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Mathematical Modeling of Social Media Addiction and Depression in a Stratified Population

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ABSTRACT

This study considers a deterministic compartmental model to analyse the dynamics between social media addiction and depression within a heterogeneous population comprising educated and non-educated individuals. The model considers critical transitions of behaviour: exposure, light use, addiction, depression, recovery, and quitting. Solutions are shown to be positive and bounded. The basic reproduction number is derived through the next-generation matrix approach. The stability analysis indicates that if the reproduction number is less than unity, then the social-media-free equilibrium is locally and globally asymptotically stable. Numerical simulation and sensitivity analysis suggest that increased transmission and media-induced depression enhance addiction and depression. At the same time, high recovery, education, and quitting rates significantly reduce the prevalence of addiction and depression. Thus, the present results emphasize the imperative need for educational intervention and mental-health strategies to help alleviate social media-related psychological disorders.

1. Introduction

Social media is an essential technology that provides a set of significant skills for people, such as but not limited to, accessing knowledge, problem-solving, entrepreneurship, self-directed learning, and online socializing, which are harnessed effectively (Baruah, 2012; Shek et al., 2008; Siddiqui and Singh, 2016; Wang et al., 2022; Kim and Lee, 2023). On the other hand, inappropriate use of these technologies might result in negative effects, with the most prominent being social media addiction (SMA) (Ali et al., 2019; Monacis et al., 2017). The emergence of online social media platforms has significantly impacted the dissemination of information as well as interpersonal interactions. The rising use of these platforms has increased the tendency of people to resort to compulsive use of social media, which has been linked to negative effects such as reduced productivity, unsuccessful interpersonal interactions, reduced academic or professional performance, irregular sleep patterns, and increased instances of anxiety and depression (Zhang et al., 2022; Xiao, et al 2020). "Social media addiction" is a phrase used to define people who habitually rely on platforms such as Instagram, Twitter, and Facebook. Because SMA has become a rising matter of serious consideration within the realm of contemporary twenty-first-century public health, more research has been initiated

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worldwide concerning the prevalence and psychological effects of SMA (Anise et al., 2013; Deborah et al., 2019; Guedes et al., 2016; Xiang et al., 2015; Chakraborty et al., 2010; Hou et al., 2019; Thomée et al., 2011). There have been a variety of models used by researchers to define the phenomenon of SMA, as well as the resulting psychological effects. For example, a compartmental dynamical model assessing the impact of social media on academic performance, with a definition of "media-free" and "addiction" equilibrium points, concluding that a basic reproduction number dictates the existence of persistent use of media, has been described by Ishaku et al. (2018). For instance, an SMA dynamical model characterized by optimal control, with a subdivision of population into susceptible, exposed, addicted, recovered, and "quitted" persons, has been described by Haileyesus et al. (2021), who concluded conditions on the local/global stability concerning the particular "addiction-free" equilibrium point, via application of the Pontryagin's Maximum Principle. Additionally, a fractional order SMA dynamical system, involving differential equations with Caputo-Fabrizio derivatives giving consideration to "memory" effects, has been illustrated by Shutaywi et al. (2023), who concluded that, when the reproduction number is small enough, the "addiction-free state" is asymptotically stable.

Recent research has increased the application of the SMA approach to include mental health factors. The work of Ali et al. (2024) included depression in an SMA approach with seven compartments, where stability analysis, sensitivity, and PRCC (Partial Rank Correlation Coefficient) analysis showed a high causal relationship between addiction and the development of depression. Similarly, Juhari et al. (2024) produced a more precise model with mild and severe stages of addiction, applying local stability conditions for addiction equilibrium points and endemic equilibrium points, with sensitivity and PRCC analysis used to examine the effect of parameters. In all cases, the population is modelled as being uniform, which neglects variability in education, socio-economic class, and vulnerability to behaviour. Kamal et al. (2025) present a mathematical model addressing social media addiction, incorporating an optimal control mechanism. The study analyzes equilibrium points, dependence-free and dependence-endemic, and calculates the basic reproduction number. Stability analysis shows that the addiction-free equilibrium is stable for $R_0 < 1$ while the endemic equilibrium maintains stability when $R_0 > 1$. The application of the Center Manifold theorem indicates a forward bifurcation at $R_0 = 1$. The model introduces time-varying awareness and punishment as control strategies and formulates an optimal control problem solved via Pontryagin's maximum principle. Numerical simulations demonstrate that an integrated approach from stakeholders and policymakers is essential for effectively managing social media addiction. Despite these advancements, most existing models assume homogeneous populations, overlooking heterogeneity in education, socio-economic status, and behavioural susceptibility. This limitation motivated the development of a more realistic and extended model that captures differences in susceptibility, exposure, addiction, and recovery across these groups. To address this gap, the study introduces a deterministic compartmental model for Social Media Addiction and Depression among Educated and Non-Educated Susceptible populations (SMADENS). The study aims to understand how addiction and depression spread through media influence and peer interaction; examine the impact of education on vulnerability and resilience;

compute threshold values such as the basic reproduction number; apply numerical simulations to reflect real-world behavioural trends; and provide evidence that can support effective policies, mental-health strategies, and digital-media regulation.

2. Model Formulation

A deterministic compartmental modeling approach is used to describe the transmission dynamics of Social Media Addiction and Depression among Educated and Non-Educated Susceptible populations at a time t. The total population $N(t)$ is sub-divided into the following sub-population: educated susceptible population $S_E(t),$ non-educated susceptible $S_N(t),$ exposed individuals $E(t),$ light social media users $A_1(t),$ addicted social media users $A_2(t),$ social media depressed individuals $D(t),$ recovered individuals $R(t),$ and social media quitters $Q(t).$ Thus, the total population is given as:

$$N(t) = S_E(t) + S_N(t) + E(t) + A_1(t) + A_2(t) + D(t) + R(t) + Q(t) \tag{1.1}$$

Individuals are recruited into the susceptible population at a rate Λ with a fraction a educated while the remaining $(1-a)$ non-educated. The educated susceptible population is increased by recovered individuals who became susceptible at the rate σ and by non-educated susceptible individuals who became educated at the rate ξ. The population will then decrease by those who became exposed at the rate $\beta_1(A_1+\theta A_2)$, by those who became quitters and those who die naturally at the rate ϕ_1 and μ respectively. Thus, the rate of change of educated susceptible individuals is given by

$$\frac{dS_E}{dt} = \Lambda(1-a) + \xi S_N + \sigma\tau R - (\beta_1(A_1+\theta A_2) + \phi_1 + \mu)S_E \tag{1.2}$$

Non-educated susceptible individuals will later be educated at the rate ξ, the population will further decrease by those who became exposed at the rate $\beta_2(A_1+\theta A_2)$, by those who became quitters at the rate ϕ_2 and those who die naturally at the rate μ. Thus, the rate of change of non-educated susceptible individuals is given by

$$\frac{dS_N}{dt} = \Lambda a - (\beta_2(A_1+\theta A_2) + \xi + \phi_2 + \mu)S_N \tag{1.3}$$

The transition rates from educated susceptible and non-educated susceptible to exposed is given by, respectively, by the following force of infection $\beta_1(A_1+\theta A_2)$ and $\beta_2(A_1+\theta A_2)$ where θ is the contact rate from addicted individuals, and β represents transmission rate from educated and non-educated susceptible individuals respectively. The rate of change of exposed individuals is given by

$$\frac{dE}{dt} = (\beta_1 S_E + \beta_2 S_N)(A_1 + \theta A_2) - (\lambda + \mu)E \tag{1.4}$$

The light social meda users' population is increased by the exposed individuals who became light users at the rate λ and is decreased by those who became social media addicted at the rate ε and natural death at the rate μ. Thus, the rate of change in light social media users is given by

$$\frac{dA_1}{dt} = \lambda E - (\varepsilon + \mu)A_1 \tag{1.5}$$

The social media addicted population is generated from light social media users at the rate ε and is decreased by those who became social media depressed at the rate $\alpha + \kappa(1-e)$, where α is the rate at which depression is brought on by media impact, also decreased by those who recovered or die naturally at the rate κe and μ respectively. Thus, the rate of change of social media addicted population is given by

$$\frac{dA_2}{dt} = \varepsilon A_1 - (\kappa + \alpha + \mu)A_2 \tag{1.6}$$

Social media depressed individuals will increase by proportion of light users at the rate $\alpha + \kappa(1-e)$. The social media depressed class will die either naturally or due to the depression at the rate μ and d respectively. While ρ is the rate at which the individuals exit from the population. So, the rate of change of social media depressed individuals is given by

$$\frac{dD}{dt} = (\alpha + \kappa(1-e))A_2 - (\rho + \mu + d)D \tag{1.7}$$

A fraction e may recover naturally from the social media addiction and move to the recovered class at the rate κ, recovered individuals will also increase by recovery from depression at the rate ρ. Also, some proportion τ of recovered individuals will move back to educated susceptible population while others became quitters, the population will also decrease by natural death μ. The rate of change of recovered individuals is given by

$$\frac{dR}{dt} = \kappa e A_2 + \rho D - (\sigma + \mu) R \tag{1.8}$$

Social media quitters will increase either by proportion of recovered individuals at the rate $\sigma(1-\tau)$ or from educated and non-educated susceptible individuals at the rate ϕ_1 and ϕ_2 respectively. The quitters will only decrease by natural death at the rate μ . The rate of change of social media quitters is given by

$$\frac{dQ}{dt} = \phi_1 S_E + \phi_2 S_N + \sigma(1-\tau) - \mu Q \tag{1.9}$$

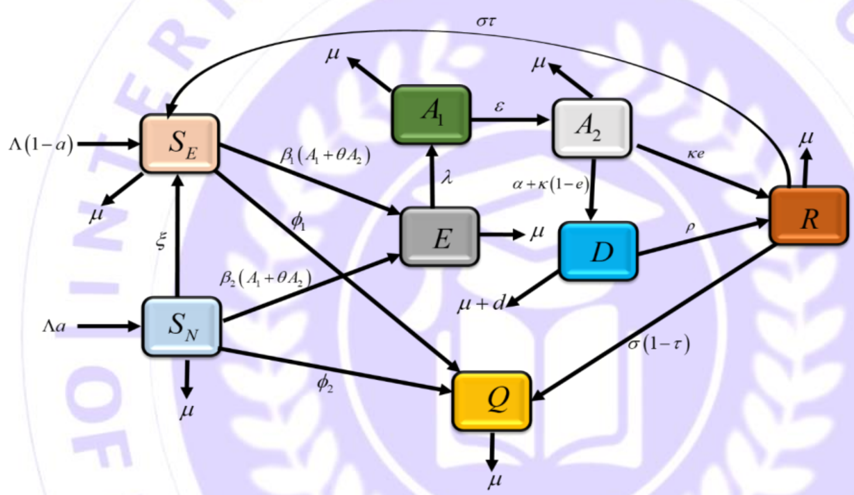


Figure 1: Schematic diagram of the model

$$\left.\begin{aligned}
\frac{dS_E}{dt} &= \Lambda(1-a) + \xi S_N + \sigma\tau R - (\beta_1(A_1+\theta A_2) + \phi_1 + \mu)S_E \\
\frac{dS_N}{dt} &= \Lambda a - (\beta_2(A_1+\theta A_2) + \xi + \phi_2 + \mu)S_N \\
\frac{dE}{dt} &= (\beta_1 S_E + \beta_2 S_N)(A_1+\theta A_2) - (\lambda+\mu)E \\
\frac{dA_1}{dt} &= \lambda E - (\varepsilon+\mu)A_1 \\
\frac{dA_2}{dt} &= \varepsilon A_1 - (\kappa+\alpha+\mu)A_2 \\
\frac{dD}{dt} &= (\alpha + \kappa(1-e))A_2 - (\rho+\mu+d)D \\
\frac{dR}{dt} &= \kappa e A_2 + \rho D - (\sigma+\mu)R \\
\frac{dQ}{dt} &= \phi_1 S_E + \phi_2 S_N + \sigma(1-\tau)R - \mu Q
\end{aligned}\right\} \quad (1.10)$$

Table 1: Parameters of the modified SMADENS Model

Parameters	Description
Λ	Recruitment rate
ζ	Rate at which non-educated susceptible individuals become educated.
λ	Exit rate from the exposed class to light social media users
μ	Natural death rate
α	The rate at which depression is brought on by media impact
β_1	Transmission rate for educated susceptible individuals
β_2	Transmission rate for non-educated susceptible individuals
σ	Exit rate from recovered class
θ	Contact rate from addicted individuals
k	Exit rate from Addicted class
ϕ_1	Rate at which Susceptible educated class became quitters
ϕ_2	Rate at which Susceptible non-educated class became quitters
ρ	Rate at which depressed individual recovered

ε	Rate at which the light users became addicted to social media
a	Proportion of recruited susceptible individuals who are not educated
e	Percentage of addicted individuals who recovered
d	Death due to social media depression
τ	Proportions of recovered individuals who became susceptible

3. Basic Properties of the Model

Theorem 1:

The closed set $\Omega = \left\{(S_E, S_N, E, A_1, A_2, D.R, Q)\epsilon\mathbb{R}^8, N \leq \frac{\Lambda}{\mu}\right\}$ is positivity invariant.

Proof:

Taking the total population at time t as:

$$N(t) = \{(S_E(t) + S_N(t) + E(t) + A_1(t), A_2(t) + D(t) + R(t) + Q(t)\}$$

Adding the equation, we get

$$\frac{dN}{dt} + \mu N \leq \prod \tag{1.11}$$

Whose solution is

$$N(t) \leq \frac{\Pi}{\mu} + \left(N_0 - \frac{\Pi}{\mu}\right)e^{-\mu t} \tag{1.12}$$

As $t \longrightarrow \infty$, Then $N(t) \longrightarrow \frac{\Pi}{\mu}$

Therefore, the closed set Ω is positively invariant and hence the proof is complete.

Theorem 2: if $S_E(0), S_N(0), E(0), A_1(0), A_2(0), D(0), R(0)$, and $Q(0)$ are all non-negative, then the solutions $S_E(t), S_N(t), E(t), A_1(t), A_2(t), D(t), R(t)$ and $Q(t)$ are all positive for $t \geq 0$

Proof:

From the equation (1.10) we have

$$\frac{dS_E}{dt} = \prod(1 - a) + \xi S_N + \sigma\tau R - (\beta_1(A_1 + \theta A_2) + \phi_1 + \mu)S_E$$

This equation can be expressed without loss of generality, after eliminating the positive term as an inequality

$$\frac{dS_E}{dt} \geq -(\beta_1(A_1 + \theta A_2) + \phi_1 + \mu)S_E \tag{1.13}$$

Integrating both sides using separation of variables, we have

$$S_E(t) \geq e^{-(\beta_1 A_1 + \beta_1\theta A_2 + \phi_1\mu)t} * e^c \tag{1.14}$$

Let $e^c = C$, so it gives

$$S_E(t) \geq Ce^{-(\beta_1 A_1 + \beta_1\theta A_2 + \phi_1\mu)t} \tag{1.15}$$

Hence, $S_E(t) > 0$

Similarly, it can be shown in the same manner that the remaining variables are no-negative. This proves that the solutions of system are positive for all $t \geq 0$.

4. Social-free Equilibrium Point of the Model

To obtain the equilibrium point, we equate the right-hand sides of the model equation (1.10) to zero and solve it. Which gives

$$S_N^* = \frac{(\prod a)}{(\xi + \phi_2 + \mu)}, \qquad S_E^* = \frac{\prod\{(1-a)(\xi + \phi_2 + \mu) + a\}}{(\xi + \phi_2 + \mu)\xi + \phi_2 + \mu}$$

$$Q^* = \frac{\prod\{\phi_1(1-a)(\xi + \phi_2 + \mu) + \xi\phi_1 a\} + \phi_2 a)(\phi_1 + \mu)\}}{(\xi + \phi_2 + \mu)(\phi_1 + \mu)} \tag{1.16}$$

Therefore, the disease-free equilibrium state of the model is given by

$E_0 = \{S_E^0, S_N^0, Q^0, 0,0,0,0,0\}$.

5. Basic Reproduction Number

Using the next-generation matrix approach (Van den Driessche and Warmouth, 2002), we obtained the basic reproduction number. Firstly, we arrange the system to get a group of infected classes only, that is $\{E, A_1, A_2\}$.

Let $F_i(x)$ be the rate of appearance of new infections in the i compartment, let $V_i(x)$ be the transition out of the compartment i by all other means.

$$F = \begin{bmatrix} 0 & (\beta_1 S_E^* + \beta_2 S_N^*) & \theta(\beta_1 S_E^* + \beta_2 S_N^*) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.17}$$

And,

$$V = \begin{bmatrix} (\lambda + \mu) & 0 & 0 \\ -\lambda & (\varepsilon + \mu) & 0 \\ 0 & -\varepsilon & (\kappa + \alpha + \mu) \end{bmatrix} \tag{1.18}$$

Thus, the spectral radius $\rho(FV^{-1})$ gives the basic reproduction number of the model as

$$R_0 = \left(\frac{\pi\beta_1\left((1-a)(\xi + \varphi_2 + \mu) + a\xi\right)}{(\xi + \varphi_2 + \mu)(\varphi_2 + \mu)} + \frac{\beta_2 \pi a}{(\xi + \varphi_2 + \mu)} \right)\left(\frac{\lambda\left(\kappa + \alpha + \mu\right) + \theta\lambda\varepsilon}{(\lambda + \mu)(\varepsilon + \mu)\left(\kappa + \alpha + \mu\right)} \right) \tag{1.19}$$

6. Global Stability of Social-free Equilibrium Point

We used Castello-Chavez theorem (Castello-Chavez *et al.,* 2002) to investigate the global asymptotic stability of the social-free state. For the theorem to work, we rewrite (1.10) in the form:

$$\left.\begin{aligned} \frac{dX}{dt} &= H(X_1, X_2) \\ \frac{dZ}{dt} &= G(X_1, X_2), G(X_1, 0) \end{aligned}\right\} \tag{1.20}$$

where $X_1 = \{S_E, S_N, D, R, Q\}$ and $X_2 = \{A_1, A_2, E\}$. Here, the components of $X_1 \in \mathbb{R}^5$ denote the uninfected individuals and the components of $X_2 \in \mathbb{R}^3$ denote the infected individuals. The disease-free equilibrium of the system now becomes $E^0 = (X^*, 0)$. To guarantee global asymptotic stability, the following two conditions must be met.

i. $F(X_1, X_2) = F(X, 0),$ (1.21)

ii. $\hat{G}(X, Z) \geq 0, \Rightarrow \hat{G}(X_1, X_2) = AX_2 - G(X_1, X_2)$ (1.22)

$$F(X_1, X_2) = \begin{bmatrix} \frac{dS_E}{dt} \\ \frac{dS_N}{dt} \\ \frac{dD}{dt} \\ \frac{dR}{dt} \\ \frac{dQ}{dt} \end{bmatrix} \tag{1.23}$$

Then,

$$F(X_1, 0) = \begin{bmatrix} \prod(1-a) + \xi S_N - (\phi_1 - \mu)S_E \\ \prod a - (\xi + \phi_2 + \mu)S_N \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1.24}$$

To show that $F(X_1, 0) \Rightarrow E^0$, we take

$$\frac{dS_E}{dt} = \prod(1-a) + \xi S_N - (\phi_1 - \mu)S_E \tag{1.25}$$

Solving it using integrating factor gives

$$S_E(t) \longrightarrow \frac{\prod(1-a) + \xi S_N}{\phi_1 + \mu} \tag{1.26}$$

Also,

$$S_N(t) \longrightarrow \frac{\prod a}{\xi + \phi_2 + \mu} \tag{1.28}$$

Therefore,

$$F\left(X_1,0\right)\rightarrow E^0=\left[S_E^0,S_N^0,0,0,0\right]$$

Then

$$G(X_1,X_2)=\begin{bmatrix}(\beta_1S_E+\beta_2S_N)(A_1+\theta A_2)-(\lambda+\mu)E\\ \lambda E-(\varepsilon+\mu)A_1\\ \varepsilon A_1-(\kappa+\alpha+\mu)\end{bmatrix}\begin{bmatrix}E\\ A_1\\ A_2\end{bmatrix} \quad (1.29)$$

And

$$A=\begin{bmatrix}-(\lambda+\mu) & (\beta_1S_E^0+\beta_2S_N^0) & \theta(\beta_1S_E^0+\beta_2S_N^0)\\ \lambda & -(\varepsilon+\mu) & 0\\ 0 & \varepsilon & -(k+\alpha+\mu)\end{bmatrix} \quad (1.30)$$

$$AX_2=\begin{bmatrix}-(\lambda+\mu)E+(\beta_1S_E^0+\beta_2S_N^0)A_1+\theta(\beta_1S_E^0+\beta_2S_N^0)A_2\\ \lambda E+-(\varepsilon+\mu)A_1\\ \varepsilon A_1-(k+\alpha+\mu)A_2\end{bmatrix} \quad 1.31)$$

$$AX_2-G(X_1,X_2)=(\beta_1S_E^0+\beta_2S_N^0)A_1+\theta(\beta_1S_E^0+\beta_2S_N^0)A_2-(\beta_1S_E+\beta_2S_N)A_1-\theta(\beta_1S_E+\beta_2S_N)A_2$$

$$G(X_1,X_2)=AX_2-G(X_1,X_2)\geq 0 \text{ since } S_E^0<S_E,\ S_N^0<S_N$$

Which implies that $\hat{G}\left(X_1,X_2\right)\geq 0$. Therefore, the conditions (i) and (ii) have been met and hence E_0^* is Globally Asymptotically Stable (GAS).

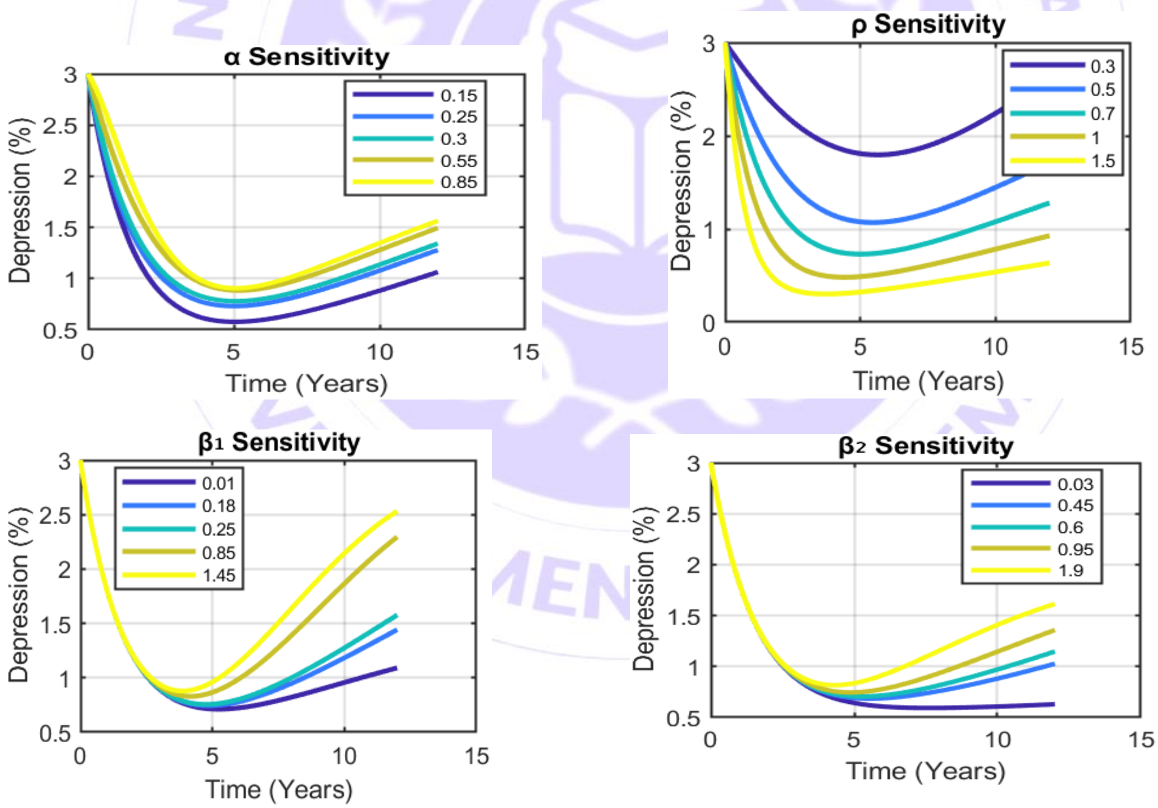
Table 2: Parameter values used for numerical simulations

Parameters	Values	References
Λ	0.5	Ali *et al.* (2024)
ζ	0.4	Assume
λ	0.25	Alemned and Alemu (2021)
μ	0.05-0.25	Alemned and Alemu (2021), Kington *et al.* (2021)
α	0.3-0.25	Ali *et al.* (2024)
β_1	0.1	Ali *et al.* (2024)
β_2	0.8	Guo & Li (2020)
σ	0.4	Alemned & Alemu (2021)
θ	0.25	Alemned & Alemu (2021)
k	0.0027	Dillon *et al.* (2002)

ϕ_1	0.01	Ali *et al.* (2024)
ϕ_2	0.07	Assume
ρ	0.7	Guo & Li (2020)
ε	0.03	Assume
a	0.2	Assume
e	0.8	Dillon *et al.* (2002)
d	0.01	Ali *et al.* (2024)
τ	0.35	Kington *et al.* (2021)

7 Numerical Simulations

To illustrate the theoretical results, numerical simulations are carried out using ODE 45 MATLAB. Model parameter values for the numerical simulations source are listed in Table 2.



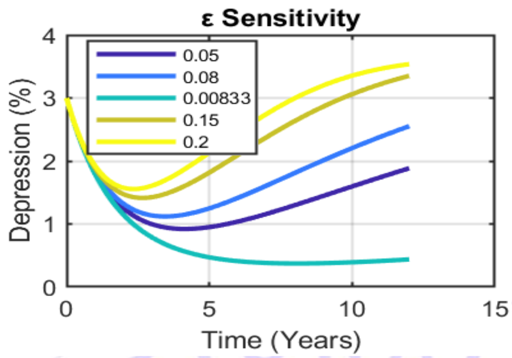


Figure 2: Sensitivity Analysis for Different Values of Model Parameters on Depressed Population

Figure 2 ascertains how changes in particular characteristics affect the prevalence of depression in the population, sensitivity analysis was carried out. Individual parameters like $\alpha, \rho, \beta_1, \beta_2$ and ε were changed while others remained unchanged. Result on the effect of α on depression prevalence shows a distinct pattern can be seen in the α-sensitivity graph; throughout the simulation period, higher values of α result in higher levels of depression. Regardless of α, depression eventually increases after first declining. Nonetheless, α has a considerable influence on the rise's size. The result of ρ -sensitivity plot shows a similar U-shaped pattern, with an initial decline followed by a gradual increase; larger ρ values result in significantly higher depression levels; this suggests that ρ may represent relapse rate or the influence of persistent triggers; higher relapse potential causes depression to accumulate over time, which is consistent with empirical evidence that people with past episodes are more likely to relapse if stressors remain present. The graph for β_2 shows that raising this parameter causes the prevalence of depression to rise more quickly. The first decline phase becomes narrow with very high β_2, suggesting resistance to early improvement. The rate of behavioural or social influences that exacerbate depression is probably regulated by β_2. This demonstrates how, if unchecked, peer pressure, societal pressure, or internet exposure situations can significantly exacerbate mental health issues. The behaviour of β_2 is mirrored in the β_1-sensitivity study, where increased β_1 leads to noticeable increases in the occurrence of depression. Transitions between moderate-risk and high-risk situations may be represented by β_1. Due to its significant impact, early-risk persons should be the focus of therapies before they develop serious mental health conditions. Effect of ε on depression shows that higher ε implies higher long-term prevalence, underscoring the significance of improving recovery pathways and access to mental health care. The ε - sensitivity graph demonstrates that higher ε results in significantly elevated depression prevalence, although the overall shape remains consistent.

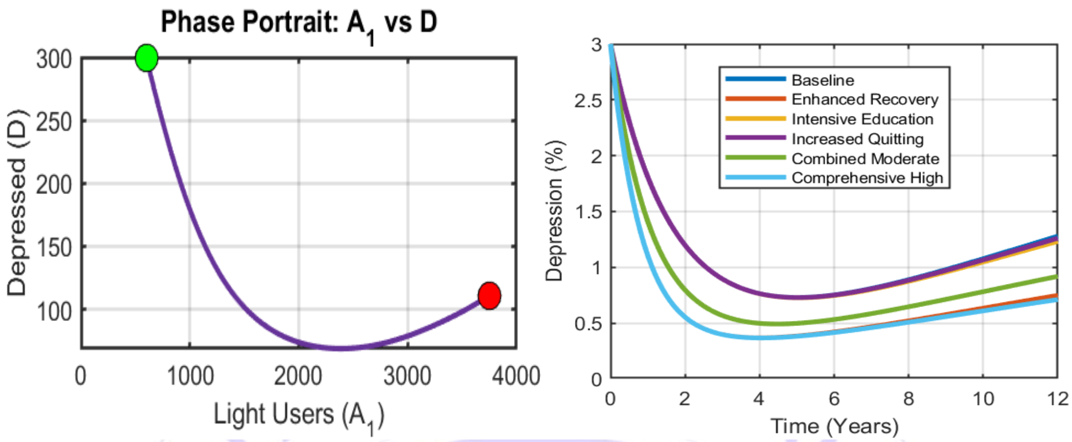


Figure 3: Phase Portrait Light Users (A_1) vs Depressed (D) Individuals and Depression (%) over time under various strategy

Figure 3 is the results of phase portrait light users (A_1) vs depressed (D) individuals and depression (%) over time under various strategy. The Figure shows a phase display of light users V and depressed individuals (D). We observed that the trajectory indicates a nonlinear relationship by first curving downward and then upward. While depression starts to rise again when the number of light users surpasses a particular threshold. The figure 3b above shows a time-series graph comparing depression prevalence over 12 years under multiple intervention strategies. We observed that in the early years, all interventions quickly lessen depression. The lowest levels of depression are regularly displayed by the Comprehensive High strategy. After years five and six, some techniques begin to slightly increase once more, most likely indicating a return to equilibrium.

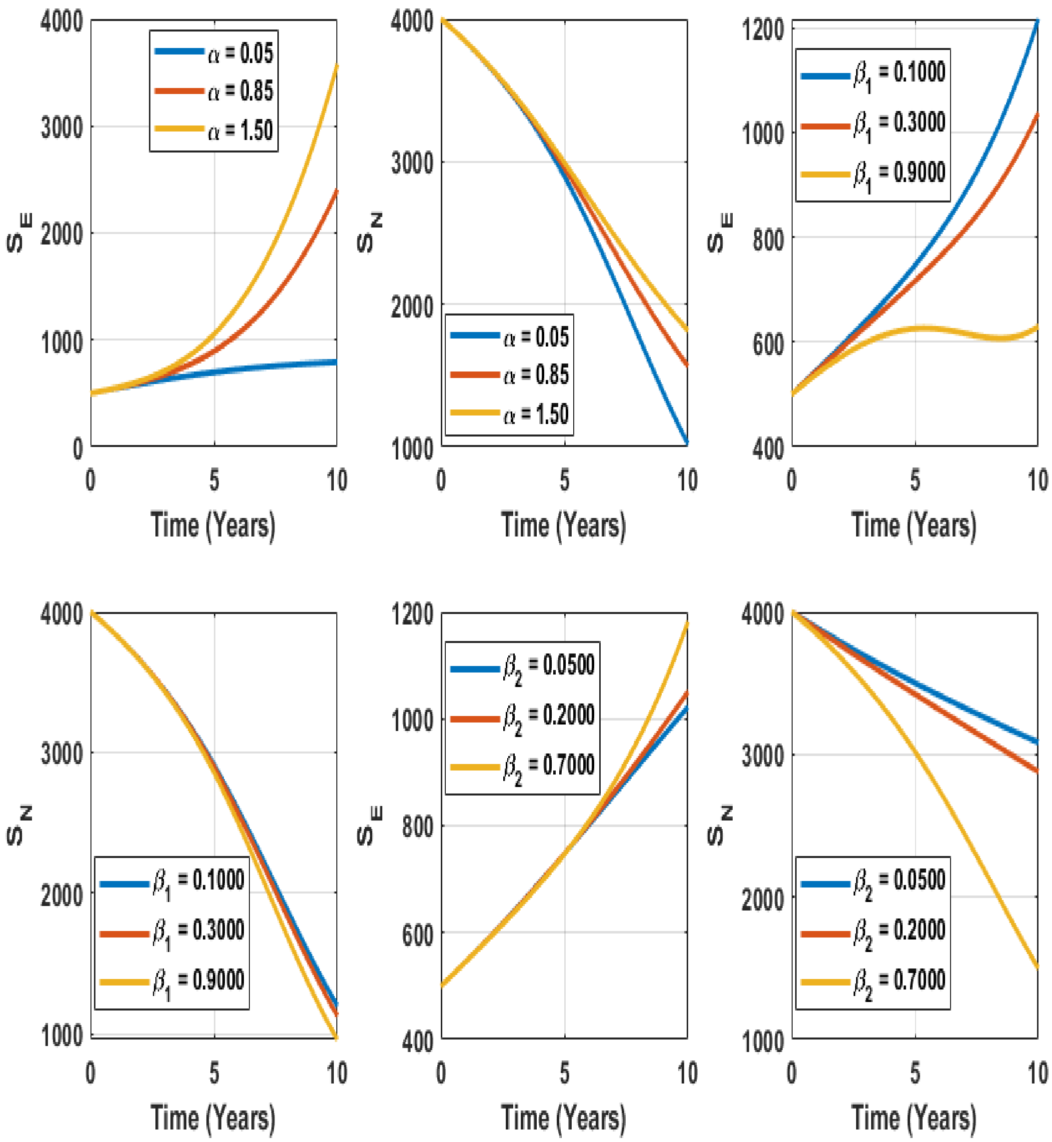


Figure 4: Time Evolution of Population Classes with Different Values of α , β_1 and β_2

Figure 4 is the time evolution of population classes with different values of α , β_1 and β_2 . The figure shows the influence of different values of α , β_1 and β_2 . on S_E and S_N in long-term dynamics. Increasing α causes the S_N population to diminish more slowly while the S_E population grows more quickly. This implies that α increases the transmission of the disease by hastening the transition into the exposed class. The S_N population declines more slowly as α increases, while the S_E population increases faster. This suggests that α accelerates the shift into the exposed class, increasing the spread of the disease.

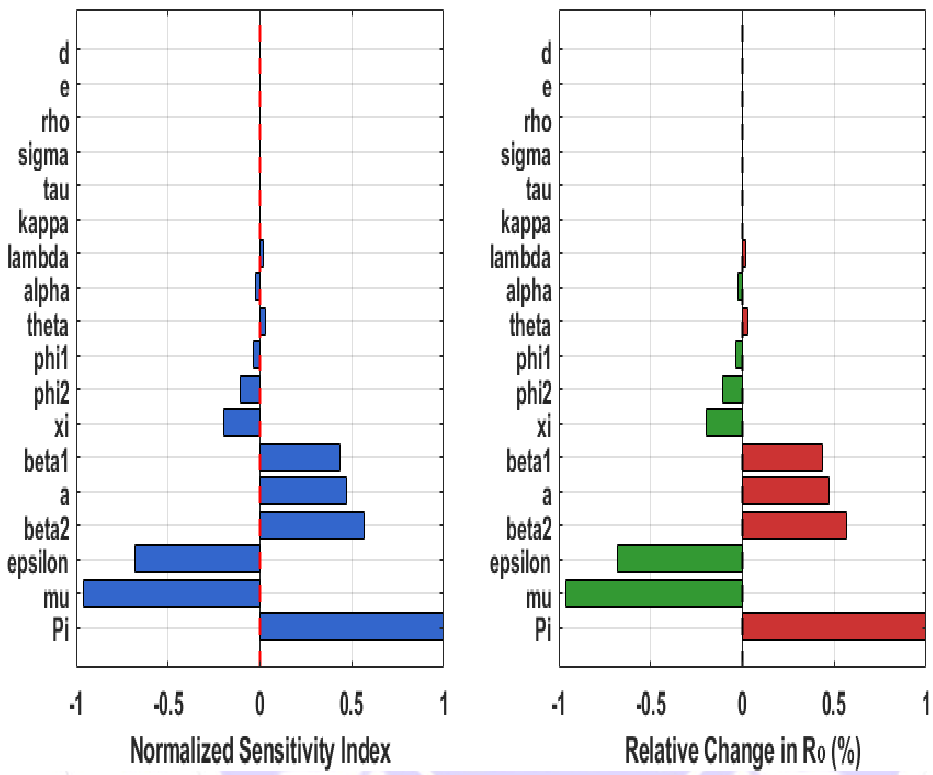


Figure 5: Normalized Sensitivity and Relative Change in R_0 .

We conducted a sensitivity analysis to look at each parameter's impact on the transmission of addiction to social media. Following the approach outlined by Blower and Dowlatabadi (1994), we utilized the index of standardized forward sensitivity. The figure 5 above shows the normalized sensitivity index and relative change in R_0. Each model parameter's unique contribution is further measured by the sensitivity bar graphs. The strongest positive influence is exerted by recruitment rate, which means that a greater population inflow expands the pool of susceptible individuals, the next strongest positive drivers of are β_1 and β_2, and ε has the biggest adverse effect, while the remaining parameters $\alpha, \rho, \kappa, \sigma, \theta, \tau, \lambda, \phi_1$ and $, \phi_2$ have little effect and stay near zero.

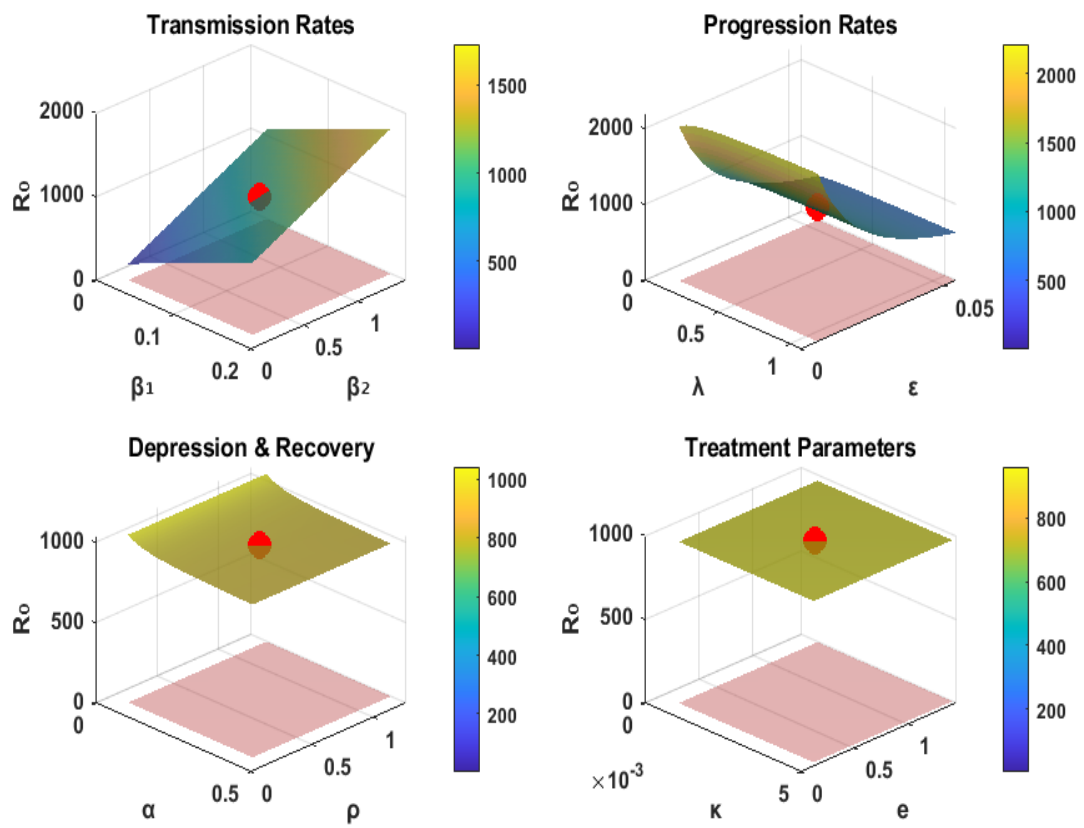


Figure 6: Two-Parameter Sensitivity Surfaces

Figure 6 is two-parameter sensitivity surfaces. In the transmission parameters (β_1, β_2), R_0 increases dramatically with either transmission rate, as the 3D surface illustrates. This synergistic impact suggests that even if only one transmission channel becomes more intense, the disease can spread quickly. In progression parameters (λ, ε), R_0 is slightly enhanced by λ, whereas it is significantly decreased by ε. The downward curvature suggests that accelerating the exit from the infectious state has a considerably greater effect on lowering R_0 than raising λ does on raising it. In depression and recovery parameters (α, ρ), although these parameters impact within-host transitions, they do not significantly alter transmission dynamics at the population level, as seen by the near-flat surface, which shows minimal influence on R_0. While in treatment parameters (κ, e), minimal variation in R_0 is also produced by treatment-associated factors.

8. Conclusion

In this research, a deterministic compartmental model has been developed for studying the addiction problem on social media and depression in a heterogeneous population with both non-educated and educated people. This deterministic compartmental model is mathematically sound, with solutions that are always positive and bounded. The basic reproduction number, which plays a significant role in studying the spread of addiction as well as depression, has been calculated, showing that for a stable media-free state, the reproduction number should be less than unity. The simulation result clearly indicates that increased contact rate and depression effect increase the prevalence of addiction, which can be reduced by recovery interventions and education. Though the result is significant, it suffers from some drawbacks, such as being a deterministic model that makes certain assumptions, not accounting for individual variation, and ignoring certain demographics. The future scope includes the use of stochastic/computational models with a fractional order with real-world inputs to increase applicability.

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