (Van den Driessche, 2017).

2.7 Local Stability of Disease Free-Equilibrium

The local stability of the disease-free equilibrium is determined by evaluating the basic reproduction number, R_0, which represents the average number of secondary infections produced by a single infected individual in a completely susceptible population. If $R_0 < 1$, the disease-free equilibrium is stable, indicating that the disease will eventually be eradicated from the population. Conversely, if $R_0 < 1$, the disease will persist and potentially spread throughout the community.

Theorem 1.3: *If* $R_0 < 1$*, the disease -free equilibrium point(s) of the system (1) is locally asymptotically stable.*

Proof: To prove theorem 1.3 of this paper, using the idea of Haile *et al.,* (2025), first apply Jacobian matrix on system (1) at disease-free equilibrium, which gives;

$$J(E^0)=\begin{bmatrix} -(\mu+\eta) & 0 & 0 & 0 & 0 & -\frac{\rho_h\beta_h\Lambda_h}{\mu+\eta} & 0 \\ \eta & -\mu & 0 & \gamma & 0 & 0 & 0 \\ 0 & 0 & -(\psi+\mu+a) & 0 & 0 & \frac{\rho_h\beta_h\Lambda_h}{\mu+\eta} & 0 \\ 0 & 0 & \psi & -(\mu+\gamma) & 0 & 0 & 0 \\ 0 & 0 & -\frac{\rho_v\beta_v\Lambda_v}{\mu+\sigma} & 0 & -(\mu+\sigma) & 0 & 0 \\ 0 & 0 & \frac{\rho_v\beta_v\Lambda_v}{\mu+\sigma} & 0 & 0 & -(b\tau+\mu+\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & b\tau & -(\mu+\delta) \end{bmatrix}.$$

$$\begin{bmatrix} -k_1 & 0 & 0 & 0 & -\frac{\rho_h\beta_h\Lambda_h}{\mu+\eta} \\ 0 & -k_2 & 0 & 0 & \frac{\rho_h\beta_h\Lambda_h}{\mu+\eta} \\ 0 & \psi & -k_3 & 0 & 0 \\ 0 & -\frac{\rho_v\beta_v\Lambda_v}{\mu+\sigma} & 0 & -k_4 & 0 \\ 0 & \frac{\rho_v\beta_v\Lambda_v}{\mu+\sigma} & 0 & 0 & -k_5 \end{bmatrix}. \tag{23}$$

where;

$k_1=(\mu+\eta)$,
$k_2=(\psi+\mu+a)$,
$k_3=(\mu+\gamma)$,
$k_4=(\mu+\sigma)$,
$k_5=(b\tau+\mu+\theta)$.

By further simplification of equation (23) gives;

$$\begin{bmatrix} -k_2 & 0 & \frac{\rho_h\beta_h\Lambda_h}{\mu+\eta} \\ -\frac{\rho_v\beta_v\Lambda_v}{\mu+\sigma} & -k_4 & 0 \\ \frac{\rho_v\beta_v\Lambda_v}{\mu+\sigma} & 0 & -k_5 \end{bmatrix}. \tag{24}$$

From equation (22), the reduction method can also be possible to 2X2 matrix, gives;

$$\begin{bmatrix} -k_2 & \frac{\rho_h\beta_h\Lambda_h}{\mu+\eta} \\ \frac{\rho_v\beta_v\Lambda_v}{\mu+\sigma} & -k_5 \end{bmatrix} \tag{25}$$

The characteristic equation is solved to determine the Jacobian matrix's eigenvalues;

$\left|J(E^0)\lambda I\right| = 0$, which is in the form as;

$$\begin{vmatrix} -k_2-\lambda & \frac{\rho_h\beta_h\Lambda_h}{\mu+\eta} \\ \frac{\rho_v\beta_v\Lambda_v}{\mu+\sigma} & -k_5-\lambda \end{vmatrix} = 0 \tag{26}$$

With the use of Maple23 software mathematical tools, the Jacobian matrix of equation (26) provides the characteristic polynomial, which yields;

$$\frac{(k_5+\lambda)(k_2+\lambda)\mu^2+(k_5+\lambda)(k_2+\lambda)(\eta+\sigma)\mu+\eta\lambda^2\sigma+(k_2+k_5)\sigma\eta\lambda-\rho_v\beta_v\Lambda_v\rho_h\beta_h\Lambda_h+\eta\sigma k_2k_5}{(\mu+\sigma)(\mu+\eta)} > 0 \tag{27}$$

Rearranging the like terms in equation (27), gives;

$k_5 + \lambda = 0 \Rightarrow \lambda_1 = -k_5, k_2 + \lambda = 0 \Rightarrow \lambda_2 = -k_2$.

Using the Routh-Hurwitz stability conditions, we can now demonstrate that all of the eigenvalues in the characteristic equation (27) have negative real parts. As a result, the expression in equation becomes;

$$\eta\sigma\lambda^2 + (k_2 + k_5)\eta\sigma\lambda + \eta\sigma k_2 k_5 - R_0 > 0 \tag{28}$$

Therefore, (28) look like quadratic equation, which can also in the form as;

$$a_1\lambda^2 + a_2\lambda + c = 0 \tag{29}$$

where;

$a_1 = \eta\sigma,$
$a_2 = (k_2 + k_5)\eta\sigma,$
$c = \eta\sigma k_2 k_5,$

According to equation (28) all of the real roots of the equation have strictly negative values if and only if $a_1 \geq 0, a_2 \geq 0$, as well as the Routh-Harwitz criteria by (Murray, 2001; Haile *et al.,* 2025). Then, $a_1, a_2 \geq 0$. This demonstrated that $a_1 \geq 0$, which shows is the sum of positive parameters. Thus, if $R_0 < 1$, the DFE is locally asymptotically stable. However, $R_0 > 0$, on the other hand, indicates that $a_1 \leq 0$, and according to Descartes's rule of signs in Haile *et al.,* (2025), the coefficient of the characteristic polynomial in equation (27), changes by exactly one sign. Thus, DFE is unstable and there is only one eigenvalue with a positive real portion. So, the proof is complete.

2.8 Global Stability of Disease Free-Equilibrium

The disease-free state, labelled as E_0. When the basic reproduction number R_0 is less than or equal to 1, this state is globally stable.

Theorem 1.4: *The disease-free equilibrium E_0 is globally asymptotically stable (meaning the system will always settle there over time) within the considered region if $R_0 < 1$. If R_0 is greater than 1, then this stability does not hold, and the disease can persist or spread.*

Proof: Using the Brauer *et al.,* (2016) theorem. Let $X(t)$ denotes as uninfected population, then $X = (S_u, S_a, R_h, S_v)$, and $Y(t)$ represents the infected $Y = (I_h, I_v, A_v)$. Thus, system (1) can be written as;

$$\frac{dX}{dt} = F(X,Y), \frac{dY}{dt} = G(X,Y); G(X,0) = 0 \tag{30}$$

Where F and G are the corresponding right-hand side of the system (1). According to Brauer *et al.,* (2016) to establish the global asymptotic stability of the disease-free equilibrium, the following two conditions D_1 and D_2 must be satisfied for $R_0 < 1$.

Therefore;

$(D_1) X^0 = (1,1,01)^T$ globally asymptotically stable for $\frac{dX}{dt} = F(X,0)$.

$(D_2) G \geq 0$ where $\widehat{G}(X,Y) = AY - G(X,Y)$ and $A = D_Y G\left(X^0, 0\right)$ is Metzler matrix

$\forall (X,Y) \in \Omega$. The first Condition is globally asymptotically stability of X^0, we have

$$\frac{dX}{dt} = F(X,0) = \begin{pmatrix} \Lambda_h - (\mu+\eta) S_u \\ \eta S_u + \gamma R_h - \mu S_a \\ -(\mu+\gamma) R_h \\ \Lambda_v - (\mu+\sigma) S_v \end{pmatrix} \tag{31}$$

Further simplification on (31), give the behaviour of each compartment as;

$$X^0 \begin{pmatrix} S_u \\ S_a \\ R_h \\ S_v \end{pmatrix} = \begin{pmatrix} 1 + S_u(0) e^{-(\mu+\eta)t} \\ 1 + S_a(0) e^{-\mu t} \\ R_h(0) e^{-(\mu+\gamma)t} \\ 1 + S_v(0) e^{-(\mu+\sigma)t} \end{pmatrix} \tag{32}$$

Now since $\lim_{x\to\infty} X(t) = X^0$ satisfying the first condition. For the second condition D_2

$$A = \begin{pmatrix} -a-\mu-\psi & \rho\beta_h S_u & 0 \\ \rho\beta_v S_v & -b\tau-\mu-\theta & 0 \\ 0 & b\tau & -\delta-\mu \end{pmatrix} \tag{33}$$

Then we compute

$$\widehat{G}(X,Y) = AY - G(X,Y) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{34}$$

It is obvious that $\widehat{G}(X,Y)=0$, thus $\widehat{G}(X,Y)\geq 0$ in D_2 is satisfied. Hence, E_0 is globally asymptotically stable, provided that $R_0<1$. This completes the proof.

2.9 Sensitivity Analysis of the model using Basic Reproduction Number R_0

The sensitivity analysis involved varying key parameters within the insecticide susceptibility model to assess their impact on the outcomes. This approach helped identify which factors most significantly affected mosquito resistance levels and the effectiveness of control measures. Therefore, table 3 is used to calculate the sensitivity analysis index with the help of R_0, which gives;

Table3: Parameter, Value and Source for Simulations

Parameter	Value	Source
Λ_h	220	Haile, *et al.,* (2025)
η	0.14	Alhaj and Nyabadza, (2025)
γ	0.057	Assumed
μ	0.01695	Estimated
ρ_v	0.0021	Opaginni and Durojaye, (2025)
ρ_h	0.0001	Assumed
β_h	0.000164	Haile, *et al.,* (2025)
β_v	0.00412	Alhaj and Nyabadza, (2025)
σ	0.21	Assumed
θ	0.00652	Haile, *et al.,* (2025)
δ	0.067	Estimated
ψ	0.0172	Alhaj and Nyabadza, (2025)
b	0.523	Opaginni and Durojaye, (2025)
Λ_v	0.042	Assumed
a	0.0143	Alhaj and Nyabadza, (2025)
τ	0.002	Haile, *et al.,* (2025)

The sensitivity analysis is used to determine the model's fitness to R_0 parameter values. This approach helped identify which factors most significantly affected mosquito resistance levels and the effectiveness of control measures. Using the approach in Chitnis *et al.,* (2008) the calculation of the normalized forward sensitivity indices of R_0, gives;

$$\Upsilon_{\iota}^{R_0}=\frac{\partial R_0}{\partial l}\times\frac{\iota}{R_0} \tag{35}$$

denote the sensitivity index of R_0 with respect to the parameter l, obtain;

Table 4: The Sensitivity Index of Basic Reproduction Number

Parameters of R_0^h	**Sensitivity Index**	**Parameters of R_0^v**	**Sensitivity Index**
Λ_h	+1.0000	Λ_v	+1.0000
β_h	+1.0000	β_v	+1.0000
ρ_h	+1.0000	ρ_v	+1.0000
μ	-0.7514	θ	-0.2664
ψ	-0.0358	σ	-0.9253
η	-0.8550	τ	-0.0409
a	-0.3578	b	-0.0409

2.9.1 Interpretation of the Sensitivity Analysis Index

The sensitivity index provides insight into which parameters have the most significant impact on the model outcomes. By analysing these indexes, this paper identifies which factors most strongly influence mosquito susceptibility to insecticides and the effectiveness of community awareness programs. The sensitivity index in Table 4, for R_0 shows the expansion of mosquitoes in the community was significantly influenced by the parameters with positive index, which are Λ_h, β_h, ρ_h, Λ_v, β_v, and ρ_v. If Λ_h, β_h, ρ_h, Λ_v, β_v, ρ_v increasing, R_0^h, R_0^v will also increasing significantly. However, an increase in the parameters μ, ψ, η, a and $\mu, \sigma, \tau, b, \theta$ will decreasing the R_0^h, R_0^v respectively, they all have an inversely proportional relationship with R_0. So, an increasing in any of negative indexes will decreasing R_0. However, the size of the decreasing will be proportionally smaller. Given R_0, the sensitivity to Λ_h, β_h, ρ_h, Λ_v, β_v, and ρ_v, it seems sensible to focus on reducing those parameters. Therefore, the sensitivity analysis of basic reproduction number of both human and mosquito, are given in figures 1and 2 respectively.

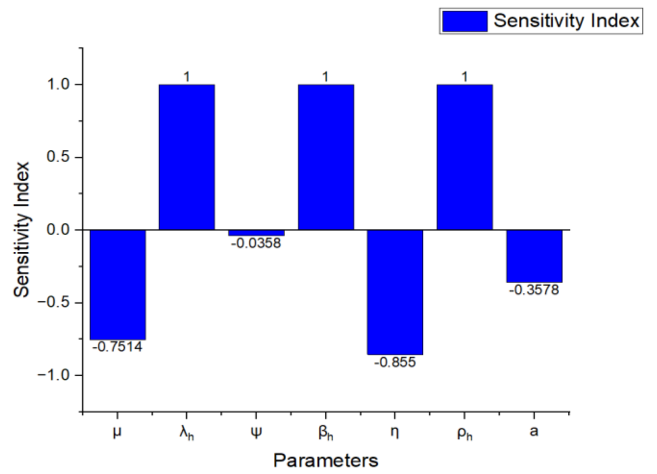

Figure 2. Sensitivity Indices of Basic Reproduction number of Human.

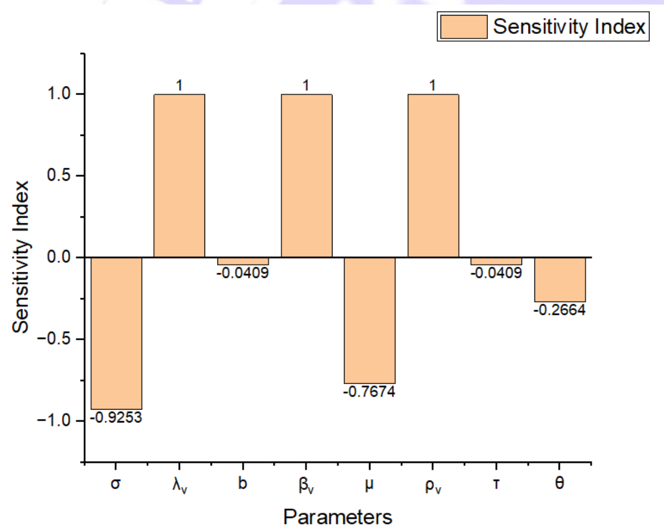

Figure 3. Sensitivity Indices of Basic Reproduction number of the Mosquito.

3. Numerical Simulations of the Model Transmission Dynamics of Mosquito

The dynamics of malaria transmission by Anopheles-gambiae mosquitoes were analysed by numerical simulations. Simulations were conducted using the Maple2023 software, which has an integrated fourth-order Runge-Kutta algorithm. The scenario is displayed on a graph as follows:

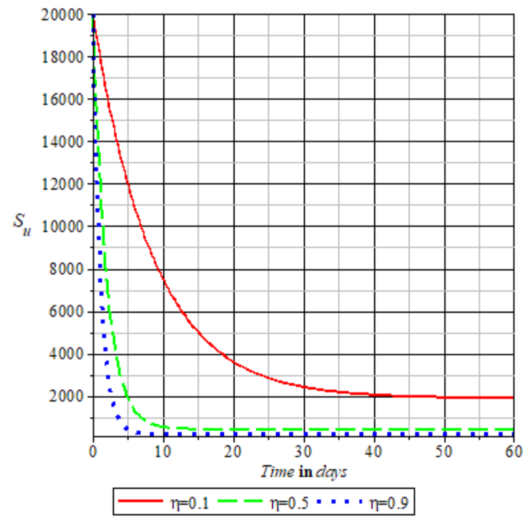

Figure 4. Graph of susceptible aware human dynamics shown with different value of η over time.

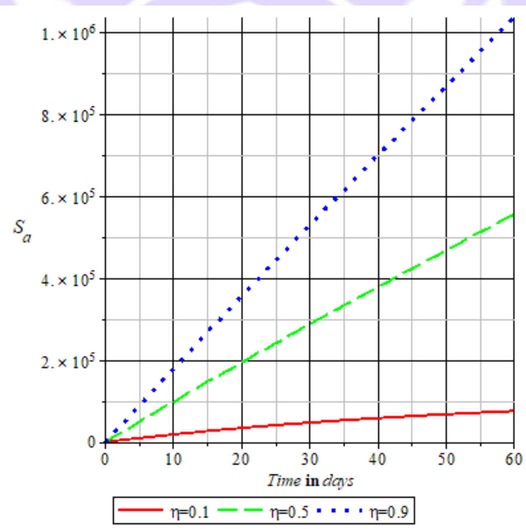

Figure 5. Graph of susceptible unaware human dynamics shown with different value of η over time.

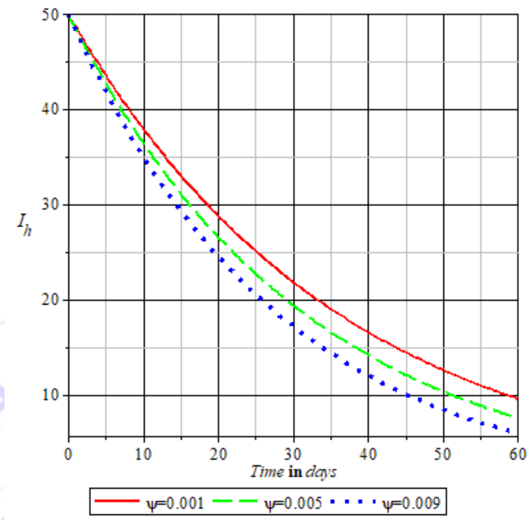

Figure 6. Graph of infected human dynamics shown with different value of ψ over time.

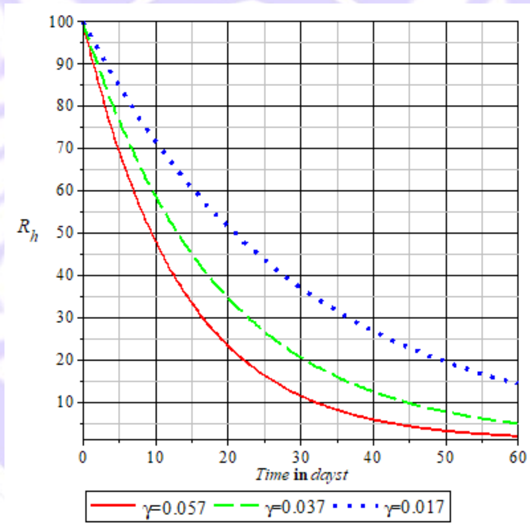

Figure 7. Graph of recovered human dynamics shown with different value of γ over time.

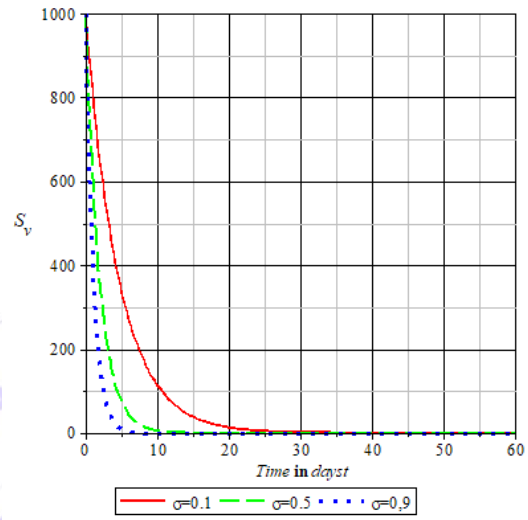

Figure 8. Graph of susceptible mosquito dynamics shown with different value of σ over time.

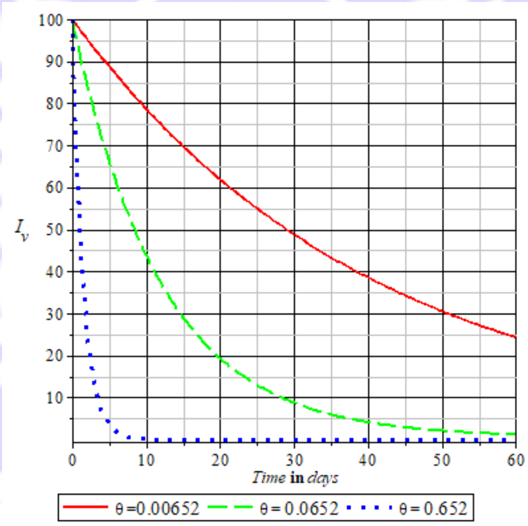

Figure 9. Graph of infected mosquito dynamics shown with different value of θ over time.

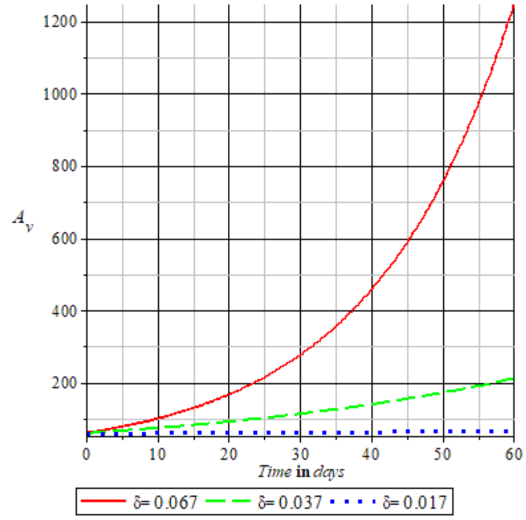

Figure 10. Graph of aquatic mosquito dynamics shown with different value of δ over time.

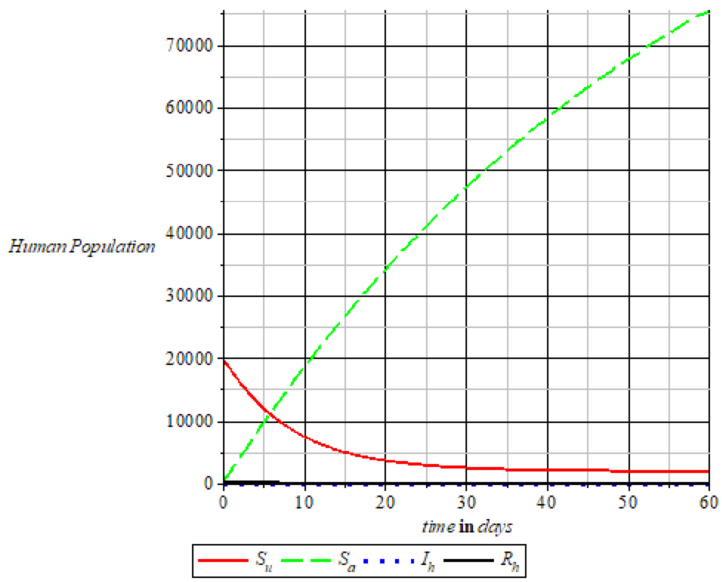

Figure 11. Graph of human population dynamics shown over time.

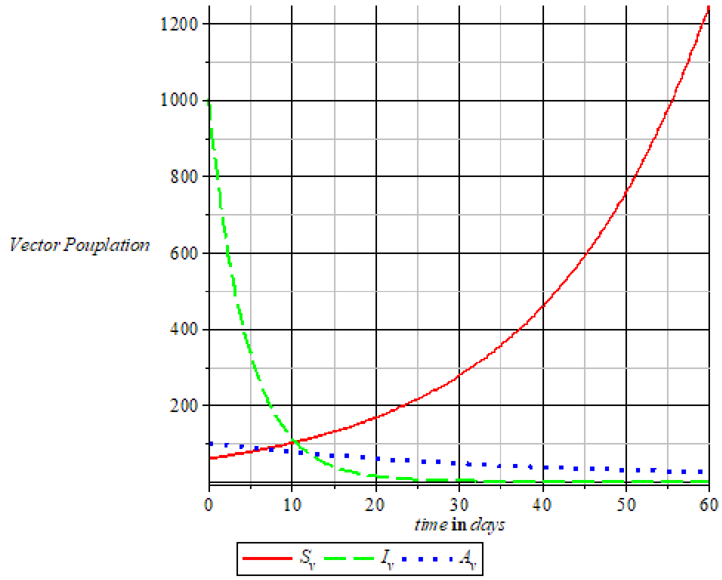

Figure 12. Graph of mosquito population dynamics shown over time.

4. Results and Discussion

The simulations result shows that since the mosquito's eradication is not yet over, the protocol must be observed. This can be deduced from Figure 4, where the unaware susceptible individuals against time decrease the population to zero with the rates of indoor insecticide spread η. Also, from the graph of aware human individuals in Figure 5, it is shown that the dynamics of aware individuals are increasing due to control and awareness with different rates of η in time being tested. This can be achieved if health workers apply media campaign strategies for enlightenment. Similarly, Figure 6 shows from the graph that up to 9% of the population of infected humans were decreasing due to campaign awareness on vector-borne diseases in a community, despite the gaps in the insecticide and traditional control by both some health workers and the community, respectively. Furthermore, the simulation results obtained, depicted in Figure 7, show the recovered individuals decreasing at an initial time with 17% and 37%, but when the mosquito protocol strategies are applied with insecticide, together with the effectiveness of traditional methods at 57%, the recovered individuals increase to become aware. Figure 8, shows a graph of susceptible mosquitoes against time with different rate of mosquito induced mortality. The disease induced mortality decreased drastically to zero which shows that the insecticide susceptibility status Anopheles-gambiae mosquitoes and community awareness of malaria is effective and the community needs to accept it. Figure 9, shows a graph of infected mosquitoes with different rates of insecticides, which indicates mosquitoes have little resistance to chemical ingredients and the strategies need some adjustment. Therefore, the scenario behaviour is similar to Figure 8. where the mosquitoes induced mortality decreasing and approached zero in time. This shows that the insecticides have a significant impact on mosquitoes. Figure 8 shows a graph of susceptible mosquitoes against time with different rates of mosquito-induced mortality. The disease-induced mortality decreased drastically to zero, which shows that the insecticide susceptibility status of Anopheles gambiae mosquitoes and community awareness of malaria are effective, and the community needs to accept it. Figure 9 shows a graph of infected mosquitoes with different rates of insecticides, which indicates mosquitoes have little resistance to chemical ingredients and the strategies need some adjustment. Therefore, the scenario behaviour is similar to Figure 8, where the mosquito-induced mortality decreased and approached zero in time. This shows that the insecticides have a significant impact on mosquitoes. Figure 10 shows a graph of aquatic mosquitoes where the mortality rate of aquatic mosquitoes due to insecticide is increasing in alignment. This result shows that the insecticides have a significant impact on both infected and healthy aquatic mosquitoes. Figure 11 shows a steady increase in the number of aware susceptible humans. Initially, the susceptible unaware human population decreasing drastically as a result of insecticides and awareness on malaria. Also, the number of infected humans decreases. However, as the number of infected humans decreasing, the aware susceptible population begins increasing. This tendency recommends that the public should accept mosquito protecting procedures to avoid direct interaction with the vectors, which can decrease the spread of malaria. From Figure 12, it can be deduced that the number of susceptible mosquitoes increases, while the number of infected and aquatic mosquitoes decreases. However, these occur due to single control measured after some time on susceptible vector. The infected and aquatic mosquitoes' population also decreases as individuals become extra aware and accept mosquito-prevention procedures in reducing mosquitos and awareness on malaria prevention.

5. Conclusion

The study highlights the importance of understanding the insecticide susceptibility status of Anopheles-gambiae mosquitoes to effectively combat malaria. It emphasizes the need for continuous monitoring of mosquito resistance patterns and increasing community awareness about malaria prevention measures. This approach is crucial for implementing effective vector control strategies and reducing the prevalence of malaria in the region. The modelling and sensitivity analysis revealed significant variations in the susceptibility of Anopheles-gambiae mosquitoes to different insecticides. Therefore, enhancing community awareness and involvement is crucial for the effective management and reduction of malaria transmission in the region. The study also concluded that Anopheles gambiae mosquitoes show changing levels of susceptibility to different insecticides, which has implications for malaria control strategies. It also highlighted the need for increased community awareness and education to effectively combat malaria transmission. Implementing targeted interventions and monitoring programmes can enhance the effectiveness of current malaria prevention efforts.

Acknowledgement:

The authors express their gratitude to for its assistance., TETFUND, and Nigerian Army University Biu, Nigeria.

Funding:

Funding for this research was assisted by TETFUND Nigeria, in collaboration with Nigerian Army University Biu under grant No (NAUB/MGT/G1/300/24).

Conflict of interest: The authors declare that there is no evidence of any conflicting financial interests or personal relationships influencing any of the works discussed in this paper.

Declaration of AI Generation: During preparation, the authors corrected errors in the manuscript's text using wordtune.com and quillBot. After using these resources to assess and make any necessary edits, the authors assume full responsibility for the publication's content.

References

Alhaj, M. S., and Nyabadza, F. (2025). A mathematical model of malaria transmission in conflict- affected regions and the implications on malaria interventions. *Scientific African*, e02746.

Ayalew, A., Molla, Y., and Woldegbreal, A. (2024). Modelling and stability analysis of the dynamics of malaria disease transmission with some control strategies. *In Abstract and Applied Analysis,* 2024(1), 8837744). Wiley.

Brauer, F., Castillo-Chavez, C., Mubayi, A., and Towers, S. (2016). Some models for epidemics of vector-transmitted diseases. *Infectious Disease Modelling*, 1(1), 79-87.

Busari, L. O., Raheem, H. O., Iwalewa, Z. O., Fasasi, K. A., and Adeleke, M. A. (2023). Investigating insecticide susceptibility status of adult mosquitoes against some class of insecticides in Osogbo metropolis, Osun State, Nigeria. *Plos one*, 18(5), e0285605.

Chitnis, N., Hyman, J. M., & Cushing, J. M. (2008). Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. *Bulletin of Mathematical Biology*, *70*, 1272-1296.

Diekmann, O., Heesterbeek, J. A. P., and Roberts, M. G. (2010). The construction of next generation matrices for compartmental epidemic models. *Journal of the Royal Society Interface*, 7(47), 873-885.

Duguay, C., Mosha, J. F., Lukole, E., Mangalu, D., Thickstun, C., Mallya, E., and Kulkarni, M. A. (2023). Assessing risk factors for malaria and schistosomiasis among children in Misungwi, Tanzania, an area of co-endemicity: A mixed methods study. *PLOS Global Public Health*, 3(11), e0002468.

Fagbohun, I. K., Idowu, E. T., Otubanjo, O. A., and Awolola, T. S. (2020). Susceptibility status of mosquitoes (Diptera: Culicidae) to malathion in Lagos, Nigeria. *Animal Research International*, *17*(1), 3541-3549.

Feachem, R. G., Chen, I., Akbari, O., Bertozzi-Villa, A., Bhatt, S., Binka, F., and Mpanju-Shumbusho, W. (2019). Malaria eradication within a generation: ambitious, achievable, and necessary. *The Lancet*, *394*(10203), 1056-1112.

Haile, G. T., Koya, P. R., and Mosisa Legesse, F. (2025). Sensitivity analysis of a mathematical model for malaria transmission accounting for infected ignorant humans and relapse dynamics. Frontiers in Applied Mathematics and Statistics, 10, 1487291.

Helikumi, M., Bisaga, T., Makau, K. A., and Mhlanga, A. (2024). Modelling the Impact of Human Awareness and Insecticide Use on Malaria Control: A Fractional-Order Approach. *Mathematics*, 12(22), 3607.

Kura, I. S., Ahmad, H., Olayemi, I. K., Solomon, D., Ahmad, A. H., and Salim, H. (2022). The status of knowledge, attitude, and practice in relation to major mosquito borne diseases among community of Niger State, Nigeria. *African Journal of Biomedical Research*, 25(3), 339-343.

Lees, R. S., Fornadel, C., Snetselaar, J., Wagman, J., and Spiers, A. (2023). Insecticides for mosquito control: improving and validating methods to strengthen the evidence base. *Insects*, 14(2), 116.

Murray, J.D. (2001). *Mathematical Biology I, An introduction, 3rd Ed*. Springer-Verlag Berlin Heidelberg.

Mwanga, G. G. (2025). Mathematical modelling and optimal control of malaria transmission with antimalarial drug and insecticide resistance. *Journal of Biological Dynamics*, 19(1), 2522345.

Opaginni, D.B., and M.O. Durojaye. (2025). Mathematical Modelling and Analysis of Malaria Transmission Dynamics with Early and Late Treatment Interventions. *Asian Research Journal of Mathematics.* 21 (7), 78-98. https://doi.org/10.9734/arjom/2025/v21i7959.

Pusawang, K., Sattabongkot, J., Saingamsook, J., Zhong, D., Yan, G., Somboon, P., and Sriwichai,P. (2022). Insecticide Susceptibility Status of Anopheles and Aedes Mosquitoes in Malaria and Dengue Endemic Areas, Thai– Myanmar Border. *Insects*, 13(11), 1035.

Ukpai, O. M., and Ekedo, C. M. (2019). Insecticide susceptibility status of Culex quinquefasciatus Diptera: Culicidae in Umudike, Ikwuano LGA Abia State,Nigeria. *International Journal of Innovation Sciences and Resources*. 6(1),114-118.

Van den Driessche, P., and Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical biosciences*, 180(1-2), 29-48.

Van den Driessche, P. (2017). Reproduction numbers of infectious disease models. *Infectious disease modelling*, 2(3), 288-303.

Wako, B. H., Dawed, M. Y., and Obsu, L. L. (2025). Mathematical model analysis of malaria transmission dynamics with induced complications. *Scientific African*, 28, e02635.

World Health Organization. (2024). WHO Malaria Policy Advisory Group (MPAG) meeting report, 4, 5 and 7 March 2024. *World Health Organization.*

World Health Organization. (2023). World malaria report 2023. *meeting report, 18– 20 April 2023*. (Accessed July, 2025), https://iris.who.int/handle/10665/374472.

World Health Organization. (2015). Global technical strategy for malaria 2016-2030. *World Health Organization*. (Accessed July, 2025).