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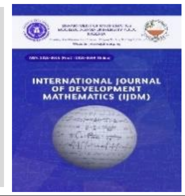

Estimators of Linear Regression Model with Non-Spherical Disturbance: Evidence from Nigerian Inflation-Trend and Economic Time Series Data

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ABSTRACT

Ordinary Least Squares (OLS) estimation loses efficiency when some of the linear regression assumptions about error terms are violated. These conditions are common in applied econometrics. The research proposes four new estimators, each with two weight structures (W_1 and W_2), to address violations of no autocorrelation and homoscedasticity in the error terms, commonly referred to as non-spherical disturbances. The proposed estimators are: the Maximum Likelihood Weighted Estimator (MLWE), the Weighted Maximum Likelihood Estimator (WMLE), the Cochrane–Orcutt Weighted Estimator (COWE), and the Weighted Cochrane–Orcutt Estimator (WCOE). Monte Carlo simulations across different sample sizes demonstrate that OLS exhibits substantial error in small samples, whereas the proposed estimators consistently maintain low error and significant efficiency gains. Among the estimators, MLWE W_2 and COWE W_2 demonstrated the strongest performance across scenarios. Application to Nigerian macroeconomic data confirms the simulation results: diagnostic tests reveal violations of OLS assumptions, and the alternative estimators delivered more precise coefficients, smaller standard errors, and higher explanatory power. These findings underscore the value of the proposed methods as practical and robust alternatives to OLS, particularly in settings where heteroscedasticity and autocorrelation co-occur.

1. Introduction

Regression analysis is a widely used statistical method for examining and quantifying relationships between variables (Gujarati & Porter, 2009; Wooldridge, 2016). At the core of this approach is the Classical Linear Regression Model (CLRM), which posits that the dependent variable is a linear combination of one or more independent variables plus a random disturbance term (Greene, 2018). Several essential assumptions about the data-generating process support the model. These include linearity between dependent and independent variables, independence of observations, exogeneity of regressors, homoscedasticity of error terms, absence of autocorrelation, and normality of the error distribution.

These assumptions enable straightforward statistical inference and ensure that the estimator possesses desirable properties, such as unbiasedness, efficiency, and consistency (Gujarati, 2005; Wooldridge, 2010). In theory, when these conditions are met, the Ordinary Least Squares (OLS) estimator is the Best Linear Unbiased Estimator (BLUE) according to the Gauss–Markov theorem. In other words, no other linear unbiased estimator will have a smaller variance than OLS. This optimality result has been documented in both classical econometrics literature and modern statistical theory (Kramer, 1980; Kleiber, 2001; Greene, 2018). The popularity of OLS in applied research stems not only from its optimality under ideal conditions but also from its computational simplicity, interpretability, and

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extensive software support. In practice, however, the strict assumptions of CLRM rarely hold perfectly. Unobserved factors often influence economic and financial data, exhibit non-stationary trends, display cyclical patterns, or are subject to measurement errors. Moreover, many economic variables are interconnected, meaning that shocks in one variable can spill over into others, thereby violating the assumptions of independence and exogeneity. (Fomby et al., 1988; Maddala, 2002; Ayinde, 2006; Ayinde & Ipinyomi, 2007). More recent applied econometric research has shown that macroeconomic and panel datasets often exhibit heteroscedasticity, autocorrelation, and cross-sectional dependence (Stock & Watson, 2020; Baltagi, 2021).

Two common and related departures from the CLRM assumptions involve the structure of the error term: heteroscedasticity and autocorrelation. Heteroscedasticity occurs when the variance of the error term is not constant across observations, which can arise in cross-sectional studies where variability increases with the level of an explanatory variable, or in financial time series where volatility clustering is common (White, 1980; Hansen, 2022). Autocorrelation, on the other hand, arises when error terms are correlated across observations, which is particularly common in time series and panel data where observations are ordered in time or space (Cochrane & Orcutt, 1949; Prais & Winsten, 1954; Hildreth & Lu, 1960). The presence of heteroscedasticity or autocorrelation renders the OLS estimator inefficient, leading to biased standard errors, invalid hypothesis testing, and misleading policy conclusions (Greene, 2018). Consequently, a variety of estimation techniques have been developed to address these issues separately. For heteroscedasticity, heteroscedasticity-consistent covariance matrix estimators (HCCMEs) such as White's robust standard errors (White, 1980) and Weighted Least Squares (WLS) are common. For autocorrelation, especially in first-order autoregressive processes AR(1), techniques such as the Cochrane–Orcutt procedure, the Prais–Winsten transformation, the Hildreth-Lu iterative method, and maximum likelihood estimators are well established (Fomby et al., 1988; Davidson & MacKinnon, 2004).

While a wide range of procedures address heteroscedasticity and autocorrelation separately, there is limited consensus on a single, practical class of estimators that jointly and efficiently address both problems in typical applied settings. Methods such as heteroscedasticity-consistent covariance estimators, Weighted Least Squares, Cochrane and Orcutt type transformations, and Heteroscedasticity and Autocorrelation Consistent (HAC) estimators like Newey and West are well established and widely used (White, 1980; Cochrane & Orcutt, 1949; Prais & Winsten, 1954; Newey & West, 1987). More recent advances in feasible generalized least squares and high-dimensional covariance regularisation offer promising ways to model complex error structures, including serial and cross-sectional dependence (Bai et. al., 2021). Even so, several practical issues remain. First, many available procedures focus on improving standard-error estimation without altering the point estimates, explicitly accounting for both heteroscedasticity and autocorrelation at the estimation stage.

Second, some methods require prior knowledge or strong assumptions about the error covariance structure, which may not be plausible in real-world datasets. Third, the finite-sample performance of theoretically appealing estimators can be poor when covariance components are estimated imprecisely. These limitations motivate the current study, which proposes and evaluates practical estimators that jointly address heteroscedasticity and autocorrelation in linear regression models, assessing their statistical properties through simulation and application to economic data.

2. Methodology

The development of the proposed estimators was based on the Maximum Likelihood Estimator approach, which provides a unified framework for estimation (Greene, 2018), and the Cochrane-Orcutt iterative procedure, which is used to address autocorrelation in regression errors (Cochrane & Orcutt, 1949). The transformed models shall result in the Maximum Likelihood Weighted Estimator, Weighted Maximum Likelihood Estimator, Cochrane-Orcutt Weighted Estimator, and Weighted Cochrane-Orcutt Estimator.

2.1 Model Formulation

In matrix form, the OLS model is written as:

$$y = X\beta + \varepsilon \tag{1}$$

where: y is an $n \times 1$ vector of dependent variables,

X is an $n \times (\mathrm{k}+1)$ matrix of explanatory variables (including a column of ones for the intercept),
β is a $(\mathrm{k}+1) \times 1$ vector of parameters,
ε is an $n \times 1$ vector of errors.

2.2 Simulation Design

Monte Carlo experiments were conducted across a range of sample sizes to evaluate the estimator's performance under heteroscedasticity and autocorrelation. The simulation considered sample sizes $n \in \{50, 100, 150, 200, 300\}$ and for each configuration, 1,000 replications were performed.

2.2.1 Generation of Error Term

To introduce both heteroscedasticity and autocorrelation in the disturbance term, we specified a non-spherical error covariance structure. First, unit-specific variances were generated from a uniform distribution: $\sigma_t^2 \sim U(1, 50)$, where $t = 1,2, \dots, n$.
Second, we defined an autoregressive correlation matrix to capture temporal dependence:

$$R_{ij} = \rho^{|i-j|}, \rho = 0.99, \tag{2}$$

so that

$$R = \begin{pmatrix} 1 & \rho & \rho^2 & \cdots \\ \rho & 1 & \rho & \cdots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}_{n \times n} \tag{3}$$

The autocorrelation parameter ρ is fixed at 0.99 to represent an extreme persistence scenario. This choice allows assessment of estimator robustness and numerical stability under extreme serial dependence and nearly violated standard regularity conditions. It is chosen to stress-test estimators' performance.
Combining these components, the full variance–covariance matrix of the error term was constructed as:

$$\Sigma_u = D_\sigma R D_\sigma, \tag{4}$$

where, $D_\sigma = diag(\sigma_1, \sigma_2, \dots, \sigma_n)$.
The resulting error vector was generated as $u \sim N(0, \Sigma_u)$, using multivariate normal draws.

2.2.2 Generation of Covariates

The covariate matrix X consisted of six independent variables, each drawn from a lognormal distribution to ensure positivity and right skewness, properties that mirror the empirical behavior of many macroeconomic variables, such as income, production, or expenditure. Specifically, for each variable $x_i, x_i \sim Lognormal(\mu_i, \sigma_i^2)$, where $i = 1,2,3, \dots 6$

2.2.3 Generation of the Dependent Variable

The dependent variable Y was generated as a linear combination of the covariates and the error term:

$$y = X\beta + u \tag{5}$$

where, $\beta = (5.7, 7.5, 3.7, 8.3, 6.3, -9.9, 5.6)'$.
This specification ensures that the simulated data exhibit both heteroscedasticity and autocorrelation, providing a realistic environment for testing the robustness of estimation under conditions commonly observed in macroeconomic data.

2.3 Research Data

Secondary data on Gross Domestic Product (GDP), Government capital and recurrent expenditure, and different sources of Government revenue, which are Oil-Revenue, Non-oil Revenue, Federation Account, and Federal retained Revenue, shall be extracted from the publication of the Central Bank of Nigeria for the past several years. The methodology adopted in this study is a linear regression model, in which GDP (a measure of economic growth) is viewed as a function of Government expenditure and various sources of revenue, spanning from the first quarter of 2010 to the second quarter of 2024. The model's assumptions shall be examined, and these have been reported to exhibit problems of both autocorrelation and heteroscedasticity (Ayinde & Lukman, 2013; Ayinde et al., 2014; Lukman et al., 2014; Ayinde et al., 2015). Thus, the model is of the form:

$$y = f(x_1, x_2, x_3, x_4, x_5, x_6) + u \tag{6}$$

where, y = Gross Domestic Product (GDP), x_1 = Recurrent Expenditure, x_2 = Non-Oil Revenue, x_3 = Federal Retained Revenue, x_4 = Capital Expenditure, x_5 = Oil Revenue, x_6 = Federal Account, u = Stochastic term.

2.4 Proposed Estimators Formation

Let

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \cdots + \beta_r x_{rt} + u_t \tag{7}$$

$$u_t = y_t - (\beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \cdots + \beta_i x_{rt})$$

$$u_t = y_t - \widehat{y_t} \tag{8}$$

where, y_t is the observed dependent (response) variable for observation *t,*
$\widehat{y_t}$ is the estimated dependent (response) variable for observation *t,*
x_{it} are the explanatory variables (regressors),
β_i are the regression coefficients (parameters to estimate),
u_t is the random error term.

To account for the disturbance, two alternative auxiliary regressions for the conditional error variance are estimated. In the first specification, $\log u_t{}^2$ is regressed on the vector of explanatory variables $(x_{1t}, x_{2t}, \ldots, x_{it})$. The fitted values from this regression are exponentiated to obtain an estimate of the conditional variance,

$$\hat{\sigma}_t^{2\,(1)} = exp^{\widehat{\log u_t}^{2\,(1)}} \tag{9}$$

from which the first set of Feasible Generalised Least Squares weights (Greene, 2018) is constructed as

$$W_1 = \left(\frac{1}{\hat{\sigma}_t^{2\,(1)}}\right)^{\frac{1}{2}} \tag{10}$$

In the second specification, a more flexible variance model is considered by augmenting the auxiliary regression with squared regressors, $(x_{1t}, x_{2t}, \ldots, x_{it}, x_{1t}{}^2, x_{2t}{}^2, \ldots, x_{it}{}^2)$. The resulting fitted values yield the conditional variance estimate

$$\hat{\sigma}_t^{2\,(2)} = exp^{\widehat{\log u_t}^{2\,(2)}} \tag{11}$$

from which the second weight is constructed as

$$W_2 = \left(\frac{1}{\hat{\sigma}_t^{2\,(2)}}\right)^{\frac{1}{2}} \tag{12}$$

Generally,

$$W_k = \left(\frac{1}{\hat{\sigma}_t^{2\,(k)}}\right)^{\frac{1}{2}} \tag{13}$$

Estimator 1: WMLE
Let $X = (x_{1t}, x_{2t}, \ldots, x_{rt})$
Assuming $y_t \sim N(X\beta, \Sigma)$, where, $\Sigma = \sigma^2 W_k{}^{-1} I$

$$f(y_t|\beta) = \frac{1}{(2\pi\sigma^2 W_k{}^{-1} I)^{\frac{n}{2}}}\, exp^{-\frac{1}{2}(y_t - X\beta)^T \Sigma^{-1}(y_t - X\beta)} \tag{14}$$

$$L(\beta|y) = \prod_{t=1}^{n} f(y_t|\beta) \tag{15}$$

Then the Weighted Maximum Likelihood Estimator (WMLE) $\hat{\beta}$ is the parameter that maximizes $\hat{\beta} = \underset{\beta}{\operatorname{argmax}}\, L(\beta|y)$

Estimator 2: MLWE
Let $Y_{wt} = W_k y_t$ and $X_w = W_k X$.
Assuming $Y_{wt} \sim N(X_w\beta, \Sigma)$, where, $\Sigma = \sigma^2 I$

$$f(Y_{wt}|\beta) = \frac{1}{(2\pi\sigma^2 I)^{\frac{n}{2}}}\, exp^{-\frac{1}{2}(Y_w - X_w\beta)^T \Sigma^{-1}(Y_w - X_w\beta)} \tag{16}$$

$$L(\beta|Y_w) = \prod_{t=1}^{n} f(Y_{wt}|\beta) \tag{17}$$

Then the Maximum Likelihood Weighted Estimator (MLWE) $\hat{\beta}$ is the parameter that maximizes $\hat{\beta} = \underset{\beta}{\operatorname{argmax}}\, L(\beta|Y)$

Estimator 3: COWE

Recall that $Y_{wt} = W_k y_t$ and $X_w = W_k X$

Step 1: Specify the weighted regression model

$$Y_{wt} = X_w\beta + \varepsilon_t,$$

Step 2: Assume the error term follows a first-order autoregressive process

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t, \qquad u_t \sim N(0, \sigma^2)$$

Step 3: Using $\hat{\rho}$, transform the dependent Y_{wt} and explanatory variables $X_w(x_{1t}, x_{2t}, \dots, x_{rt})$ into new variables $Y_{wt}{}^*$and $X_w{}^*(x_{1t}{}^*, x_{2t}{}^*, \dots, x_{rt}{}^*)$ respectively.

Define $Y_{wt}{}^* = Y_{w_t} - \hat{\rho}\ Y_{w_{t-1}}$,

$x_{it}{}^* = x_{i_t} - \hat{\rho}\ x_{i_{t-1}}$, $\quad i = 1, 2, \dots, r$

Step 4: Estimate the transformed regression model

$$Y_{wt}{}^* = X_w{}^*\beta + \varepsilon_t$$

Step 5: Using the residuals ε_t from the transformed model, re-estimate $\hat{\rho}$.

Steps 2 to 4 are repeated until $\hat{\rho}$ converges, meaning successive estimates of ρ stops changing in any significant way. At convergence, the resulting coefficient estimates are the Cochrane Orcutt Weighted Estimates (COWE).

Estimator 4: WCOE

Step 1: Specify the weighted regression model

$$Y_t = X\beta + \varepsilon_t,$$

Step 2: Assume the error term follows a first-order autoregressive process

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t, \qquad u_t \sim N(0, \sigma^2)$$

Step 3: Using $\hat{\rho}$ and W_k, transform the dependent Y_t and explanatory variables $X(x_{1t}, x_{2t}, \dots, x_{rt})$ into new variables $Y_t{}^*$and $X^*(x_{1t}{}^*, x_{2t}{}^*, \dots, x_{rt}{}^*)$ respectively.

Define $Y_t{}^* = W_k(Y_t - \hat{\rho}\ Y_{t-1})$,

$x_{it}{}^* = W_k(x_{i_t} - \hat{\rho}\ x_{i_{t-1}})$, $\quad i = 1, 2, \dots, r$

Step 4: Estimate the transformed regression model

$$Y_t{}^* = X^*\beta + \varepsilon_t$$

Step 5: Using the residuals ε_t from the transformed model, re-estimate $\hat{\rho}$.

Steps 2 to 4 are repeated until $\hat{\rho}$ converges, meaning successive estimates of ρ stops changing in any significant way. At convergence, the resulting coefficient estimates are the Weighted Cochrane-Orcutt Estimates (WCOE).

2.5 Evaluation Metrics

To assess the performance of the estimators under different disturbance structures (heteroscedasticity and autocorrelation), several evaluation metrics were employed. These include the Mean Squared Error (MSE), Relative Efficiency (RE) with respect to the Ordinary Least Squares (OLS) estimator, Bias, Mean Absolute Error (MAE), and Adjusted Coefficient of Determination (R^2).

The Mean Squared Error measures the average squared deviation of an estimated parameter from its true value. It captures both the variance and the squared bias of the estimator, providing an overall measure of estimation accuracy.

$$MSE(\hat{\beta}) = \frac{1}{R}\sum_{r=1}^{R}(\hat{\beta}_r - \beta)^2 \tag{18}$$

where, $\hat{\beta}$ is the estimate from the r^{th} replication, β is the true parameter value, and R is the number of replications. A lower MSE indicates higher estimation precision.

Relative efficiency compares the performance of an estimator to a benchmark estimator (typically OLS) based on their MSEs. It indicates the efficiency gain relative to OLS.

$$RE(\hat{\beta}) = \frac{MSE_{OLS}}{MSE_{\hat{\beta}}} - 1 \tag{19}$$

A value of RE > 0 suggests that the estimator is more efficient than OLS, while RE < 0 implies lower efficiency.
Bias measures the systematic deviation of the estimated parameter from its true value across repeated samples. It reflects the estimator's accuracy (unbiasedness).

$$Bias(\hat{\beta}) = \frac{1}{R}\sum_{r=1}^{R}(\hat{\beta}_r - \beta) \quad (20)$$

A bias value close to zero indicates that the estimator is approximately unbiased.

The Mean Absolute Error quantifies the average magnitude of estimation errors, disregarding their direction. Unlike MSE, it is less sensitive to large deviations and provides a robust measure of average accuracy.

$$MAE(\hat{\beta}) = \frac{1}{R}\sum_{r=1}^{R}\left|\hat{\beta}_r - \beta\right| \quad (21)$$

Lower MAE values indicate that the estimator performs more consistently across replications.

The Adjusted Coefficient of Determination (Adjusted R^2) measures the regression model's explanatory power while accounting for the number of predictors. It penalizes unnecessary inclusion of variables and provides a more accurate assessment of model fit.

$$Adjusted\ R^2 = 1 - \frac{(1 - R^2)(n - 1)}{(n - k - 1)} \quad (22)$$

where $R^2 = 1 - \frac{\Sigma(y_i - \hat{y}_i)^2}{\Sigma(y_i - \bar{y})^2}$ n is the sample size, and k is the number of explanatory variables. A higher Adjusted R^2 indicates better goodness-of-fit after accounting for model complexity.

3. Results

Each of the four estimators was combined with the two weighting schemes, as shown in *Eqs. 10* and *12*, yielding a total of eight estimators. Their performances were compared in terms of Mean Squared Error (MSE), efficiency relative to the ordinary least squares estimator (R.E. to OLS), Bias, and Mean Absolute Error (MAE).

3.1 OLS under Heteroscedasticity and Autocorrelation

The OLS estimator maintains unbiasedness across samples but becomes inefficient when the error structure is non-spherical. Across all sample sizes, OLS recorded large Mean Squared Errors (MSEs), which declined only moderately as sample sizes increased from small to large (Table 1). Although the bias remains relatively small (around −0.06), the large MSE and Mean Absolute Error (MAE) indicate that ignoring the true disturbance structure inflates the estimator's variance substantially. This confirms the theoretical result that OLS is no longer the Best Linear Unbiased Estimator (BLUE) in the presence of heteroscedasticity or autocorrelation, as the covariance structure introduces inefficiency and unreliable inference.

3.2 Proposed Estimators under Heteroscedasticity and Autocorrelation

The estimators (MLWE, WMLE, COWE, and WCOE) explicitly incorporate the non-spherical structure through weighting matrices. The Maximum Likelihood Weighted Estimators (MLWE_W1 and MLWE_W2) exhibit a significant improvement in efficiency, with MSE values decreasing to approximately 4.9 and 4.49, respectively, at n = 50. The corresponding relative efficiency gains (R.E.) over OLS exceed 2,000% in smaller samples and remain above 1,000% as the sample size increases (Table 1). This strong performance reflects their ability to correct for unequal variances and autocorrelation in the disturbance process, yielding estimators that are both consistent and asymptotically efficient. The Weighted Maximum Likelihood Estimators (WMLE_W1 and WMLE_W2) also achieve stable and low MSEs (≈ 4.64) across all sample sizes. Their invariance across sample sizes suggests that the weighting matrix was correctly specified and efficiently estimated. The constancy and low MSE confirm their robustness in handling non-spherical error structures.

The Cochrane–Orcutt Weighted Estimators (COWE_W1 and COWE_W2) perform comparably to the MLWE variants, recording MSE values between 4.90 and 3.96 and R.E. gains above 1,100% relative to OLS. These results demonstrate the effectiveness of an iterative weighted scheme for addressing non-spherical disturbances via transformed models. As the sample size increases, both bias and MAE stabilize, indicating consistent estimation and improved efficiency.

On the other hand, the Weighted Cochrane–Orcutt Estimators (WCOE_W1 and WCOE_W2) exhibit instability in smaller samples, with extremely large MSE and bias values (e.g., 1.67×10^{17} at n = 50). However, as n increases, the estimators gradually stabilize, and their performance converges toward efficiency comparable to that of the other weighted methods (Table 1). This pattern emphasizes the sensitivity of high-order weighted corrections to sample size and model specification. WCOE performs better with larger than with smaller sample sizes.

3.3 Application to Real-Life Data

To illustrate practical relevance, the estimators were applied to real-life data. Before using the new estimators, we fitted an OLS model to the real-life dataset. The results of diagnostic tests for the OLS model applied to the real dataset indicate strong evidence of autocorrelation in the residuals, as shown by the Durbin–Watson statistic (1.4776, p = 0.0097), which violates the classical regression assumption of independence. Likewise, the Goldfeld–Quandt test statistic (2.5234, p = 0.0174) indicates heteroscedasticity, i.e., the error variance is not constant across observations. These findings suggest that the OLS estimator is inefficient in this context, as it fails to account for both serial correlation and heteroscedasticity. The presence of these violations provides strong justification for applying the newly proposed estimators, which are designed to be more robust to such data irregularities.

Each estimator reports the regression coefficients along with their corresponding standard errors in parentheses, as well as the adjusted R-squared value, which reflects the model's overall explanatory power (Table 2). The OLS model yielded an adjusted R-squared value of 0.9183, serving as the baseline for comparison. Even with significant coefficients, the presence of heteroscedasticity and autocorrelation suggests that OLS estimates may be inefficient. The proposed MLWE estimators demonstrated superior performance. MLWE_W1 yielded an adjusted R^2= 0.9553, while MLWE_W2 achieved the highest explanatory power at 0.9967. Similarly, COWE_W1 and COWE_W2 exhibited nearly identical performance, with adjusted R^2 values of 0.9553 and 0.9967, respectively. These estimators consistently produced larger coefficient magnitudes than OLS and reduced standard errors, indicating greater efficiency. The WMLE provided mixed results. WMLE_W1 performed comparably to OLS with an adjusted R^2=0.9161, while WMLE_W2 underperformed with R^2=0.9016. In contrast, the WCOE were the weakest performers. WCOE_W1 produced a negative adjusted R^2, indicating a worse fit than the null model, whereas WCOE_W2 improved to 0.8019 but still fell below the OLS benchmark.

Table 1. Simulation Results: Average MSE, Bias, and MAE across Estimators

Estimators	Evaluation Metrics	Sample Size (n) 50	100	150	200	300
OLS	MSE	1.09×10^{2}	8.04×10^{1}	7.35×10^{1}	6.17×10^{1}	5.13×10^{1}
	Bias	-6.46×10^{-2}	-4.41×10^{-2}	-1.06×10^{-1}	-6.26×10^{-2}	-3.60×10^{-2}
	MAE	2.95	2.64	2.53	2.33	2.15
MLWE_W1	MSE	4.98	4.68	4.58	4.25	4.00
	Bias	-7.52×10^{-1}	-7.23×10^{-1}	-7.45×10^{-1}	-7.26×10^{-1}	-7.17×10^{-1}
	MAE	7.64×10^{-1}	7.43×10^{-1}	7.48×10^{-1}	7.33×10^{-1}	7.19×10^{-1}
	R.E. to OLS	2.09×10^{1}	1.62×10^{1}	1.51×10^{1}	1.35×10^{1}	1.18×10^{1}
MLWE_W2	MSE	4.49	4.44	4.43	4.36	4.23
	Bias	-7.84×10^{-1}	-7.78×10^{-1}	-7.77×10^{-1}	-7.72×10^{-1}	-7.61×10^{-1}
	MAE	7.84×10^{-1}	7.78×10^{-1}	7.77×10^{-1}	7.72×10^{-1}	7.61×10^{-1}
	R.E. to OLS	2.33×10^{1}	1.71×10^{1}	1.56×10^{1}	1.32×10^{1}	1.11×10^{1}
WMLE_W1	MSE	4.64	4.64	4.64	4.64	4.64
	Bias	-8.14×10^{-1}	-8.14×10^{-1}	-8.14×10^{-1}	-8.14×10^{-1}	-8.14×10^{-1}
	MAE	8.14×10^{-1}	8.14×10^{-1}	8.14×10^{-1}	8.14×10^{-1}	8.14×10^{-1}

	R.E. to OLS	2.25×10^{1}	1.63×10^{1}	1.48×10^{1}	1.23×10^{1}	1.00×10^{1}
WMLE_W2	MSE	4.64	4.64	4.64	4.64	4.64
	Bias	-8.14×10^{-1}	-8.14×10^{-1}	-8.14×10^{-1}	-8.14×10^{-1}	-8.14×10^{-1}
	MAE	8.14×10^{-1}	8.14×10^{-1}	8.14×10^{-1}	8.14×10^{-1}	8.14×10^{-1}
	R.E. to OLS	2.25×10^{1}	1.63×10^{1}	1.48×10^{1}	1.23×10^{1}	1.00×10^{1}
COWE_W1	MSE	4.90	4.59	4.48	4.21	3.96
	Bias	-7.43×10^{-1}	-7.26×10^{-1}	-7.41×10^{-1}	-7.26×10^{-1}	-7.17×10^{-1}
	MAE	7.60×10^{-1}	7.42×10^{-1}	7.43×10^{-1}	7.32×10^{-1}	7.18×10^{-1}
	R.E. to OLS	2.13×10^{1}	1.65×10^{1}	1.54×10^{1}	1.37×10^{1}	1.19×10^{1}
COWE_W2	MSE	4.49	4.41	4.40	4.34	4.21
	Bias	-7.84×10^{-1}	-7.77×10^{-1}	-7.76×10^{-1}	-7.72×10^{-1}	-7.61×10^{-1}
	MAE	7.84×10-1	7.77×10-1	7.76×10-1	7.72×10-1	7.61×10-1
	R.E. to OLS	2.33×10^{1}	1.72×10^{1}	1.57×10^{1}	1.32×10^{1}	1.12×10^{1}
WCOE_W1	MSE	1.67×10^{17}	1.47×10^{14}	7.00×10^{1}	5.63×10^{1}	4.66×10^{1}
	Bias	-7.08×10^{6}	1.45×10^{5}	-9.34×10^{-2}	-3.66×10^{-2}	-3.11×10^{-2}
	MAE	8.96×10^{6}	1.45×10^{5}	2.41	2.18	2.04
	R.E. to OLS	-1.00	-1.00	5.00×10^{-2}	9.59×10^{-2}	9.97×10^{-2}
WCOE_W2	MSE	1.29×10^{18}	4.63×10^{17}	2.98×10^{15}	6.31×10^{1}	5.13×10^{1}
	Bias	4.25×10^{6}	-2.17×10^{6}	7.64×10^{5}	-3.55×10^{-2}	-4.03×10^{-2}
	MAE	3.52×10^{7}	1.40×10^{7}	7.64×10^{5}	2.31	2.14
	R.E. to OLS	-1.00	-1.00	-1.00	-2.23×10^{-2}	-5.00×10^{-4}

Note. MSE = Mean Squared Error; MAE = Mean Absolute Error; R.E. = Relative Efficiency; OLS = Ordinary Least Square; MLWE = Maximum Likelihood Weighted Estimator; WMLE = Weighted Maximum Likelihood Estimator; COWE = Cochrane–Orcutt Weighted Estimator; WCOE = Weighted Cochrane–Orcutt Estimator; W1 = Weight scheme 1; W2 = Weight scheme 2.
Source: Author's computation

Table 2. Real Life Data: Estimation Results

	OLS	MLWE1	MLWE2	WMLE1	WMLE2	COWE1	COWE2	WCOE1	WCOE2
β_0 (SE)	1.80×10^{7} (2.40×10^{6})	6.82×10^{-6} (9.45×10^{-7})	1.47×10^{-6} (3.76×10^{-7})	1.70×10^{7} (2.44×10^{6})	1.63×10^{7} (2.64×10^{6})	6.82×10^{-6} (9.45×10^{-7})	1.47×10^{-6} (3.76×10^{-7})	2.80×10^{-6} (8.64×10^{6})	1.16×10^{-6} (3.74×10^{6})
β_1 (SE)	4.63 (1.65)	4.42 (1.67)	17.00 (2.10)	5.69 (1.67)	8.19 (1.81)	4.42 (1.67)	17.00 (2.10)	4.31 (5.91)	7.01 (2.56)
β_2 (SE)	5.53 (4.05)	17.60 (3.12)	-5.39×10^{-2} (3.17)	7.53 (4.11)	7.44 (4.45)	17.60 (3.12)	-5.39×10^{-2} (3.17)	5.53 (14.60)	13.10 (6.31)
β_3 (SE)	2.48 (3.17)	-15.90 (5.06)	6.75 (2.26)	2.57 (3.22)	3.09 (3.48)	-15.90 (5.06)	6.75 (2.26)	5.10 (11.40)	14.20 (4.94)
β_4 (SE)	4.36 (2.01)	-1.76 (2.54)	-4.96 (2.85)	4.03 (2.04)	0.44 (2.21)	-1.76 (2.54)	-4.96 (2.85)	5.72 (7.22)	5.38 (3.13)
β_5 (SE)	-9.13 (2.63)	5.80 (1.60)	2.30 (0.69)	-6.37 (2.67)	-4.48 (2.89)	5.80 (1.60)	2.30 (0.69)	0.11 (9.45)	2.58 (4.10)
β_6	5.04	-3.17	-4.57	1.31	-1.28	-3.17	-4.57	-0.86	-8.73

(SE)	(3.87)	(2.22)	(0.99)	(3.92)	(4.25)	(2.22)	(0.99)	(13.90)	(6.03)
AdjR^2	0.9183	0.9553	0.9967	0.9161	0.9016	0.9553	0.9967	-0.0555	0.8019

Note. SE = Standard Error in parentheses; AdjR² = Adjusted R-squared.
Source: Author's computation

4. Discussion

The research examined the performance of several estimators under non-spherical disturbances, in which the error process exhibits both heteroscedasticity and autocorrelation. The Monte Carlo evidence clearly shows that the Ordinary Least Squares (OLS) estimator, while remaining approximately unbiased, suffers from substantial efficiency losses and inflated mean squared errors when the classical assumptions of constant variance and serially uncorrelated errors are violated. This inefficiency of OLS under general error covariance structures has been highlighted in recent econometric work that proposes alternatives such as generalized automatic least squares to achieve efficiency gains over OLS and WLS when heteroscedasticity is present (Gafarov, 2023). The weighted estimators, particularly the Maximum Likelihood Weighted Estimators (MLWE), Weighted Maximum Likelihood Estimators (WMLE), and Cochrane–Orcutt Weighted Estimators (COWE), achieved substantial reductions in mean squared error (MSE) and mean absolute error (MAE), notable gains in relative efficiency, and higher adjusted coefficients of determination in the empirical application which aligns with the broader literature on feasible GLS and related weighted estimation methods, which demonstrate improved performance relative to OLS in the presence of heteroscedasticity and autocorrelation when the error covariance structure is accounted for (Moriya & Noda, 2025). The results, therefore, confirm that incorporating a weighted structure into both the maximum likelihood framework and the Cochrane–Orcutt procedure yields consistent and efficient parameter estimates when the error covariance matrix is correctly specified.

From a practical standpoint, these results have important implications for applied researchers working with time-series or panel data characterized by volatility clustering and serial dependence. In such settings, reliance on OLS may lead to inefficient estimation and misleading inference. In contrast, weighted likelihood-based or weighted transformation methods can significantly improve estimation accuracy, inference reliability, and overall model fit, even in moderately sized samples. This analysis assumes the correct specification of the weighting matrix and the underlying error-correlation structure. In other practical applications, misspecification of these components may reduce the efficiency gains of the weighted estimators and, in some cases, introduce additional bias, particularly when the true form of heteroscedasticity or autocorrelation is unknown.

5. Conclusion

Reaffirm a central econometric principle: ignoring heteroscedasticity and autocorrelation leads to inefficient estimation and unreliable inference, while appropriately weighted-type estimators restore efficiency and accuracy. These results provide a strong empirical justification for adopting Maximum Likelihood Weighted Estimators (MLWE), Weighted Maximum Likelihood Estimators (WMLE), and Cochrane–Orcutt Weighted Estimators (COWE) procedures in models with non-spherical disturbances, particularly in applied economic and time-series analyses where such violations are common.

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