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Statistical Analysis of Fuzzy Latin Square Design with Exponential Trapezoidal Fuzzy Numbers on Analgesia in Clinical Trial

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ABSTRACT

This study evaluates and validates an innovative framework integrating Fuzzy Latin Square Design (LSD) with Exponential Trapezoidal Fuzzy Numbers (ETZFNs) to enhance the statistical analysis of clinical trials characterized by subjective, vague, or uncertain data, specifically in postoperative pain management. This research extends classical LSD by incorporating fuzzy set theory, explicitly representing data uncertainty through ETZFNs. A retrospective dataset of 64 postoperative Visual Analog Scale (VAS) scores was transformed into ETZFNs, capturing the range and most plausible values of pain perception. An 8×8 balanced Latin Square was constructed, blocking for eight surgical types (rows) and eight patient age groups (columns), with eight analgesic regimens as treatments. The ETZFNs were defuzzified into crisp median values using Rezvani's formula, enabling conventional statistical analysis. The integrated fuzzy-statistical framework successfully translated statistical outcomes into clinically interpretable, actionable recommendations for pain management.

1. Introduction

The Latin Square Design (LSD) is a widely employed statistical method in experimental research that effectively controls for three sources of variability, typically row-wise and column-wise factors and treatment such as time, location, or environmental conditions. This design is particularly beneficial when evaluating the effects of multiple treatments while mitigating the influence of potential confounders. In a standard Latin square arrangement, each treatment appears exactly once in every row and every column, thereby ensuring balanced comparisons and minimizing systematic bias (Montgomery, 2017). Exponential Trapezoidal Fuzzy Numbers (ETZFNs) are advanced mathematical constructs designed to represent uncertain and imprecise information across various domains, such as decision-making, risk assessment, and control systems. Unlike conventional crisp numbers, fuzzy numbers incorporate degrees of uncertainty, providing a more nuanced and realistic representation of real-world phenomena. ETZFNs are distinguished by their unique structure, which integrates the characteristics of trapezoidal fuzzy numbers with an exponential decay function. This formulation enhances their ability to model complex scenarios where uncertainty is not uniformly distributed (Rezvani et al., 2015). Analgesics, commonly known as pain relievers, represent a vital class of medications used to reduce pain and enhance the quality of life for individuals suffering from various acute and chronic pain conditions. These medications are broadly categorized into two main groups: non-opioid analgesics and opioid analgesics. Non-opioid analgesics, such as acetaminophen and nonsteroidal anti-inflammatory drugs (NSAIDs), are typically used for managing mild to moderate pain. They function by inhibiting inflammatory pathways and altering pain perception within the central nervous system. In contrast, opioid analgesics including morphine and oxycodone are generally reserved for more severe pain and exert their effects by binding to specific opioid receptors in the brain and spinal cord, thereby effectively blocking pain signals (Brunton et al., 2018).

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The incorporation of an exponential function adds a dynamic component to the fuzzy membership structure, enabling ETZFNs to reflect varying degrees of membership in their domain. This characteristic is particularly valuable in contexts where the probability of outcomes changes nonlinearly, such as medical decision-making and clinical risk assessment (Rezvani et al, 2015). Consequently, ETZFNs provide a powerful framework for extracting meaningful insights from uncertain data in healthcare, finance, and engineering (Rezvani, 2013). The incorporation of an exponential function adds a dynamic component to the fuzzy membership structure, enabling ETZFNs to reflect varying degrees of membership in their domain. This characteristic is particularly valuable in contexts where the probability of outcomes changes nonlinearly, such as medical decision-making and clinical risk assessment (Rezvani et al, 2015). Consequently, ETZFNs provide a powerful framework for extracting meaningful insights from uncertain data in healthcare, finance, and engineering (Rezvani, 2013).

The topic of Fuzzy Latin Square Design with Exponential Trapezoidal Fuzzy Numbers (ETZFNs) in analgesia research highlights the integration of fuzzy logic into experimental design, particularly to evaluate the efficacy of analgesics. Traditional statistical methods often fail to capture the inherent uncertainties and individual variability associated with human responses to pain management. By incorporating exponential trapezoidal fuzzy numbers, researchers can more accurately model these uncertainties, offering a nuanced and realistic representation of patient responses. The Latin Square Design (LSD), recognized for its strength in controlling two sources of variability, provides a robust framework for organizing treatments systematically. The fusion of LSD with ETZFNs enables a more sophisticated statistical approach, enhancing the interpretation of how various analgesics perform under diverse conditions. This methodological innovation not only improves the reliability of experimental findings but also advances the field of fuzzy statistics and its application in medical and clinical research (Rezvani et al., 2015)

Torres-Escobar et al, (2022). conducted a prospective, randomized, double-blind clinical trial to compare the analgesic efficacy of epidural ropivacaine combined with fentanyl versus morphine in adult patients undergoing surgical cleaning of contaminated lower-body wounds. Pain levels were measured using the Visual Analog Scale (VAS), alongside hemodynamic parameters, side effects, and the need for rescue analgesia. The findings indicated significantly superior analgesic outcomes for the ropivacaine--morphine combination across all postoperative time points. Although the study demonstrated strong methodological rigor, limitations included a relatively small sample size and the absence of long-term follow-up or pharmacoeconomic evaluation. The conceptual framework integrates anesthesiology practice, pain management theory, and pharmacological principles to optimize postoperative pain control. Bodjanova, (2005) proposed median-based measures for characterizing fuzzy numbers, introducing the median value and median interval as alternatives to traditional descriptors. The study demonstrated that median-based representations offer robust and interpretable summaries, particularly for skewed fuzzy distributions. Nevertheless, the work remained theoretical, with limited discussion of algorithmic implementation or real-world applications.

The framework integrates fuzzy set theory, defuzzification concepts, and statistical distribution theory. Rezvani et al. (2014) proposes a novel method for ranking exponential trapezoidal fuzzy numbers (ETZFN) using a defined TRD (Total Ranking Distance) metric, which integrates the weighted average representative and the weighted width of fuzzy numbers. Wu (2007) presents a novel approach for applying analysis of variance (ANOVA) to imprecise (fuzzy) data by transforming fuzzy measurements into their h-level sets.

2. Methodology

2.1 Fuzzy Statistical Design

This study adopts a Fuzzy Latin Square Design (Fuzzy LSD) that integrates classical experimental design principles with fuzzy logic techniques. The framework consists of the following components:

1. **Classical LSD Components**: Row blocking (surgery type), column blocking (age group), and treatment factor (analgesic type).
2. **Fuzzy Logic Extension**: Incorporation of Exponential Trapezoidal Fuzzy Numbers (ETZFNs) to model uncertainty and vagueness in postoperative pain scores.
3. **Two-Phase Analysis**:
    - Fuzzy modeling and defuzzification
    - Statistical inference and validation

2.2 Design Specifications

• **Design Type**: 8 × 8 Latin Square Design (balanced design)

• **Replication**: Each analgesic appears exactly once in each row and each column • **Blocking Variables**: **Rows**: Eight surgery types (Exploratory laparotomy, Herniorrhaphy, ORIF, Caesarean section, Mastectomy, Myomectomy, Craniotomy, Urethroplasty). **Columns**: Eight age groups (18-22, 23-27, . . . , 53and above years). **Treatments**: Eight analgesic regimens (Paracetamol, Morphine, Fentanyl, Ketorolac, Tramadol, Pregabalin, Diclofenac, Ketamine)

2.3 Data Collection

Source and Nature of Data

• **Source**: Secondary data from the Analgesia and Critical Care Unit, Federal Teaching Hospital, Gombe

• **Study Period**: Retrospective data covering the year 2025

• **Population**: Adult patients (≥ 18 years) undergoing specified surgical procedures

• **Sample Size**: 64 observations (8 × 8 Latin Square)

2.4 Inclusion Criteria

• Patients aged 18 years and above

• Patients who underwent one of the specified surgical procedures

• Receipt of standardized postoperative analgesic regimen

• Complete pain assessment records

• No documented allergy to administered analgesics

2.5 Method of Data Analysis.

Definition 1. (Generalized Fuzzy Number) A **generalized fuzzy number** $\tilde{A}$ is a fuzzy set defined on the real line R characterized by a membership function $\mu_{\tilde{A}}: R \to [0, 1]$ satisfying the following properties:

$\mu_{\tilde{A}}$ is continuous.

$\mu_{\tilde{A}}(x) = 0$ for all x □ $(-\infty, a]$.

$\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on [a, b].

$\mu_{\tilde{A}}(x) = w$ for all x □ [b, c), where w □ (0, 1].

$\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on [c, d].

$\mu_{\tilde{A}}(x) = 0$ for all x □ $[d, \infty)$.

where a, b, c, d □ R with $a \le b \le c \le d$, and $L : [a, b] \to [0, w]$, $R : [c, d] \to [w, 0]$ are continuous functions.

Definition 2. ETZFNs Membership Function

The exponential trapezoidal fuzzy membership function is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x < a, \\ we^{k_1(x-a)}, a \le x < b, \\ w, b \le x \le c, \\ we^{-k_2(x-c)}, c < x \le d, \\ 0, x > d, \end{cases}$$

where $0 < w \le 1$, a, b, c, d □ R.

The generalized exponential trapezoidal fuzzy number is denoted as:

$$\tilde{A} = (a, b, c, d; w)E \quad (1)$$

2.6 Scalar Cardinality of ETZFNs

A fuzzy set Ã, the scalar cardinality is

$$|\tilde{A}| = \sum x \in X \mu_{\tilde{A}}(x), \quad (2)$$

where $\mu_{\tilde{A}}(x)$ is the membership value of element x in set Ã.

Figure 1: Exponential Trapezoidal Fuzzy Number

2.7 Median Value of ETZFNs

Let an ETZFN be defined as:

$$\tilde{A} = [a, b, c, d; \omega]E \quad (3)$$

where a and d define uncertainty bounds, b and c represent the most plausible pain interval, and w ∈ (0,1] denotes a normal fuzzy number.

The fuzzy median $m_{\tilde{A}}$ is given by:

$$m(\tilde{A}) = (a + d)/2 + w/(2\times e) \times [(c - d)(1 - e) - (b - a)(e - 1)] \quad (4)$$

where e ≈ 2.71828 is Euler's constant.

2.8 Defuzzification of the LSD Matrix

Given a fuzzy Latin Square matrix

$$\hat{Y} = \left[\overline{Y}_{ij}\right], \quad (5)$$

where: each element is an Enhanced Trapezoidal Fuzzy Number (ETZFN):

$$\overline{Y}_{ij} = [a_{ij}, b_{ij}, c_{ij}, d_{ij}; w_{ij}]_E, \quad (6)$$

To obtain a crisp defuzzified matrix, we apply median defuzzification:

$$Y_{median} = [m_{y_{ij}}], \quad (7)$$

where $m_{y_{ij}}$ is the median of $\overline{Y}_{ij}$

The resulting crisp matrix has the structure:

$$Y_{median} = \begin{bmatrix} m_{\overline{Y}_{11}} & m_{\overline{Y}_{11}} & \cdots & m_{\overline{Y}_{1t}} \\ m_{\overline{Y}_{21}} & m_{\overline{Y}_{22}} & \cdots & m_{\overline{Y}_{2t}} \\ \vdots & \vdots & \ddots & \vdots \\ m_{\overline{Y}_{t1}} & m_{\overline{Y}_{t2}} & \cdots & m_{\overline{Y}_{tt}} \end{bmatrix}$$

Each element is a crisp representative of its corresponding ETZFN.

2.9 Statistical Analysis of the Latin Square Design

The linear model of the fuzzy Latin Square design is

$$\tilde{y}_{ijs} = \tilde{\mu} + \tilde{r}_i + \tilde{c}_j + \tilde{t}_s + \tilde{e}_{ijs} \quad (8)$$

Where$\tilde{y}_{ijs}$ is the variable corresponding to ith row, jth column, and under sth treatment, $\tilde{\mu}$, $\tilde{r}$, $\tilde{c}$, $\tilde{t}$, (i, j, s=1, 2, …., 8) are fixed effects shall denote in order the general mean, row, column and effects of treatment and $\tilde{e}_{ijs}$, the error variable, assumed to be distributed with zero mean and a constant variance σ^2 independently and normal

The total sum of squares (TSS) is

$$TSS = \sum_i \sum_j y_{ij}^2 - \frac{G^2}{k^2} \quad (9)$$

The row sum of squares (RSS) is

$$RSS = \sum_i \frac{R_i^2}{k} - \frac{G^2}{k^2} \quad (10)$$

The columns sum of squares (CSS) is

$$CSS = \sum_j \frac{C_j^2}{k} - \frac{G^2}{k^2} \quad (11)$$

The treatments sum of squares (TrtSS) is

$$TrtSS = \sum_s \frac{T_s^2}{k} - \frac{G^2}{k^2} \quad (12)$$

Error sum of squares (ESS) is by subtraction

$$ESS = TSS - RSS - CSS - TrtSS \quad (13)$$

Degree of freedom of TSS, RSS, CSS is (k - 1) and for ESS is (k-1)(k-2)

The mean sum of squares is obtained as

$$\text{For Row is MSSR} = \frac{RSS}{(k-1)}, \quad (14)$$

$$\text{For Column is MSSC} = \frac{CSS}{(k-1)}, \quad (15)$$

$$\text{For Treatment is MSSTrt} = \frac{TrtSS}{(k-1)}, \text{ and} \quad (16)$$

$$\text{For Error is MSSE} = \frac{ESS}{(k-1(k-2)}. \quad (17)$$

The test statistic that will used is

$$\text{For Row is FR} = \frac{MSS_R}{MSS_E}, \quad (18)$$

$$\text{For Column is FC} = \frac{MSS_C}{MSS_E} \text{ and} \quad (19)$$

$$\text{For treatment is FTrt} = \frac{MSS_{Trt}}{MSS_E} \quad (20)$$

When the null hypothesis H_0 remains true, the test statistic F follows an F-distributed with degree of freedom (k - 1) and (k - 1)(k - 2) denoted as F[(k - 1),(k - 1)(k - 2)].

2.10 Decision rules of F-ratio

The null hypothesis H_0 is true, then it is verified that F has the representative of the F_t distribution with degree of freedom for rows, columns and treatments [(k - 1), (k - 1)(k - 2)]. the significance of the decision rule at the level α is based on various statement:

If $F_c \geq F_t$ then we reject the null hypothesis H_0

If $F_c < Ft$ then we accept the null hypothesis H_0

Table 1: Latin Square Design of ANOVA

SV	Df	SS	MSS	F-ratio
Rows	(k - 1)	RSS = $\sum_i k \quad k^2$	MSSR = $(k-1)$	FR = MSS_E
Columns	(k - 1)	CSS = $\sum_j k \quad k^2$	MSSC = $(k-1)$	FC = MSS_E

Treatments	(k - 1)	TrtSS = $\sum_s \overline{k} - \overline{k^2}$	MSSTrt = $(k-1)$	FTrt = $\overline{MSS_E}$
Error	(k - 1)(k - 2)	ESS by subtraction	MSSE = $(k-1(k-2)$	
Total	(k2 - 1)	TSS = $\sum_i \sum_j y_{ij} - \overline{k^2}$		

Table 2: TZFNs for VAS Score

Age Group	Exploratory Laparotomy	Herniorrhaphy	ORIF	Caesarean Section	Mastectomy	Myomectomy	Craniotomy	Urethroplasty
18-22	Morphine (4, 5, 6, 6)	Tramadol (3, 3, 3, 4)	Pregabalin (7, 7, 8, 9)	Fentanyl (1, 2, 3, 3)	Diclofenac (5, 6, 7, 8)	Paracetamol (5, 5, 6, 7)	Ketorolac (4, 5, 5, 6)	Ketamine (6, 6, 7, 8)
23-27	Tramadol (4, 5, 6, 7)	Pregabalin (6, 6, 6, 7)	Fentanyl (3, 4, 5, 5)	Diclofenac (4, 5, 5, 6)	Paracetamol (6, 7, 8, 9)	Ketorolac (4, 4, 5, 6)	Ketamine (6, 6, 6, 7)	Morphine (3, 4, 4, 5)
28-32	Pregabalin (7, 8, 8, 9)	Fentanyl (1, 2, 2, 3)	Diclofenac (6, 7, 8, 9)	Paracetamol (4, 4, 5, 6)	Ketorolac (4, 5, 6, 7)	Ketamine (5, 6 ,7, 7)	Morphine (3, 3, 4, 5)	Tramadol (3, 4, 5, 6)
33-37	Fentanyl (3, 3, 4, 5)	Diclofena (4, 4, 5, 5)	Paracetamol (7, 8, 8, 9)	Ketorolac (4, 5, 5, 6)	Ketamine (6, 7, 8, 8)	Morphine (3, 3, 4, 5)	Tramadol (3, 4, 4, 5)	Pregabalin (6, 6 ,7, 8)
38-42	Diclofenac (6, 7, 8, 9)	Paracetamol (3, 4, 5, 6)	Ketorolac (5, 7, 7, 8)	Ketamine (5, 6, 6, 6)	Morphine (3, 3, 4, 5)	Tramadol (3, 3, 4, 5)	Pregabalin (7, 7, 8, 8)	Fentanyl (2, 3, 4, 4)
43-47	Paracetamol (7, 8, 9, 9)	Ketorolac (4, 5, 6, 6)	Ketamine (7, 8, 9, 9)	Morphine (2, 2, 3, 4)	Tramadol (3, 4, 5, 6)	Pregabalin (6, 6, 7, 8)	Fentanyl (2, 2, 3, 4)	Diclofenac (5, 6, 7, 8)
48-52	Ketorolac (5, 6, 7, 8)	Ketamine (4, 4, 5, 6)	Morphine (4, 5, 6, 6)	Tramadol (3, 4, 4, 4)	Pregabalin (6, 7, 8, 9)	Fentanyl (2, 2, 3 ,3)	Diclofenac (5, 6, 7, 8)	Paracetamol (6, 7, 7, 8)
53 and above	Ketamine (7, 8, 8, 9)	Morphine (2, 2, 3, 4)	Tramadol (4, 5, 6, 7)	Pregabalin (6, 7, 8, 9)	Fentanyl (2, 3, 4, 5)	Diclofenac (4, 5, 6, 6)	Paracetamol (6, 7, 8, 8)	Ketorolac (4, 5, 6, 7)

Table 3: Fuzzy Latin Square Design with Median Values (Defuzzified)

Age Group	Exploratory Laparotomy	Herniorrhaphy	ORIF	Caesarean Section	Mastectomy	Myomectomy	Craniotomy	Urethroplasty
18-22	Morphine (4.68)	Tramadol (3.82)	Pregabalin (8.32)	Fentanyl (1.68)	Diclofenac (6.5)	Paracetamol (6.32)	Ketorolac (5.0)	Ketamine (7.32)
23-27	Tramadol (5.5)	Pregabalin (6.82)	Fentanyl (3.68)	Diclofenac (5.)	Paracetamol (7.5)	Ketorolac (5.32)	Ketamine (6.82)	Morphine (4.0)
28-32	Pregabalin (8.0)	Fentanyl (2.0)	Diclofenac (7.5)	Paracetamol (5.32)	Ketorolac (6.32)	Ketamine (5.68)	Morphine (4.32)	Tramadol (4.5)
33-37	Fentanyl (4.32)	Diclofenac (4.5)	Paracetamol (8.0)	Ketorolac (5.0)	Ketamine (6.68)	Morphine (4.32)	Tramadol (4.0)	Pregabalin (7.32)
38-42	Diclofenac (7.5)	Paracetamol (4.5)	Ketorolac (6.18)	Ketamine (5.18)	Morphine (4.32)	Tramadol (4.32)	Pregabalin (7.5)	Fentanyl (2.68)
43-47	Paracetamol (7.68)	Ketorolac (4.68)	Ketamine (7.68)	Morphine (3.32)	Tramadol (4.5)	Pregabalin (7.32)	Fentanyl (3.32)	Diclofenac (6.5)
48-52	Ketorolac (6.5)	Ketamine (5.32)	Morphine (4.68)	Tramadol (3.18)	Pregabalin (7.5)	Fentanyl (2.5)	Diclofenac (6.5)	Paracetamol (7.0)
53and above	Ketamine (8.0)	Morphine (3.32)	Tramadol (5.5)	Pregabalin (7.5)	Fentanyl (3.5)	Diclofenac (4.68)	Paracetamol (6.68)	Ketorolac (5.5)

Note: Values in parentheses represent median values (defuzzified) for each ETZFN.

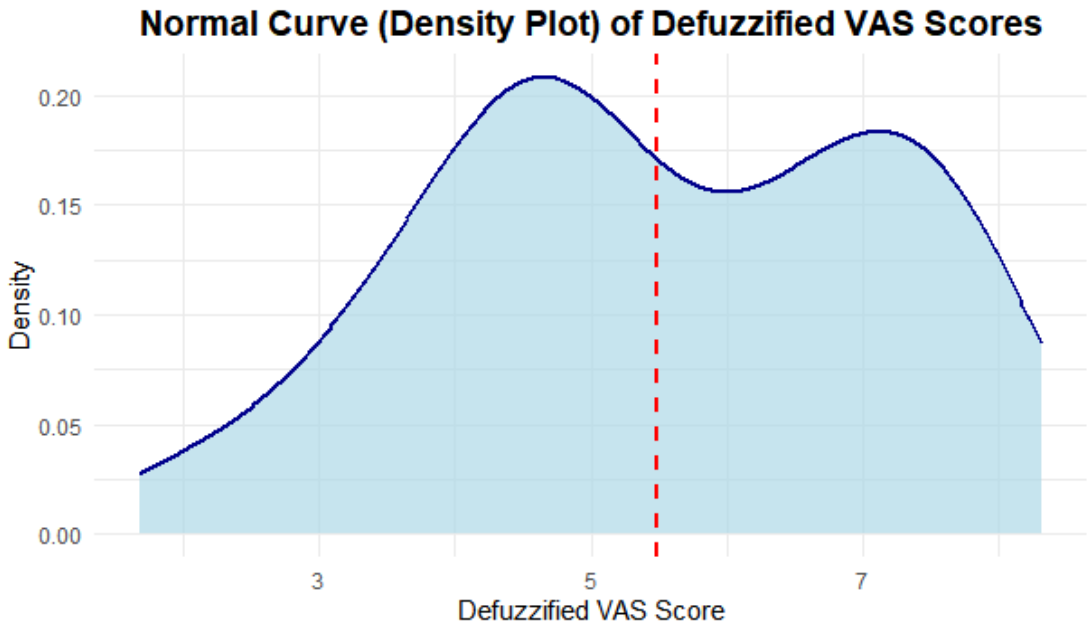

Figure 2: Exponential Trapezoidal Fuzzy Number for Tramadol (Best Performer)

Figure 2 shows the trapezoidal shape with exponential slopes. The median value represents the defuzzified point that divides the area under the membership function into two equal parts, serving as a crisp representative for statistical analysis.

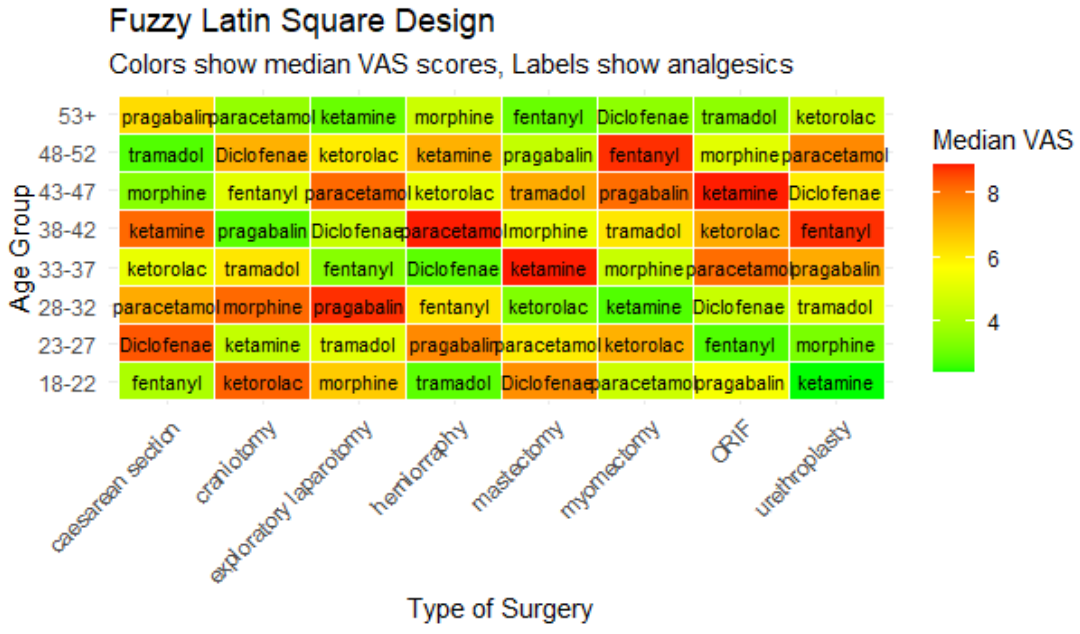

Figure 3. Fuzzy Latin Square

3. Statistical Analysis of Defuzzified Data

3.1 ANOVA Results for Fuzzy Latin Square Design

The defuzzified median values were analyzed using traditional ANOVA for Latin Square Design. The results test the significance of three factors: age group (column blocking), surgery type (row blocking), and analgesic type (treatment).

Table 4: ANOVA for the Latin Square Design with FUZZY Median Value

Source of Variation	Df	Sum Square	Mean Square	F value	p-value	Significance
Type of analgesics	7	127.929	18.2756	74.8992	0.0000022	***
Age group	7	1.234	0.1763	0.7225	0.6536	
Type of surgery	7	47.475	6.7821	27.7952	0.000013	***
Residuals	42	10.248	0.244			

Table 4 shows that the analysis of variance (ANOVA) results obtained from the 8 × 8 Latin Square Design indicate that the type of analgesic has a highly significant effect on postoperative pain scores ($F(7,42) = 74.90$, $p = 0.000013$), demonstrating that pain outcomes vary substantially across the different analgesic regimens. The type of surgery also exhibits a statistically significant effect on pain scores ($F(7,42) = 27.80$, $p = 0.0000022$), suggesting that differences in surgical procedures are associated with systematically different levels of postoperative pain. In contrast, the age group of patients does not show a statistically significant influence on pain scores after accounting for analgesic type and surgery type ($F(7,42) = 0.72$, $p = 0.654$), indicating that age-related differences in pain perception are not pronounced within the controlled structure of the study. The relatively small residual mean square (0.244) reflects low unexplained variability, implying that the model fits the data well. Overall, these findings demonstrate that the proposed fuzzy Latin Square framework effectively isolates the primary sources of variation in postoperative pain.

Figure 4. Residuals vs Fitted, Q-Q, Scale-Location, and Leverage Plots Assessing Model Assumptions in Postoperative Pain Analysis

The figure 3 show that Residual diagnostics indicate that the model provides an appropriate fit to the data. The **Residuals vs Fitted plot** shows a largely random scatter around zero, suggesting no substantial deviations from linearity and only minor heteroscedasticity. The **Q-Q plot** demonstrates that the standardized residuals closely follow the 45-degree reference line, indicating approximate normality, with only slight deviations at the tails. The **Scale-Location plot** confirms relatively uniform spread of residuals across fitted values, supporting the assumption of homoscedasticity. Additionally, the **Residuals vs Factor Levels plot** for age groups shows no systematic patterns,

suggesting that the model assumptions hold across categorical subgroups. Collectively, these diagnostic results support the validity of the model assumptions and indicate that the findings derived from this analysis are reliable and interpretable.

3.2 Assumption Validation

The validity of the parametric tests was supported by diagnostic checks. The Shapiro-Wilk test on the model residuals ($W = 0.989$, $p = 0.883$) indicated no significant departure from normality. Furthermore, Levene's test for homogeneity of variances across analgesic groups was non-significant ($F(7, 42) = 0.909$, $p = 0.5065$), confirming that the assumption of equal variances was met. These results validate the use of ANOVA and linear regression for the defuzzified data.

3.3 Multiple Linear Regression Analysis

A complementary regression analysis was conducted to assess the predictive power of the factors.

Table 5: Multiple Linear Regression of VAS Score on Predictors

Predictor	Estimate (β)	Std. Error	t	p-value	Significance
Intercept	5.189	1.065	17.947	<0.0001	***
Type of Surgery					
Craniotomy	2.715	0.909	2.988	0.0047	**
Exploratory laparotomy	2.000	0.909	8.098	0.0001	***
Herniorrhaphy	-1.822	0.909	-2.005	0.0514	.
Mastectomy	1.159	0.909	4.691	0.0194	***
Myomectomy	0.7071	0.909	0.778	0.4407	
ORIF	2.091	0.909	8.468	<0.0001	***
Urethroplasty	3.343	0.909	3.679	0.0006	***
Age Group					
23–27	0.4599	0.909	0.506	0.6154	
28–32	-0.0001	0.909	0.000	1.000	
33–37	0.8459	0.909	0.253	0.8015	
38–42	1.0092	0.0909	0.382	0.7642	
43–47	1.2559	0.909	1.382	0.1246	
48–52	0.7343	0.909	0.808	0.4236	
53 and above	1.4241	0.909	1.567	0.1246	

Predictor	Estimate (β)	Std. Error	t	p-value	Significance
Type of Analgesics					
Fentanyl	-3.125	0.909	-12.653	<0.0001	***
Ketamine	1.8394	0.909	2.024	0.0493	*
Ketorolac	-2.2374	0.909	-2.462	0.079	*
Morphine	-2.308	0.909	-9.344	<0.0001	***
Paracetamol	1.6713	0.909	1.839	0.0729	*
Pregabalin	1.022	0.909	4.136	0.0002	***
Tramadol	-1.841	0.909	-7.456	0.0004	***

Model Summary:

i. Residual standard error = 0.494 (df = 42)

ii. Multiple $R^2 = 0.9452$, Adjusted $R^2 = 0.9717$

iii. $F(21,42) = 34.47$, $p = <0.0001$

Interpretation:

The table 5 show that multiple linear regression model predicting VAS (Visual Analog Scale) scores shows a strong overall fit, with a high multiple R^2 of 0.9452 and an adjusted R^2 of 0.9717, indicating that approximately 97% of the variability in pain scores is explained by the predictors in the model. The model is statistically significant overall ($F(21,42) = 34.47$, $p < 0.0001$), meaning that the predictors together reliably explain differences in VAS scores.

Starting with the intercept, the baseline VAS score when all predictors are at reference levels is estimated at 5.189 and is highly significant ($p < 0.0001$).

Regarding the type of surgery, several categories have statistically significant effects on VAS scores compared to the reference category. Craniotomy ($\beta = 2.715$, $p = 0.0047$), Exploratory Laparotomy ($\beta = 2.000$, $p = 0.0001$), mastectomy ($\beta = 1.159$, $p = 0.0194$), ORIF (open reduction and internal fixation; $\beta = 2.091$, $p < 0.0001$), and Urethroplasty ($\beta = 3.343$, $p = 0.0006$) are all associated with significantly higher pain scores. Herniorrhaphy shows a marginally non-significant negative association ($\beta = -1.822$, $p = 0.0514$), suggesting it may be related to lower pain, but this effect is borderline. Myomectomy does not show a significant effect ($p = 0.4407$).

For the age groups, none of the categories show statistically significant associations with VAS scores, as all p-values are above the 0.05 threshold. This indicates that age group does not appear to be a strong predictor of pain scores in this model. In terms of type of analgesics, several medications significantly affect VAS scores. Fentanyl ($\beta = -3.125$, $p < 0.0001$), morphine ($\beta = -2.308$, $p < 0.0001$), pregabalin ($\beta = 1.022$, $p = 0.0002$), and Tramadol ($\beta = -1.841$, $p = 0.0004$) show significant associations. Fentanyl, Morphine, and Tramadol are associated with significantly lower VAS

scores, indicating better pain relief, whereas Pregabalin is associated with higher pain scores in this model. Ketamine ($\beta = 1.8394$, $p = 0.0493$) is also significant but at a less stringent level, associated with increased pain scores. Ketorolac and Paracetamol show trends towards significance but do not quite reach conventional thresholds (p-values 0.079 and 0.073 respectively), suggesting a possible but uncertain association. The residual standard error is low (0.494), indicating that the model predictions are quite close to the observed values on average.

Post-hoc Pairwise Comparisons

Tukey-adjusted pairwise comparisons confirmed statistically significant differences between some of the analgesic pairs ($p < 0.05$), reinforcing that rankings represent trends rather than definitive superiority.

Table 6: Post-Pairwise Comparisons

Contrast	estimate	SE	df	t.ratio	p.value
Pregabalin - Tramadol	2.862	0.246	42	11.591	<0.0001
Paracetamol - Tramadol	2.295	0.246	42	9.294	<0.0001
Paracetamol - Pregabalin	-0.567	0.246	42	-2.296	0.3194
Morphine – Tramadol	-0.466	0.246	42	-1.888	0.5661
Morphine - Pregabalin	-3.329	0.246	42	-13.479	<0.0001
Morphine - Paracetamol	-2.762	0.246	42	-11.183	<0.0001
Ketorolac – Pramadol	1.233	0.246	42	4.993	0.0003
Ketorolac - Pregabalin	-1.629	0.246	42	-6.598	<0.0001
Ketorolac - Paracetamol	-1.062	0.246	42	-4.301	0.0023
Ketorolac – Morphine	1.699	0.246	42	6.881	<0.0001
Ketamine – Tramadol	2.341	0.246	42	9.479	<0.0001
Ketamine - Pregabalin	-0.521	0.246	42	-2.111	0.4244
Ketamine - Paracetamol	0.045	0.246	42	0.184	1.0000
Ketamine – Morphine	2.807	0.246	42	11.368	<0.0001
Ketamine – Ketorolac	1.108	0.246	42	4.486	0.0013
Fentanyl – Tramadol	-1.286	0.246	42	-5.197	0.0001
Fentanyl – Pregabalin	-4.146	0.246	42	-16.788	<0.0001
Fentanyl - Paracetamol	-3.579	0.246	42	-14.492	<0.0001
Fentanyl – Morphine	-0.817	0.246	42	-3.308	0.0370
Fentanyl – Ketorolac	-2.516	0.246	42	-10.190	<0.0001
Kentanyl – Ketamine	-3.625	0.246	42	-14.677	<0.0001
Diclofenac – Tramadol	1.841	0.246	42	7.455	<0.0001
Diclofenac - Pregabalin	-1.021	0.246	42	-4.136	0.0038
Diclofenac - Paracetamol	-0.454	0.246	42	-1.839	0,5980
Diclofenac - Morphine	2.307	0.246	42	9.343	<0.0001
Diclofenac - Ketorolac	0.608	0.246	42	2.462	0.2395

Diclofenac - Ketamine	-0.5	0.246	42	-2.024	0.4785
Diclofenac - Fentanyl	3.125	00.246	42	12.652	<0.0001

Interpretation

The post-hoc pairwise comparison results provide detailed insight into the magnitude and statistical significance of differences in postoperative pain control among the analgesics. Overall, the findings reinforce a clear hierarchy of effectiveness. Fentanyl shows consistently and significantly lower VAS scores than most other analgesics, including Tramadol, Pregabalin, Paracetamol, Ketorolac, Ketamine, and Diclofenac (all $p<0.001$), and is also significantly superior to morphine ($p=0.037$), confirming its position as the most effective analgesic. Morphine demonstrates significantly better pain control than Pregabalin and Paracetamol ($p<0.0001$), but its difference from Tramadol is not statistically significant, suggesting comparable efficacy between these two agents. Tramadol performs significantly better than Pregabalin and Paracetamol, while being inferior to Fentanyl and Morphine, placing it in an intermediate effectiveness tier. Ketorolac shows significantly worse pain control than Morphine and Fentanyl but significantly better outcomes than Pregabalin, Paracetamol, and Ketamine, supporting its classification as a moderate-performing analgesic. Diclofenac performs significantly worse than Fentanyl and Morphine, but its differences from Paracetamol and Ketamine are not statistically significant, indicating similar moderate efficacy among these agents. Ketamine and Pregabalin generally exhibit higher pain scores and fewer significant advantages over other drugs, with several non-significant contrasts, highlighting their limited standalone analgesic effectiveness.

4. Conclusion

Based on the comprehensive analysis undertaken in this study, it is concluded that the integration of Exponential Trapezoidal Fuzzy Numbers within a Latin Square Design presents a viable and enhanced framework for analyzing clinical trial data characterized by imprecision and subjectivity. The proposed Fuzzy LSD methodology successfully achieved its objectives by: Formally incorporating fuzzy data to model clinical uncertainty and eextending the traditional LSD with fuzzy logic principles. The study conclusively identified significant differences in analgesic efficacy among the eight regimens tested. The finding that type of surgery significantly influenced pain scores, while patient age did not because all the age group are adult, underscores the complex, The findings demonstrate the multi-factorial nature of pain perception and highlights the importance of demographic factors in pain management. Therefore, the Fuzzy LSD with ETZFNs is not merely a theoretical exercise but a practical analytical tool that bridge the gap between rigid statistical models and the fuzzy reality of clinical practice, ultimately supporting more nuanced and evidence-based decisions in pain management and beyond.

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