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A Numerical Method for the Solution of Linear Volterra Integral Equations

Ganiyu Ajileye[a*], Aduroja O. Olamiposi[b], Grace I. Amakoromo[a] and Adewole M. Ajileye[b]

[a]*Department of Mathematics and Statistics, Federal University Wukari, Taraba State, Nigeria.*
[b]*Department of Mathematics, University of Ilesa, Ilesa, Osun State, Nigeria.*

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ABSTRACT

The goal of this paper is to present a numerical method for solving the Volterra integral equations using the collocation approximation method with a power series polynomial. The modeled problem is converted into an algebraic equation system and solved using the standard collocation points. After establishing the approach's uniqueness and convergence, numerical examples were used to evaluate its effectiveness. The results show that the method outperforms others.

1. Introduction

Many problems in mathematical physics can be solved using integral equations. These equations can also be found as reformulations of other mathematical problems, such as partial differential equations and ordinary differential equations. As a result, studying integral equations and methods for solving them is extremely valuable in practice. In recent years, there has been a growing interest in the Volterra integral equations developing in several domains of physics and engineering, such as potential theory and Dirichlet problems, electrostatics, particles transportation problems in astrophysics, contact problems, and diffusion problems (Zarnan, 2018).

Many researchers explored and considered numerical solutions to the Volterra integral equation, including: Bernstein method (Adhraa and Ayal, 2019), Adomian decompositions method (Khan and Bakoda, 2013), Collocation method by (Ajileye and Amoo, 2023; Agbolade and Anake, 2017), Hybrid linear multistep method (Mehdiyera, Ibrahim and Imanova, 2019), Chebyshev-Galerkin method (Issa and Saleh, 2017), Lagrange Interpolation (Shoukralla and Ahmed, 2020), Least-Squares Method (Al-Humedi and Shoushan ,2021), Chebyshev polynomials (MaadadiandRahmoune, 2018), Optimal Auxiliary Function Method (OAFM) (Zada, Al-Hamami, Nawaz, Jehanzeb, Morsy, Abdei-Aty and Nisar, 2021) and many more.

This paper consider Volterra integral equation of the second kind

$$y(x) + \lambda \int_0^x k(x,t)\, y(t) dt = f(x) \tag{1}$$

where $k(x,t)$ is the Volterra integral kernel, λ is the parameter given, $f(x)$ is a known function, and $y(x)$ is the unknown function to be determined.

2. Materials and Methods

In this section, we implement an approximation approach for the numerical solution of Volterra integral equations.

2.1 Method of Solution

Let the solution of equation (1) be approximated by

$$y(x) = \sum_{m=0}^{M} a_m x^m = \emptyset(x)\boldsymbol{A} \tag{2}$$

* Corresponding author. Tel.: +2348034906427
E-mail address: ajileye@fuwukari.edu.ng (G. Ajileye).
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where $\emptyset(x) = [1\ x x^2 \cdots x^M]$, $\boldsymbol{A} = [a_0 a_1 \cdots a_M]^T$

Substituting equation (2) into equation (1) gives

$$\emptyset(x)\boldsymbol{A} = \lambda \int_0^x k(x,t)\,(\emptyset(t)\boldsymbol{A})dt + f(x) \tag{3}$$

$$\left(\emptyset(x) - \lambda \int_0^x k(x,t)\,\emptyset(t)dt\right)\boldsymbol{A} = f(x) \tag{4}$$

Equation (4) can be in the form

$$M(x)\boldsymbol{A} = f(x) \tag{5}$$

where

$$M(x) = \emptyset(x) - \lambda \int_0^x k(x,t)\,\emptyset(t)dt$$

Collocating equation (5) at x_i using standard collocation points equation

$$x_i = a + \frac{(b-a)i}{M}, i = 0, 1, 2 \ldots \ldots . M$$

$$M(x_i)\boldsymbol{A} = f(x_i) \tag{6}$$

where.

$$M(x_i) = \begin{bmatrix} M_0(x_0) & M_1(x_0) & M_2(x_0) & \cdots & M_N(x_0) \\ M_0(x_1) & M_1(x_1) & M_2(x_1) & \cdots & M_N(x_1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ M_0(x_N) & M_1(x_N) & M_2(x_N) & \cdots & M_N(x_N) \end{bmatrix}, f(x_i) = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_N) \end{bmatrix}$$

To obtain the numerical result, we solve the system of equations (6) for the unknown values and substitute the results into the approximate solution.

2.2 Numerical Examples

In this section, we present numerical examples to evaluate the method's applicability and accuracy. Let $y_n(x)$ and $y(x)$ be the approximate and exact solutions, respectively. $Error_N = |y_n(x) - y(x)|$

Example 1: Considering Volterra integral equation

$$y(x) - \int_0^x (t-x)\,y(t)dt = x \tag{7}$$

Exact solution: $y(x) = sinx$

Source: (Zarnan, 2018)

Solution 1

The approximate solution of equation (7) at N=5 gives

$$y_5 = -3.552713678801 * 10^{-14} + 0.999978043885x + 0.244040421e - 3x^2 - 0.167616536724x^3 + 0.1612963621e - 2x^4 + 0.7252526964e - 2x^5$$

Table 1: Exact, approximate and absolute error values for example 1

X	**Exact**	**Our Method$_{N=5}$**	**Error$_5$**	**Error$_{\text{Zarnan } et\ al.=5}$**
0.2	0.198669330800	0.198669339600	8.80e-9	3.31e-4
0.4	0.389418342300	0.389418363600	2.13e-8	6.42e-4
0.6	0.564642473400	0.564642505600	3.22e-8	9.14e-4
0.8	0.717356090900	0.717356132100	4.12e-8	1.13e-3
1.0	0.841470984800	0.841471037900	5.31e-8	1.28e-3

Example 2: Considering Volterra integral equation

$$y(x) - \int_0^x (x-t)\, y(t)dt = x + 1 \tag{8}$$

Exact solution: $y(x) = e^x$

Source: (Shoukralla and Ahmed, 2020)

Solution 2

The approximate solution of equation (8) at N=5 gives

$$y_5 = 1.000000000000 + 1.000082628954x + .499069314450x^2 + 0.170409090701x^3 + 0.34866686678e - 1x^4 + 0.13854370802e - 1x^5$$

Table 2: Exact, approximate and absolute error values for example 2

x	Exact	Our Method$_{N=5}$	Error$_5$	Error$_{Shoukralla\ et\ al.=5}$
0.2	1.221402758000	1.221402752000	3,40e-08	9.149e-08
0.4	1.491824698000	1.491824780000	8.20e-08	6.031e-06
0.6	1.822118800000	1.822118933000	1.33e-07	7.08e-05
0.8	2.225540928000	2.225541113000	1.85e-07	4.10e-04
1.0	2.718281828000	2.718282092000	2.67e-07	1.61e-03

3. Results and Discussion

This section discusses the numerical findings developed from the solved examples using the proposed numerical approach.

The result obtained for Example 1, as shown in Table 1, is that the approximate solution at $N = 5$ gives $y_5 = -3.552713678801 * 10^{-14} + 0.999978043885x + 0.244040421e - 3x^2 - 0.167616536724x^3 + 0.1612963621e - 2x^4 + 0.7252526964e - 2x^5$

. The numerical result converged to an exact solution, and this confirmed that our method performed better than the method proposed by (Zarnan *et al.*, 2018) at the same value of N.

In numerical Example 2, as shown in Table 2, the approximate solution at $N = 5$ gives

$$y_5 = 1.000000000000 + 1.000082628954x + .499069314450x^2 + 0.170409090701x^3 + 0.34866686678e - 1x^4 + 0.13854370802e - 1x^5$$

The numerical results give a better result compared to the result obtained by (Shoukralla*et al.*, 2020) at $N = 5$.

4. Conclusions

The collocation method was examined for the numerical solution of Volterra integral equations in this work. This approach is reliable, effective, and straightforward to compute. Maple 18 is used for all of the computations in this work. The reliability of the method is demonstrated by considering some problems.

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