



## Patriarchy Family Tree Graph from Tiv to Kumator (Seember)

Babarinsa Olayiwola<sup>a\*</sup> and Ngule S. Sandra<sup>a</sup>

<sup>a</sup>Department of Mathematics, Federal University Lokoja, P.M.B 1154, Kogi, Nigeria

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### ABSTRACT

Tree graphs, as acyclic and connected structures, provide a framework for modelling hierarchical relationships, such as genealogical lineages. This study constructs and analyzes a patriarchal family tree of the Tiv ethnic group, tracing lineage from Tiv to Kumator, through the application of the Dsatur graph coloring algorithm. Each vertex represents an individual, while edges denote parent-child relationships, forming an acyclic connected graph. By implementing Dsatur, optimal vertex coloring was achieved such that no two adjacent vertices share the same color, confirming the tree's bipartite nature. The resulting family tree revealed a chromatic number of two, consistent with theoretical predictions for acyclic graphs, and a longest lineage path of fifteen generations. Aside from establishing structural relationships, the model determines how patriarchal lineage rules influence genealogical continuity, illustrating where dynastic extensions cease due to the absence of male descendants.

## 1. Introduction

Graph theory involves the study of mathematical formations used to model relationships between objects. These relationships can be modelled to form graphs using an adjacency matrix. An adjacency matrix,  $A(G) = [a_{i,j}]$ , is a square matrix indexed by the vertices  $\{v_1, v_2, \dots, v_n\}$ , denoted as

$$a_{i,j} = \begin{cases} 1 & \text{if } v_i v_j \text{ is an edge of } G \\ 0 & \text{otherwise} \end{cases}$$

The adjacency matrix is useful not only in constructing the graph itself but also in determining the connectivity of a graph (Babarinsa, 2022). A graph  $G$  comprises an ordered pair of vertices  $V(G) = \{v_1, v_2, \dots, v_n\}$  and edges  $(G) = \{e_1, e_2, \dots, e_n\}$  that connect them (Babarinsa and Kamarulhaili, 2019; Diestel, 2025). A simple (undirected) graph is a graph without loops (Diestel, 2025). A loop is an edge that links a vertex to itself. A pendant is a vertex that joins exactly one other vertex; otherwise, it is an isolated vertex (Vasudev, 2006).

A tree graph is a connected, acyclic graph in which every pair of vertices is linked by a unique path (Marcus, 2020). This absence of cycles makes it a fundamental structure in graph theory, widely recognized for its hierarchical

\*Corresponding author. Tel.: +2348060032554

E-mail address: [babarinsa.olayiwola@fulokoja.edu.ng](mailto:babarinsa.olayiwola@fulokoja.edu.ng) (Babarinsa Olayiwola)

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nature. The historical roots of tree graphs can be traced to the 19th century, particularly in the works of Arthur Cayley, who used them in chemical enumeration, and James Joseph Sylvester, who applied them in logical analysis (Anellis and Abeles, 2016). Their formulation provided the mathematical foundation for hierarchical modelling, from genealogical charts such as family trees to the abstract study of non-cyclic structures. The tree list is a data structure in which nodes/vertices are organized in a hierarchical manner, allowing nested levels of information. As one of the simplest yet most powerful graph structures, tree structures remain central to theoretical and applied mathematics.

The applications of tree graphs are broad and interdisciplinary. In computer science, they underpin essential data structures such as binary search trees, heaps, AVL trees, B-trees, and file directory systems, enabling efficient searching, sorting, and indexing (Medjedovic and Tahirovic, 2022). In artificial intelligence and machine learning, decision trees are central for classification and prediction, while in compiler design, syntax trees parse programming languages into a structured form (Galitsky, 2013). Biology employs phylogenetic trees to study evolutionary relationships and inheritance patterns, while linguistics uses parse trees to map grammatical structures. In organizational management, tree graphs model authority and communication hierarchies, and in networking, spanning trees ensure efficient routing without cycles (Hussain and Islam, 2007). Across these domains, the clarity and efficiency of trees is aided by graph algorithm. A graph algorithm is a procedure that solves problems on a graph  $G = (V, E)$  such as Dsatur graph coloring, in representing hierarchical, non-cyclic relationships explain their enduring importance in both theory and practice.

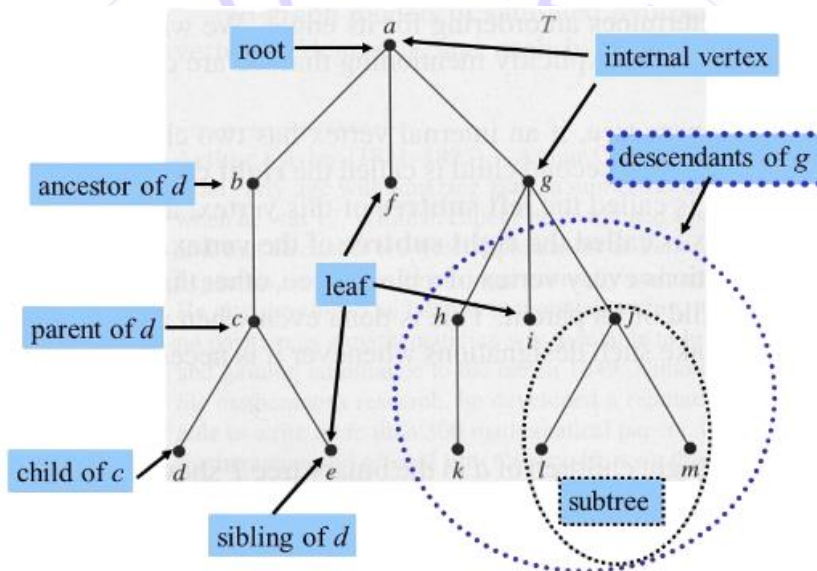
Graph coloring in the context of trees concerns the assignment of labels (often called colors) to vertices such that adjacent vertices do not share the same color (Tuza, 1997). This area of study originates from the Four-Color Problem posed in 1852 by Francis Guthrie, which led to significant developments in graph theory and was finally resolved in 1976 by Appel and Haken with a computer-assisted proof. For trees, vertex coloring is straightforward: every tree is bipartite and thus requires at most two colors (Malaguti & Toth, 2010). This property arises because trees contain no odd cycles, ensuring that they can always be partitioned into two disjoint sets of vertices where each set is colored uniformly. Edge coloring, another variant, is also tractable for trees due to their simple structure, making trees an important testing ground for coloring algorithms.

Section 2 discusses the graph tree, while Section 3 applies the graph coloring algorithm, specifically the Dsatur algorithm, for solving the family tree graph of the Tiv dynasty. Then we construct and analyze the tree list of Tiv dynasty to determine the chromatic number, terminal descendant, and diameter of the tree.

## 2. Preliminaries on Tree Graph

This section gives the background and terminology of the graph tree, which are needed in analyzing Section 3. A tree is a connected undirected graph with no simple circuits. A circuit or cycle is a path that starts and ends at the same vertex as all other vertices. A path is a sequence of edges that begins at a vertex of a graph and travels from one vertex to another vertex of the graph, such as a graph tree (Marcus, 2020). Thus, a tree may be a rooted tree or a binary tree. A rooted tree is a tree in which one vertex has been designated as the root and every edge is directed away from

the root (Elgot, Bloom, and Tindell, 1978). However, a binary tree is a special type of tree structure where each vertex has at most two children. These children are usually referred to as the left child and the right child (Valiente, 2002). A subtree is the subgraph of the tree consisting of a vertex in the tree and its descendants and all edges incident to these descendants. A parent is a unique vertex  $U(G)$ , such that there is a directed edge from  $U(G)$  to  $V(G)$  called a child. Vertices that have children are called internal vertices. Any vertex of a rooted tree with no children is termed a leaf. Siblings are vertices with the same parent  $U(G)$ . An ancestor is the path from the root to the vertex, excluding the vertex itself and including the root. However, descendants are the vertices that have  $V(G)$  as an ancestor, see (Klapisich-Zuber, 1991; Klawitter, 2020; Rahman, 2017). These definitions are illustrated in Figure 2.1.



**Figure 2.1** Sample of a family tree graph (Arumugam, Brandstädt, Nishizeki, and Thulasiraman, 2016)

Every tree  $T = (V, E)$  in graph theory is inherently bipartite, since its acyclic nature eliminates the possibility of odd cycles. By choosing any vertex as a root and partitioning all others according to the parity of their distance from it, placing even levels in one set  $V_1$  and odd levels in another  $V_2$ —each edge connects vertices of opposite parity, satisfying

$$V = V_1 \cup V_2, V_1 \cap V_2 = \emptyset, \text{ and } E \subseteq V_1 \times V_2.$$

This property directly determines the chromatic number of a tree: except for the trivial single-vertex case ( $\chi = 1$ ), all nontrivial trees have a chromatic number  $\chi(T) = 2$ , as two colors suffice to distinguish adjacent vertices (Mömke, Popa, Roshany, Ruderer, and Vincze 2025). The bipartite partition thus provides an optimal coloring, making trees the simplest structures for testing vertex coloring algorithms and establishing chromatic bounds in larger networks. Structurally, the degree of vertices governs the tree's branching pattern and connectivity. Since every tree satisfies  $|E| = |V| - 1$ , it follows that

$$\sum_{v \in V} \deg(v) = 2|E| = 2(|V| - 1),$$

giving an average degree

$$\bar{d} = \frac{2(|V|-1)}{|V|},$$

which is always less than 2. Vertices of degree 1 (leaves) define terminal branches, while those with higher degrees  $\text{degree}(\text{deg}(v) \geq 2)$  serve as internal or ancestral nodes (Wang, 2014). The diameter of a tree, defined as the length of the longest path between any two vertices

$$D(G) = \max(d(u, v)).$$

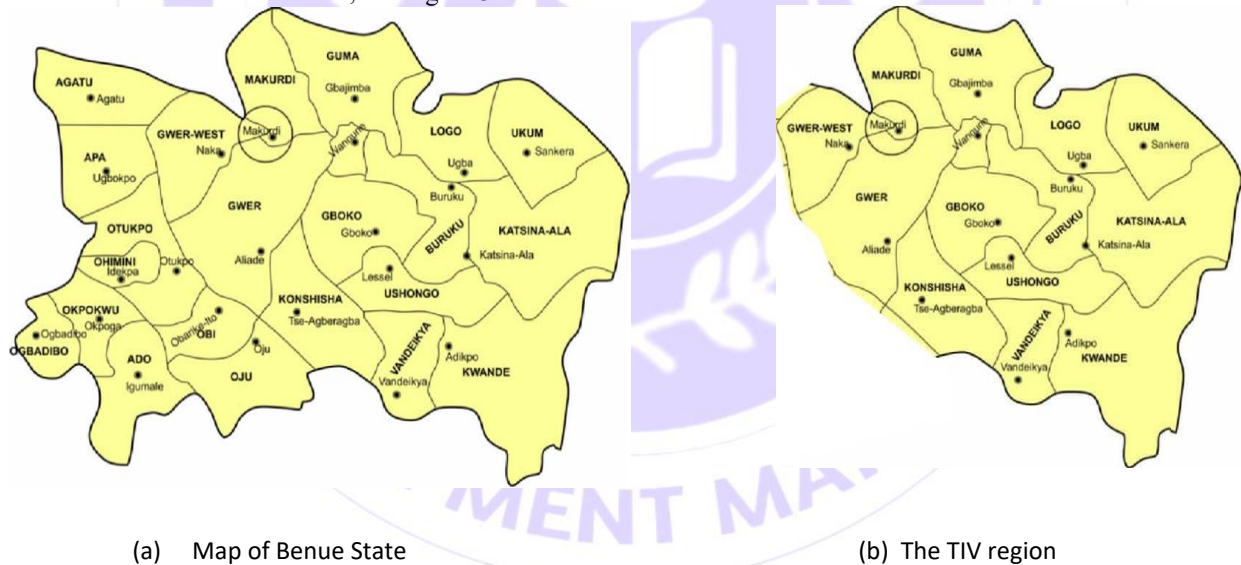
For  $u, v \in V$ .

These degree distributions determine the tree's shape, influence its diameter, and, in applied contexts such as genealogical modelling, quantify how lineage rules, like male-line inheritance, mathematically prune or extend the family structure.

### 3. Methodology

The family tree of Tiv to Seember considered in the article is patriarchal, hence female child is cut off from the family tree once she marries and changes her name to her husband's name. In addition, a forest from the Tiv is not considered as it will be bigger than the scope of this research.

To begin with, the family tree of Tiv need to start from the region where the tribe is located in Nigeria. The Tiv region covers two-thirds of Benue State, see Figure 3.1.



**Figure 3.1** A map of Benue State (a) and TIV region (b) (Ade, 2014)

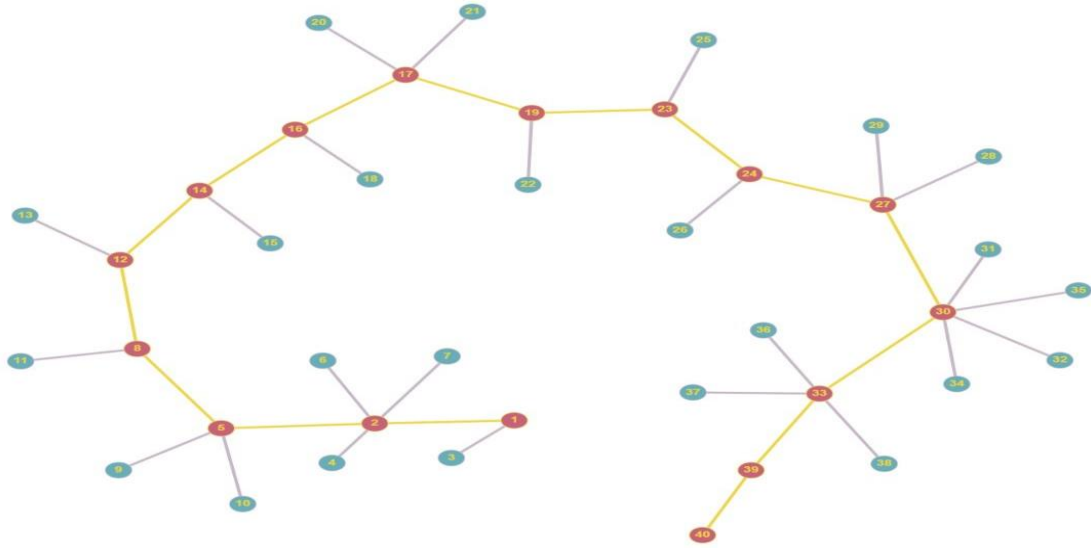
We consider using a graph coloring (Dsatur) algorithm to draw the family tree graph, which consists of the parents and children. This algorithm is imperative as it helps us to assign colors to each person in the family tree of Table 3.1.

**Table 3.1** Tree list of Tiv to Kumator

<b>Parent</b>	<b>Children</b>
<i>Tiv</i>	Ipusu Chongo
<i>Ipusu</i>	Shitile Kparev Ukum Tongou
<i>Kparev</i>	Kwande Jerichira Jemngbagh
<i>Kwande</i>	Barakuv Barajov
<i>Barajov</i>	Mata Gwa
<i>Gwa</i>	Atirkyese Sember
<i>Sember</i>	Agir Ivende
<i>Agir</i>	Anyam Kuha Too
<i>Anyam</i>	Ayaku V av
<i>Vav</i>	Soon Agbe
<i>Soon</i>	Ge Avihitwev
<i>Avihitwev</i>	Yeke Achigbe





**Figure 3.2** Colored tree graph of Tiv to Kumator**Figure 3.3** A tree graph showing the longest path between Tiv to Kumator.

From Figure 3.2, the constructed Tiv–Kumator family tree graph, denoted  $G = (V, E)$ , consists of  $|V| = 40$  vertices representing family members and  $|E| = 39$  edges ( $|E| = |V| - 1$ ) representing parent–child relations. The absence of cycles was verified from the adjacency matrix  $A(G)$ , where the trace  $tr(A^k) = 0$  for  $2 \leq k \leq |V|$ , confirming acyclicity. The degree sequence ranges from  $deg(v_i) = 1$  for leaves (terminal descendants) to  $deg(v_i) = 4$  for internal ancestral vertices, which indicates a moderately branched structure typical of genealogical trees. Applying the Dsatur algorithm yielded a chromatic number,  $\chi(G) = 2$ , consistent with the bipartite property of trees and confirms the coloring’s optimality and minimal chromatic requirement.

From Figure 3.3, the diameter of the tree was found to be  $D(G) = 15$ , corresponding to the Tiv–Kumator lineage path. This indicates the genealogical hierarchy of a direct descent chain of fifteen generations. The average degree of the tree shows that each ancestor connects to approximately two descendants on average, is

$$d = \frac{2 \times 39}{40} = 1.95,$$

Furthermore, by representing the patriarchal restriction (female-line termination) as a vertex deletion rule,  $G' = G - v_f$ , where  $v_f$  denotes female vertices who exit the lineage upon marriage, the resulting subgraph  $G'$  remains connected but pruned. This formalizes the observed genealogical truncation under Tiv patriarchal norms and mathematically demonstrates how sociocultural rules affect tree topology.

## 4. Conclusion

This study successfully modelled the Tiv patriarchal lineage from Tiv to Kumator using tree graph theory and the Dsatur coloring algorithm. The resulting graph, with a chromatic number of two and a maximum path length of fifteen, confirms the theoretical properties of tree graphs while providing a rigorous computational representation of family succession. The research reveals how graph-based methods can effectively capture real-world hierarchical and cultural structures, particularly in contexts where lineage is gender-dependent. The patriarchal rule of succession within the Tiv family tree creates natural terminations in the graph where male heirs are absent. Overall, this work demonstrates that the integration of graph algorithms with sociocultural genealogy provides an analytical tool for mapping heritage, understanding lineage discontinuities, and preserving ancestral knowledge within complex family systems. This study is beneficiary to individuals who wish to trace their lineage in either the patriarchal or matriarchal level.

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