International Journal of Development Mathematics Vol 2 Issue 4 Page No. 131 - 136

INTERNATIONAL JOURNAL OF DEVELOPMENT MATHEMATICS

ISSN: 3026-8656 (Print) | 3026-8699 (Online)

journal homepage: https://ijdm.org.ng/index.php/Journals

A [7 4 2] Linear Code Due to the Cayley Table for $n=7$ for the Generated Points of the (123)/(132) - Avoiding Patterns of the Non – Associative AUNU Schemes

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ARTICLE INFO

Article history:
Received 20 August 2025
Received in revised form 30 October 2025
Accepted 20 November 2025

Keywords:
Cayley tables, AUNU Scheme, Hamming codes, Standard Generator matrix, Reduced row Echelon form

MSC 2020 Subject classification:
05C45

ABSTRACT

In this communication, we present the construction of a [7 4 2] - linear code which is an extended code of the [6 4 1] code and is in one-to-one correspondence with the known [7 4 3] - Hamming code. Our construction is due to the Cayley table for $n=7$ of the generated points of W as permutations of the (132) *and* (123) -avoiding patterns of the non-associative AUNU schemes as reported by the authors earlier. First, the Cayley table for $n=7$ is converted to a matrix say G in the binary system using *Modular* arithmetic. The matrix G which is of order 5×7 is then shown to generate a linear code of size $M=16$. Next, echelon row operations are used to transform G into a standard generator matrix say G^I, for a [6 4 1] code. Finally, the [6 4 1] code is extended to obtain the desired [7 4 2]-linear Code which is in one-to-one correspondence with the known [7 4 3]- Hamming code.

1. Introduction

Historically, Claude Shannons paper titled, A Mathematical theory of Communication, in the early 1940s signified the beginning of coding theory and the first error-correcting code to arise was the presently known Hamming $[7 \quad 4 \quad 3]$ code, discovered by Richard Hamming in the late 1940s (David and John-Lark, 2011)). As it is central, the main objective in coding theory is to devise methods of encoding and decoding so as to effect the total elimination or minimization of errors that may have occurred during transmission due to disturbances in the channel. Since then, several other codes that have found suitable applications had evolved due to intensive research in this revealing area of applied algebra and discrete mathematics, termed coding theory. Some of the notable applicable discovered codes includes; the Golay codes, Reed Muller codes, Bose Chauduri codes (BHC) codes, Reed Solomon codes, Hadamard codes e.t.c.

The acronym AUNU which stands for AUDU and AMINU was first reported in Ibrahim and Audu (2005), also, see www.algebragroup.org. AUNU numbers form a sequence of natural numbers that occur in some counting problems, Ibrahim and Sloane (2006) Integer Sequence A119626 in the On-line Encyclopedia of Integer Sequence, the

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https://doi.org/10.62054/ijdm/0204.08

papers in OEIS often involving recursively defined objects. The n^{th} Aunu number is given by the recursive relation $A_n = \frac{p_{n \geq 3} - 1}{2}$, for details see also Ibrahim and Sloane (2006) integer sequence A123642, Ibrahim. And Sloane et al. (2007) integer sequence A128929 and Ibrahim and Sloane (2007) integer sequence A*128984*.

The special class of the (132) and (123) - avoiding Patterns of AUNU has found applications in various areas of applied Mathematics. The authors had reported the application of the adjacency matrix of Eulerian graphs due to the (132) - avoiding patterns of AUNU numbers in the generation and analysis of some classes of linear and cyclic codes as reported in Chun, lbrahim and Garba (2016a) and Chun, lbrahim and Garba (2016b) respectively. In Ibrahim, Chun and Garba (2016), the Authors utilized the Cayley tables for $n = 5$ as in Ibrahim and Abubakar (2016) to derive a standard form of the generator /parity check matrix for some code and also the construction of a [49 16 6] - linear code (Chun, 2018). In this article, we enumerate the construction of a [7 4 2] – linear Code from the Cayley table for $n = 7$ of the generated points of W (a non empty set of *n* points) as permutations of the (132) and (123) -avoiding patterns of the non-commutative AUNU schemes as reported in (Ibrahim and Abubakar (2016). The [7 4 2] linear code is shown to be an extended code of the [6 4 1] code and is in one-to-one correspondence with the [7 4 3]-Hamming Code.

2. Methodology

We consider the Cayley table below, which is constructed using $A_n(132)$ for $n = 7$ as in Ibrahim and Abubakar (2016).

Table 1: Cayley table for $n = 7$ showing generated points of W as permutations of (132) and (123)-avoiding patterns of AUNU scheme under the action of Θ.

Θ	1	2	3	4	5	6	7
1	1	3	5	7	2	4	6
2	1	4	7	3	6	2	5
3	1	5	2	6	3	7	4
4	1	6	4	2	7	5	3
5	1	7	6	5	4	3	2

We now convert the entries of the Cayley **Table 1** above to the binary system using $Modulus\,2\ \ arithmetic$. The table1, thus becomes the Table 2 below:

Table 2: Cayley table for $n = 7$ showing converted entries of table 1 into modulus 2 arithmetic.

Θ	1	2	3	4	5	6	7
1	1	1	1	1	0	0	0
2	1	0	1	1	0	0	1
3	1	1	0	0	1	1	0
4	1	0	0	0	1	1	1
5	1	1	0	1	0	1	0

The above table thus becomes the matrix G below

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Clearly, all possible linear combinations of the rows of G generate a linear Code say C of length $n = 7$ and size $M = 16$ with the following Code words;

0 0 0 0 0 0 0	0 0 1 1 1 1 0	1 0 0 1 0 1 1	1 1 0 0 1 1 0
0 1 0 0 0 0 1	0 1 0 1 1 0 1	1 0 0 0 1 1 1	1 1 0 1 0 1 0
0 0 1 0 0 1 0	0 1 1 0 0 1 1	1 0 1 1 0 0 1	1 1 1 0 1 0 0
0 0 0 1 1 0 0	0 1 1 1 1 1 1	1 0 1 0 1 0 1	1 1 1 1 0 0 0

Now, since G has five rows and generates a code with sixteen code words instead of $2^5 = 32$ code words, we seek a generator matrix for C. Deleting the first row and last column of G, we obtain a Matrix say

$$G^{II} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Next, we apply the following series of echelon row operations on G^{II}, we have;

$$i. R_2 = R_1 + R_2 \;\; ii. R_3 = R_1 + R_3 \;\; iii. R_2 = R_3 + R_2 \;\; iv. R_4 = R_1 + R_4 \;\; v. R_4 = R_4 + R_2 \;\; vi. R_4 = R_3 + R_4$$
$$vii. R_1 = R_1 + R_3 \;\; viii. R_3 = R_4 + R_3 \;\; respectively\, as\, follows;$$

$$G^{II}=\begin{bmatrix}1&0&1&1&0&0\\1&1&0&0&1&1\\1&0&0&0&1&1\\1&1&0&1&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&1&1&1&1\\1&0&0&0&1&1\\1&1&0&1&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&1&1&1&1\\0&0&1&1&1&1\\1&1&0&1&0&1\end{bmatrix}$$

$$\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\1&1&0&1&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&1&1&0&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&0&1&0&0&1\end{bmatrix}$$

$$\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&0&0&1&1&0\end{bmatrix}\rightarrow\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&0&0&1&1&0\end{bmatrix}\rightarrow\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&0\\0&0&1&0&0&1\\0&0&0&1&1&0\end{bmatrix}$$

$$=\left[\begin{array}{cccc|cc}1&0&0&0&1&1\\0&1&0&0&0&0\\0&0&1&0&0&1\\0&0&0&1&1&0\end{array}\right]=G^{I}$$

Observe that G^{I} above is a generator matrix in standard form, $i.e\ G^{I}=(I_k\,|X_{k\times n-k})$, but G^{I} which is a generator matrix for C has words (rows) of length $n=6$, while C has word length $n=7$. Note also that all words in C has even weight. Therefore extending the rows of G^{I} by adding one digit each to make the number of nonzero digits even in each row of G^{I} even, we obtain

$$G^{I}=\left[\begin{array}{cccccc|c}1&0&0&0&1&1&1\\0&1&0&0&0&0&1\\0&0&1&0&0&1&0\\0&0&0&1&1&0&0\end{array}\right]$$

Which is the required generator matrix for our code C.

3. Results

The Cayley table for $n = 7$ of the generated points of the (132) and (123)-avoiding patterns of the non-associative AUNU schemes has been used to construct a [7 4 2]-linear code with generator matrix G^I which is an extended code of the [6 4 1] code. The generated code which has size $M = 16$ is clearly seen to inherit a one-one correspondence with the famous [7 4 3] - Hamming code as both codes has the same number of codewords (16) and the same code length of (7) with the only difference in respect of their minimum distance of 2 amd 3 respectively making an isomorphism between the two codes obvious.

4. Conclusion

The Cayley tables generated from points of W (a non-empty set of n points) as permutations of the (132) and (123)-avoiding patterns of the non-associative AUNU schemes has pointed another direction in coding theory to the generation and analysis of the class of the hamming-related codes. Further study of the Cayley tables as in Ibrahim A.A. and Abubakar S.I.(2016) for $n > 7$ *i.e* $n = 11, 13, 17, . . .$ is recommended with the aim of generalization of application of these cayley tables in codes generation,

AKNOWLEDGEMENT

We acknowledge the support enjoyed in this research endeavour by the Institution Based Research (IBR) TETFund Grand 2025, through the Plateau State University Bokkos , Jos, Nigeria.

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