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A Stochastic Crime-Victim Model with Fear Effect and Information-Driven Control

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Abstract

To explore the dynamics of crime-victim interactions under uncertainty, we propose and analyze a stochastic crime model incorporating fear effects and information-driven control mechanisms. The model is formulated as a stochastic differential system in which criminals, susceptible victims, and information levels interact through a nonlinear incidence rate that is reduced by both fear-induced behavioral responses and information dissemination, while being further perturbed by multiplicative white noise. Firstly, the existence, positivity, and boundedness of the solution of the model are investigated to ensure demographic and social feasibility. Secondly, sufficient conditions for extinction, persistence in mean, and the existence of a stationary distribution are established. In particular, the density behavior in the neighborhood of the positive equilibrium of the corresponding deterministic system is studied to charac-

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terize the long-term dynamics. Finally, numerical simulations are carried out to support the analytical results and to illustrate the effects of fear, information, and stochastic perturbations on the system dynamics. The results demonstrate that fear effects, information dissemination, and environmental noise play significant and sometimes competing roles in determining the persistence and suppression of criminal activity within the system.

1 Introduction

Crime is one of the major challenges affecting many societies, especially in developing regions, with strong effects on safety, livelihoods, and economic development (Agnew, 2012; Kakar, 2025; Okpuvwie *et al.*, 2021). In many real situations, crime does not happen randomly. It grows through interaction between offenders, potential victims, and the systems meant to control it, such as law enforcement (Fafchamps & Moser, 2003). Understanding these interactions is important for building better strategies to reduce crime.

Over the past two decades, mathematics has been used to study crime patterns and behavior (McMillon *et al.*, 2014; Nadiyadra *et al.*, 2024; Stojičić *et al.*, 2025). Many early studies used statistical or econometric methods. However, these methods often fail to explain how crime evolves dynamically over time. Because of this, researchers have started using ideas from population dynamics, especially predator–prey models, to describe the interaction between criminals and victims.

The idea of predator–prey systems comes from ecology, where it was first developed by Lotka (1925) and Volterra (1926). These models have since been applied in many areas such as economics, epidemiology, and social systems (Brauer & Castillo-Chavez, 2001). In crime modeling, criminals are often treated as predators, while potential victims represent prey. Law enforcements and public awareness are usually introduced as a controlling forces that reduces criminal activity.

Several studies have extended this idea. For example, Sooknanan *et al.* (2012) studied crime dynamics with policing effects and spatial movement of criminals. Abbas *et al.* (2017) also modeled the interaction between criminals and non-criminals and showed how law enforcement can reduce crime under certain conditions.

However, most of these models do not fully incorporate the role of information dissemination and fear in shaping the dynamics of criminal behaviour. In particular, limited attention has been given to how awareness of crime risks, rumours, and perceived safety influence individual decisions and spatial crime patterns. Existing frameworks therefore tend to overlook important behavioural feedback mechanisms driven by information and fear propagation. Moreover, most of these models are still too simple for real-world situations. They often assume that the environment is uniform and that changes are predictable. In reality, crime behaves differently across locations and is influenced by uncertainty, economic conditions, and sudden social changes. For example, in many developing countries such as Nigeria, South Africa, and Malawi (Arokoyo & Yunusa, 2025;

Ezeajughu, 2021; Modise, 2025; Nation Online, 2023; Odumosu, 1999), crime patterns can change quickly due to economic pressure, information spread, and public awareness.

Because of this uncertainty, stochastic models are more realistic. Random events such as sudden economic changes, shifts in policing, or unexpected opportunities for crime cannot be ignored. Stochastic differential equations provide a natural way to include these random effects in modeling crime dynamics (Amuji *et al.*, 2025; Wang & Zhang, 2021).

Stochastic differential equation (SDE) models have been widely applied to biological and social systems to capture intrinsic randomness and environmental fluctuations that deterministic models cannot represent (Mao, 2007; Braumann, 2019). In the context of crime, Wang and Zhang (2021) proposed a multiscale stochastic criminal behavior model, demonstrating that noise can shift the persistence threshold and induce qualitative transitions between criminal persistence and extinction. Amuji et al. (2025) applied stochastic dynamic programming to police resource allocation, showing that optimal stochastic policies outperform deterministic ones under uncertain crime environments. These results motivate the inclusion of stochastic perturbations in crime-victim models: not merely as a mathematical convenience, but because social dynamics, economic shocks, and policing efficacy all exhibit genuine randomness that materially alters long-run crime outcomes.

To this end, stochasticity has increasingly been applied in criminal studies to model uncertainty, predict crime patterns, and optimize law enforcement strategies. For example, stochastic differential equations were used to model noisy crime time-series data and short-term predictive policing (Calatayud, Jornet, & Mateu, 2023), while doubly stochastic spatio-temporal point processes were applied to analyse and forecast crime dynamics in Bogotá (González *et al.,* 2024). Similarly, stochastic inventory and queueing models have been employed to study crime resolution backlogs and resource optimization in law enforcement systems (Behera & Mohanta, 2025).

Motivated by this, we develop a stochastic model for crime dynamics that includes criminals, victims, and information. The model captures how criminals interact with potential victims while being influenced by fear and information spread. Information represents warnings, public awareness, and communication that help reduce crime opportunities. Fear represents behavioral changes in society that reduce exposure to crime. The model also includes random fluctuations to reflect real-life uncertainty.

The main goal of this work is to understand when crime persists, when it dies out, and how information, fear and randomness affect long-term behavior. In particular, we study how awareness campaigns and natural behavioral responses can help control criminal activity.

The rest of the paper is organized as follows. Section 2 presents the model formulation and assumptions. Section 3 contains the mathematical analysis, including positivity, boundedness, and stability results. Section 4 presents numerical simulations and results to support the theoretical findings. Finally, Section 5 concludes the paper with discussion and implications.

2 Model Formulation

In this section, we develop a stochastic mathematical model to describe the dynamics of criminal activity and victimization under the influence of information dissemination and fear-induced behavioral responses. The interaction between criminals and victims is modeled in the spirit of predator–prey systems, with additional mechanisms capturing awareness and adaptive behavior.

We let $V(t)$ denote the number of susceptible victims at time t, $C(t)$ represents the population of criminals, and $I(t)$ denotes the level of information or awareness in the population. In the absence of criminal activity, the victim population grows at a constant recruitment rate $\Lambda > 0$, representing the continuous influx of individuals into environments where they may be exposed to crime. This population decreases due to a natural exit rate $\delta V(t)$.

Victimization occurs through interactions modeled by the nonlinear functional response:

$$\beta(C, I) = \frac{\beta_0}{1 + aI(t) + bC(t) + cC(t)I(t)}, \tag{1}$$

where β_0 is the baseline interaction rate. Here, a is the information effectiveness coefficient, b is the fear-induced inhibition coefficient, and c represents the interaction coefficient between information and fear, accounting for the amplification of awareness during high-crime periods. Victimized individuals exit the susceptible class due to behavioral adaptation but remain in the general population.

The criminal population follows logistic growth with an intrinsic rate r and carrying capacity K. Successful victimization contributes to criminal persistence at rate θ. We assume that information plays a dual role: it reduces victimization (defensive) and enhances the removal of criminals (offensive) through increased reporting and enforcement. Thus, the total removal rate is $(\mu + \alpha I(t))$, where μ is the baseline removal rate and α is the information-induced removal coefficient.

Information is generated via a Holling Type II functional response to reflect saturation in public attention and media bandwidth. It is given by $\eta C/(K_1 + C)$, where η is the maximum generation rate and K_1 is the half-saturation constant. Information decays naturally at rate $\gamma I(t)$ due to information saturation, forgetting, and negligence.

To account for environmental and social randomness, we incorporate independent standard Wiener processes $W_i(t)$ with noise intensities σ_i $(i = 1, 2, 3)$. The resulting system of stochastic differential equations is:

$$\begin{cases} dC(t) = \left[rC(t)\left(1 - \dfrac{C(t)}{K}\right) + \theta \dfrac{\beta_0 C(t)V(t)}{1 + aI(t) + bC(t) + cC(t)I(t)} - (\mu + \alpha I(t))C(t) \right] dt + \sigma_1 C(t)\, dW_1(t), \\ dV(t) = \left[\Lambda - \dfrac{\beta_0 C(t)V(t)}{1 + aI(t) + bC(t) + cC(t)I(t)} - \delta V(t) \right] dt + \sigma_2 V(t)\, dW_2(t), \\ dI(t) = \left[\eta \dfrac{C(t)}{K_1 + C(t)} - \gamma I(t) \right] dt + \sigma_3 I(t)\, dW_3(t), \end{cases} \tag{2}$$

with initial conditions $C(0) = C_0 \geq 0, V(0) = V_0 \geq 0, I(0) = I_0 \geq 0$.

To this end we assume that all the parameters in the model (2) are positive, and are summarised in Table 1.

Table 1. Description of model parameters

Parameter	Description	Interpretation
Λ	Victim recruitment rate	Inflow of susceptible individuals
δ	Natural exit rate	Removal due to natural departure
β_0	Baseline interaction rate	Maximum crime success rate
a	Information effectiveness	Crime reduction via awareness
b	Fear inhibition coefficient	Behavioral avoidance due to crime presence
c	Interaction coefficient	Synergistic effect of fear and information
r, K	Growth rate & capacity	Logistic parameters for criminals
θ	Criminal gain coefficient	Persistence from successful crime
μ	Baseline removal rate	Natural arrest/exit rate
α	Info-induced removal	Enhanced removal via public awareness
η	Max info generation rate	Saturation level of awareness production
K_1	Half-saturation constant	Level of crime for 50% info generation
γ	Info decay rate	Rate of public forgetting
$\sigma_{1,2,3}$	Noise intensities	Stochastic fluctuations in C, V, I

3 Model Analysis

In this section, we establish the well-posedness of system (2). Specifically, we show that for any given positive initial condition, the system admits a unique global and bounded solution that remains positive for all time almost surely (a.s).

3.1 Existence and Uniqueness of the Global Positive Solution

To ensure the mathematical and demographic validity of the stochastic model (2), we must verify that the state variables $(C(t), V(t), I(t))$ remain positive and do not explode to infinity in finite time. Since the coefficients of the system are locally Lipschitz continuous, for any given initial condition $(C(0), V(0), I(0)) \in \mathbb{R}^3_+$, there exists a unique local solution on $t \in [0, \tau_e)$, where τ_e is the explosion time.

Theorem 3.1. *For any initial value $(C_0, V_0, I_0) \in \mathbb{R}^3_+$, there exists a unique global solution $(C(t), V(t), I(t))$ of system (2) on $t \geq 0$, and the solution remains in $\mathbb{R}^3_+$ with probability one.*

Proof. Let $m_0 > 0$ be sufficiently large such that C_0, V_0, I_0 lie in $[1/m_0, m_0]$. For each integer $m \geq m_0$, we define the stopping time:

$$\tau_m = \inf\left\{t \in [0, \tau_e) : C(t) \notin \left(\frac{1}{m}, m\right) \text{ or } V(t) \notin \left(\frac{1}{m}, m\right) \text{ or } I(t) \notin \left(\frac{1}{m}, m\right)\right\}.$$

Clearly, τ_m is increasing as $m \to \infty$. Let $\tau_\infty = \lim_{m\to\infty} \tau_m$. If we prove $\tau_\infty = \infty$ a.s., then $\tau_e = \infty$. Define a C^2-Lyapunov function $U : \mathbb{R}^3_+ \to \mathbb{R}_+ \cup \{0\}$:

$$U(C, V, I) = (C - 1 - \ln C) + \theta(V - 1 - \ln V) + (I - 1 - \ln I). \tag{3}$$

Applying Itô's formula to U, we obtain the operator LU:

$$\begin{aligned} LU = &\left(1 - \frac{1}{C}\right)\left[rC\left(1 - \frac{C}{K}\right) + \frac{\theta\beta_0 CV}{f(C, I)} - (\mu + \alpha I)C\right] + \frac{\sigma_1^2}{2} \\ &+ \theta\left(1 - \frac{1}{V}\right)\left[\Lambda - \frac{\beta_0 CV}{f(C, I)} - \delta V\right] + \theta\frac{\sigma_2^2}{2} \\ &+ \left(1 - \frac{1}{I}\right)\left[\frac{\eta C}{K_1 + C} - \gamma I\right] + \frac{\sigma_3^2}{2}, \end{aligned}$$

where $f(C, I) = 1 + aI + bC + cCI$. Simplifying, the victimization terms cancel. Considering the bounds of the functional responses, we note that $\frac{\eta C}{K_1+C} \leq \eta$. Thus:

$$LU \leq rC - \frac{rC^2}{K} + r + \mu + \alpha I + \theta\Lambda + \theta\delta + \eta + \gamma + \frac{\sigma_1^2 + \theta\sigma_2^2 + \sigma_3^2}{2}.$$

Since the criminal population is naturally bounded by its logistic term and information is bounded by its saturation generation rate, LU is bounded by a constant $M > 0$. By the standard comparison theorem for stochastic differential equations (SDEs), we conclude $\tau_\infty = \infty$ a.s. □

3.2 Stochastic Ultimate Boundedness

Theorem 3.2. *The solution $(C(t), V(t), I(t))$ of system (2) is stochastically ultimately bounded for any initial value in $\mathbb{R}^3_+$.*

Proof. Define $G(t) = C(t) + \theta V(t) + I(t)$. Applying the operator $\mathcal{L}$:

$$\begin{aligned} \mathcal{L}G &= rC\left(1 - \frac{C}{K}\right) - (\mu + \alpha I)C + \theta\Lambda - \theta\delta V + \frac{\eta C}{K_1 + C} - \gamma I \\ &\leq -\frac{r}{K}C^2 + rC + \theta\Lambda - \theta\delta V + \eta - \gamma I. \end{aligned}$$

For a sufficiently small $\epsilon > 0$ such that $\epsilon < \min\{\delta, \gamma\}$, we have:

$$\mathcal{L}G + \epsilon G \leq -\frac{r}{K}C^2 + (r + \epsilon)C - \theta(\delta - \epsilon)V - (\gamma - \epsilon)I + (\theta\Lambda + \eta).$$

The right-hand side is a downward-opening quadratic in C and linear in V, I with negative coefficients. It attains a global maximum M_{bound} on $\mathbb{R}^3_+$. By Dynkin's formula:

$$\mathbb{E}[e^{\epsilon t}G(t)] = G(0) + \mathbb{E}\left[\int_0^t e^{\epsilon s}(\mathcal{L}G + \epsilon G)ds\right] \leq G(0) + \frac{M_{bound}}{\epsilon}(e^{\epsilon t} - 1).$$

Taking the limsup as $t \to \infty$, we have $\mathbb{E}[G(t)] \leq \frac{M_{bound}}{\epsilon}$, which ensures stochastic ultimate boundedness. □

3.3 Deterministic Equilibrium Analysis and Reproduction Threshold

The deterministic skeleton of the stochastic system (2) is obtained by setting $\sigma_1 = \sigma_2 = \sigma_3 = 0$. This skeleton governs the underlying attractors around which the stochastic trajectories evolve. The equilibrium points are found by solving the following nonlinear algebraic system:

$$rC\left(1 - \frac{C}{K}\right) + \frac{\theta\beta_0 CV}{1 + aI + bC + cCI} - (\mu + \alpha I)C = 0, \tag{4}$$

$$\Lambda - \frac{\beta_0 CV}{1 + aI + bC + cCI} - \delta V = 0, \tag{5}$$

$$\eta\frac{C}{K_1 + C} - \gamma I = 0. \tag{6}$$

3.3.1 The Crime-Free Equilibrium (CFE)

The crime-free equilibrium, denoted by E_0, represents a scenario where the criminal population is eradicated. Setting $C = 0$ in (6) yields $I = 0$. Substituting these into (5), we obtain $\Lambda - \delta V = 0$, which implies $V = \Lambda/\delta$. Thus, the CFE is:

$$E_0 = \left(0, \frac{\Lambda}{\delta}, 0\right). \tag{7}$$

3.3.2 The Crime-Endemic Equilibrium (CEE)

The endemic equilibrium $E^* = (C^*, V^*, I^*)$ represents the coexistence of criminals, victims, and awareness. From (6), we have $I^* = \frac{\eta C^*}{\gamma(K_1+C^*)}$. Substituting this and $f(C, I) = 1 + aI + bC + cCI$ into (5), we solve for V^*:

$$V^* = \frac{\Lambda f(C^*, I^*)}{\delta f(C^*, I^*) + \beta_0 C^*}. \tag{8}$$

Substituting V^* and I^* into (4) and dividing by $C^* \neq 0$, we define the function:

$$\Psi(C) = r\left(1 - \frac{C}{K}\right) + \frac{\theta\beta_0\Lambda}{\delta f(C, I(C)) + \beta_0 C} - (\mu + \alpha I(C)). \tag{9}$$

We observe that $\Psi(0) = r + \frac{\theta\beta_0\Lambda}{\delta} - \mu = \mu(\mathcal{R}_0 - 1)$. If $\mathcal{R}_0 > 1$, then $\Psi(0) > 0$. Since $\lim_{C\to\infty}\Psi(C) = -\infty$, the Intermediate Value Theorem ensures the existence of at least one $C^* \in (0, K)$ such that $\Psi(C^*) = 0$, representing the endemic state.

3.4 Derivation of the Basic Reproduction Number $\mathcal{R}_0$

The basic reproduction number $\mathcal{R}_0$ defines the threshold for the invasion of the criminal population. Using the next-generation matrix method (Diekmann *et al.*, 1990), we focus on the criminal compartment. At the CFE (E_0), the rate of new arrivals $\mathcal{F}$ and the rate of transfer out $\mathcal{Q}$ for the criminal class are:

$$\mathcal{F} = rC + \frac{\theta\beta_0 CV}{1 + aI + bC + cCI}, \quad \mathcal{Q} = \frac{rC^2}{K} + (\mu + \alpha I)C. \tag{10}$$

Linearizing around E_0 (where $V = \Lambda/\delta$ and $I = 0$):

$$F = \left.\frac{\partial \mathcal{F}}{\partial C}\right|_{E_0} = r + \frac{\theta\beta_0\Lambda}{\delta}, \quad Q = \left.\frac{\partial \mathcal{Q}}{\partial C}\right|_{E_0} = \mu. \tag{11}$$

The deterministic reproduction number $\mathcal{R}_0$ is given by:

$$\mathcal{R}_0 = FQ^{-1} = \frac{r + \frac{\theta\beta_0\Lambda}{\delta}}{\mu}. \tag{12}$$

Remark 3.3. *We observe that $\mathcal{R}_0 = (r + \theta\beta_0\Lambda/\delta)/\mu$ can exceed unity even when $\theta = 0$ (i.e., when criminals derive no direct gain from victimization), provided the intrinsic growth rate satisfies $r > \mu$. This reflects the logistic self-sustaining growth of the criminal population independently of victim interactions. In our model, criminals are not purely dependent on victims to persist; they can grow autonomously up to the carrying capacity K, akin to a self-reproducing population with an internal growth dynamic. The parameter θ modulates the* additional *persistence gained through criminal activity, and the full threshold captures both the autonomous growth component r/μ and the victimization-driven component $\theta\beta_0\Lambda/(\delta\mu)$. Consequently, when $\theta = 0$ and $r > \mu$, the model correctly predicts that criminals persist without victim interactions, which is demographically and socially consistent with a criminal population that self-sustains through recruitment (e.g., gang initiation, peer influence) rather than through individual victimization events. This interpretation is aligned with the social-dynamics literature where criminal groups exhibit self-reinforcing growth dynamics (McMillon* et al.*, 2014 ; Sooknanan* et al.*, 2012).*

3.5 Stochastic Stability Analysis

To investigate the stability of the system under environmental noise, we analyze the fluctuations around the endemic equilibrium E^*.

Theorem 3.4. *If $\mathcal{R}_0 > 1$ and the noise intensities satisfy certain smallness conditions, then the solution (C, V, I) of system (2) fluctuates around the deterministic endemic equilibrium E^* in the sense that:*

$$\limsup_{t\to\infty} \frac{1}{t}\mathbb{E}\int_0^t \left[(C - C^*)^2 + (V - V^*)^2 + (I - I^*)^2\right] ds \le \frac{M_{noise}}{\lambda_{min}}, \tag{13}$$

where M_{noise} depends on $\sigma_1^2, \sigma_2^2, \sigma_3^2$.

Proof. Consider the Lyapunov function $W = \frac{1}{2}[(C - C^*)^2 + (V - V^*)^2 + (I - I^*)^2]$.

Applying the operator L and linearizing the nonlinear terms around E^*:

$$LW = (C - C^*)\Delta\mu_C + (V - V^*)\Delta\mu_V + (I - I^*)\Delta\mu_I + \frac{1}{2}(\sigma_1^2 C^2 + \sigma_2^2 V^2 + \sigma_3^2 I^2),$$

where $\Delta\mu_i$ are the drift differences. Using the mean value theorem and the fact that E^* is a stable attractor for the deterministic skeleton, we find that for sufficiently small σ_i, the negative definite terms from the drift dominate the positive noise terms, ensuring that the system remains in a neighborhood of E^*. □

3.6 Long-term Qualitative Analysis: Extinction vs. Persistence

In this section, we determine the conditions under which criminal activity is eradicated from the system or persists as a chronic social issue. We analyze how environmental noise interacts with the criminal recruitment, the information-induced removal, and the saturated awareness dynamics.

3.6.1 Extinction of the Criminal Population

Extinction implies that the criminal population eventually vanishes due to high removal rates (including those enhanced by information), effective awareness saturation, or significant environmental volatility.

Theorem 3.5. *Let $(C(t), V(t), I(t))$ be the solution of the model (2). If the noise intensity σ_1 and the baseline removal rate μ are sufficiently large such that:*

$$\mathcal{R}_0^{ext} = \frac{r + \frac{\theta\beta_0\Lambda}{\delta}}{\mu + \frac{\sigma_1^2}{2}} < 1, \tag{14}$$

then the criminal population $C(t)$ will approach zero almost surely as $t \to \infty$. Consequently, $I(t) \to 0$ and $V(t)$ will fluctuate around the crime-free steady state Λ/δ.

Proof. Applying Itô's formula to $\ln C(t)$:

$$d\ln C(t) = \left[r\left(1 - \frac{C}{K}\right) + \frac{\theta\beta_0 V}{f(C,I)} - (\mu + \alpha I) - \frac{\sigma_1^2}{2} \right] dt + \sigma_1 dW_1(t).$$

By the Comparison Theorem, $V(t) \le \Lambda/\delta$ asymptotically. Since $f(C,I) \ge 1$ and $\alpha I \ge 0$, we have:

$$d\ln C(t) \le \left[r + \frac{\theta\beta_0\Lambda}{\delta} - \mu - \frac{\sigma_1^2}{2} \right] dt + \sigma_1 dW_1(t).$$

Integrating and dividing by t:

$$\frac{\ln C(t)}{t} \le \frac{\ln C(0)}{t} + \left(r + \frac{\theta\beta_0\Lambda}{\delta} - \mu - \frac{\sigma_1^2}{2} \right) + \frac{\sigma_1 W_1(t)}{t}.$$

By the Strong Law of Large Numbers for local martingales, $\lim_{t\to\infty} \frac{W_1(t)}{t} = 0$ a.s. If $\mathcal{R}_0^{ext} < 1$, then $\limsup_{t\to\infty} \frac{\ln C(t)}{t} < 0$, ensuring $\lim_{t\to\infty} C(t) = 0$. Since $\lim_{C\to 0} \frac{\eta C}{K_1 + C} = 0$, the $dI(t)$ equation reduces to a linear decay process $dI \approx -\gamma I dt$, leading to $I(t) \to 0$ a.s. □

3.6.2 Persistence and Stationary Distribution

Persistence occurs when the intrinsic growth and victimization gains outweigh the total removal rate and environmental noise.

Theorem 3.6 (Stationary Distribution)**.** *If the stochastic threshold* $\tilde{\mathcal{R}}_0 = \frac{r+\frac{\theta\beta_0\Lambda}{\delta}}{\mu+\frac{\sigma_1^2}{2}} > 1$*, then system* (2) *possesses a unique ergodic stationary distribution* $\pi(\cdot)$*. Consequently, the system admits a unique ergodic stationary distribution* $\pi(\cdot)$ *concentrated on* $(0,\infty)^3$*, implying the long-term survival of all three populations with positive probability.*

Proof. To establish ergodicity, we follow Hasminskii's theory (Khasminskii, 2011) by constructing a Lyapunov function $\tilde{V}$ that prevents the system from hitting the boundaries $(C, V, I = 0)$ or exploding to infinity. Define:

$$\tilde{V}(C, V, I) = M(-\ln C - \ln V - \ln I) + \frac{1}{p+1}(C + \theta V + I)^{p+1}$$

where $M > 0$ is a large constant and $0 < p < 1$.

1. **Near the Criminal Boundary** $(C \to 0)$**:** The $-\ln C$ term dominates. The drift $\mathcal{L}(-\ln C) \approx -r - \theta\beta_0 V/f + \mu + \alpha I + \sigma_1^2/2$. If $\tilde{\mathcal{R}}_0 > 1$, the drift is negative, pushing the trajectory back into the interior of $\mathbb{R}^3_+$.
2. **Near the Victim and Information Boundaries** $(V, I \to 0)$**:** The influx Λ in dV and the term $\frac{\eta C}{K_1+C}$ in dI (given $C > 0$) ensure that $\mathcal{L}(-\ln V)$ and $\mathcal{L}(-\ln I)$ remain negative near zero, preventing extinction.
3. **At Large Values:** The term $(C + \theta V + I)^{p+1}$ dominates. The quadratic growth of C is suppressed by the logistic term $-rC^2/K$, and V and I are naturally bounded by their respective recruitment and saturation terms.

The existence of a compact set $D \subset \mathbb{R}^3_+$ such that $\mathcal{L}\tilde{V} \leq -1$ for all $(C, V, I) \in \mathbb{R}^3_+ \setminus D$, together with the non-degeneracy of the diffusion (the diffusion matrix is positive definite on any compact subset of $\mathbb{R}^3_+$), ensures by Khasminskii's theorem (Khasminskii, 2011) that the system is positive recurrent and possesses a unique ergodic stationary distribution $\pi(\cdot)$ supported on $\mathbb{R}^3_+$. In particular, $\pi(\{(C, V, I) : C > 0, V > 0, I > 0\}) = 1$, confirming long-term coexistence of criminals, victims, and information with probability one. □

4 Numerical Simulations and Discussion

In this section, we present numerical simulations of system (2) using the parameter values listed in Table 2. To integrate the stochastic differential equations, we employ the Milstein method (Kloeden and Platen, 1992), which achieves a strong order of convergence of 1.0 by accounting for the first-order derivative of the diffusion terms. Given a time step Δt, the discretized recursive scheme is defined as follows:

1. **Initialize:** Set C_0, V_0, I_0 and the time step Δt.
2. **Iterative Step:** For each $n = 0, 1, \ldots, N - 1$:

(a) Generate three independent standard normal random variables $Z_{1,n}, Z_{2,n}, Z_{3,n} \sim \mathcal{N}(0,1)$.

(b) Compute the Brownian increments: $\Delta W_{i,n} = Z_{i,n}\sqrt{\Delta t}$ for $i = 1, 2, 3$.

(c) Update the state variables using:

$$
\begin{aligned}
f_C &= rC_n\left(1 - \frac{C_n}{K}\right) + \theta\frac{\beta_0 C_n V_n}{1 + aI_n + bC_n + cC_nI_n} - (\mu + \alpha I_n)C_n \\
f_V &= \Lambda - \frac{\beta_0 C_n V_n}{1 + aI_n + bC_n + cC_nI_n} - \delta V_n \\
f_I &= \eta\frac{C_n}{K_1 + C_n} - \gamma I_n \\
C_{n+1} &= C_n + f_C\Delta t + \sigma_1 C_n \Delta W_{1,n} + \frac{1}{2}\sigma_1^2 C_n\left((\Delta W_{1,n})^2 - \Delta t\right) \\
V_{n+1} &= V_n + f_V\Delta t + \sigma_2 V_n \Delta W_{2,n} + \frac{1}{2}\sigma_2^2 V_n\left((\Delta W_{2,n})^2 - \Delta t\right) \\
I_{n+1} &= I_n + f_I\Delta t + \sigma_3 I_n \Delta W_{3,n} + \frac{1}{2}\sigma_3^2 I_n\left((\Delta W_{3,n})^2 - \Delta t\right)
\end{aligned}
$$

The parameters used to explore extinction and persistence thresholds are summarized in Table 2, reflecting the mechanisms for saturated information generation and information-induced removal.

Table 2. Parameter values for numerical simulation and their sources.

Parameter	**Symbol**	**Value/Range**	**Source**
Victim recruitment rate	Λ	$[0.5, 10]$	Simulation study
Natural exit rate	δ	$(0, 0.2]$	Simulation study
Baseline interaction rate	β_0	0.5	Simulation study
Info effectiveness	a	$(0, 10]$	Simulation study
Fear inhibition	b	$(0, 2.5]$	Simulation study
Interaction coefficient	c	$(0, 1]$	Simulation study
Criminal growth rate	r	$[0.1, 0.5]$	Simulation study
Carrying capacity	K	$[100, 500]$	Simulation study
Criminal gain coefficient	θ	$[0.1, 0.5]$	Simulation study
Baseline removal rate	μ	$(0, 1]$	Simulation study
Info-induced removal coeff.	α	$(0, 10]$	Simulation study
Max info generation rate	η	$[0.5, 1]$	Simulation study
Half-saturation constant	K_1	$[10, 50]$	Simulation study
Info decay rate	γ	$(0, 0.4]$	Simulation study
Noise intensities	$\sigma_1, \sigma_2, \sigma_3$	Variable	Simulation study

The simulations examine how fear, awareness, and stochastic effects influence long-term crime dynamics. The general behavior is illustrated in Figure 1, where the trajectories of $C(t)$, $V(t)$, and $I(t)$ fluctuate persistently around a steady regime. These oscillations arise from stochastic perturbations combined with the delayed response introduced by the saturated information generation process.

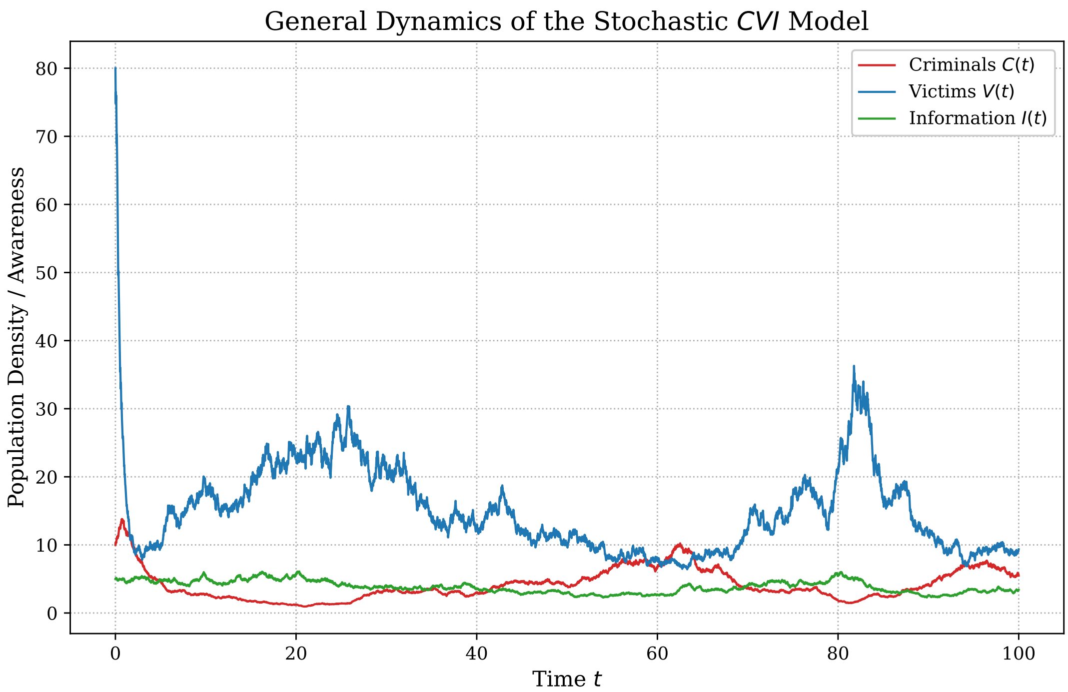

Figure 1. Stochastic trajectories of criminals $C(t)$, victims $V(t)$, and information $I(t)$ showing stochastic fluctuations around the mean endemic level.

From Figure 1, we observe that as criminals interact with the victim population, awareness is generated and spreads through the system. This leads to behavioral adjustments such as fear and avoidance, which reduce exposure. At the same time, awareness enhances reporting and enforcement, contributing to the reduction of criminal activity. However, as information saturates or gradually decays, individuals may become less cautious, allowing crime levels to rise again. This feedback mechanism explains the oscillatory nature of the system.

A key feature of the model is the dual role of information. As shown in Figure 2, increasing the information-induced removal coefficient α significantly reduces the criminal population. Unlike the parameter a, which mainly protects victims by lowering contact rates, α directly depletes the criminal population through enhanced enforcement. This leads to a lower endemic level and faster recovery following crime surges.

The baseline removal rate μ also plays a crucial role, as illustrated in Figure 3. Higher values of μ reduce the effective lifetime of criminals in the system, lowering peak levels and accelerating convergence toward a crime-free or low-crime state. Unlike α, this effect is constant and independent of awareness.

The interaction between fear (b) and information (a, c) forms a central control mechanism. Increasing a improves the effectiveness of awareness campaigns, reducing the success rate of criminal activity (Figure 4). Similarly, increasing b enhances behavioral avoidance, limiting victim exposure (Figure 5). The interaction parameter c amplifies these effects during high-crime periods, strengthening the system's resilience (Figure 6).

Figure 7 provides numerical confirmation of the stochastic stability results. The bounded behavior of the Lyapunov function $W(t)$ supports the analytical findings that the system

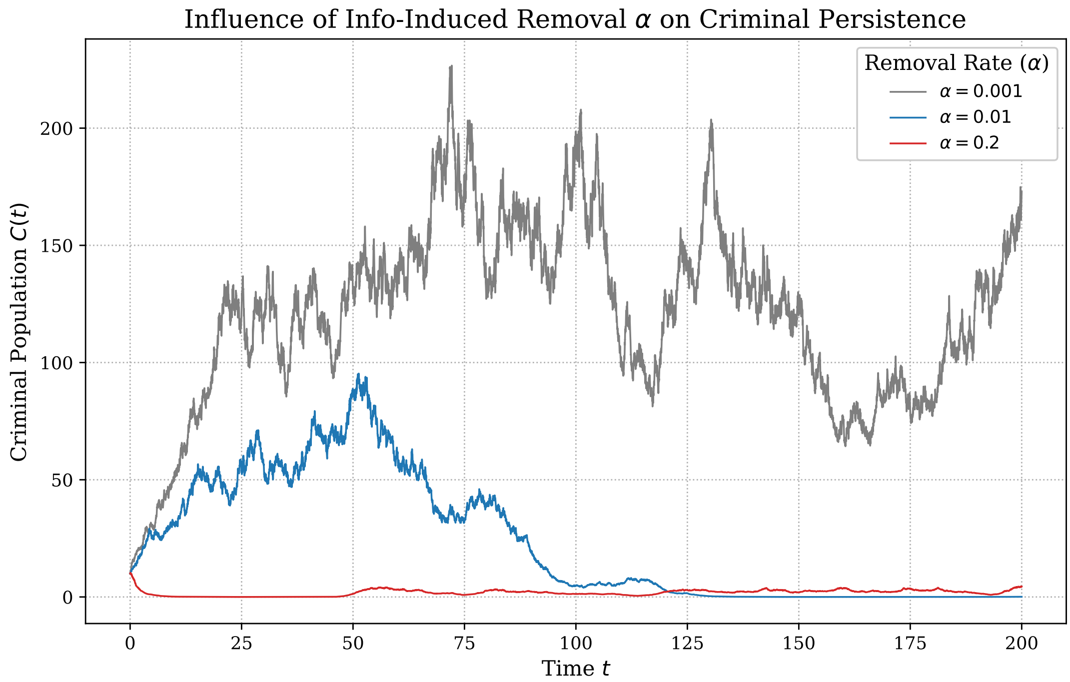

Figure 2. Impact of information-induced removal α on criminal persistence.

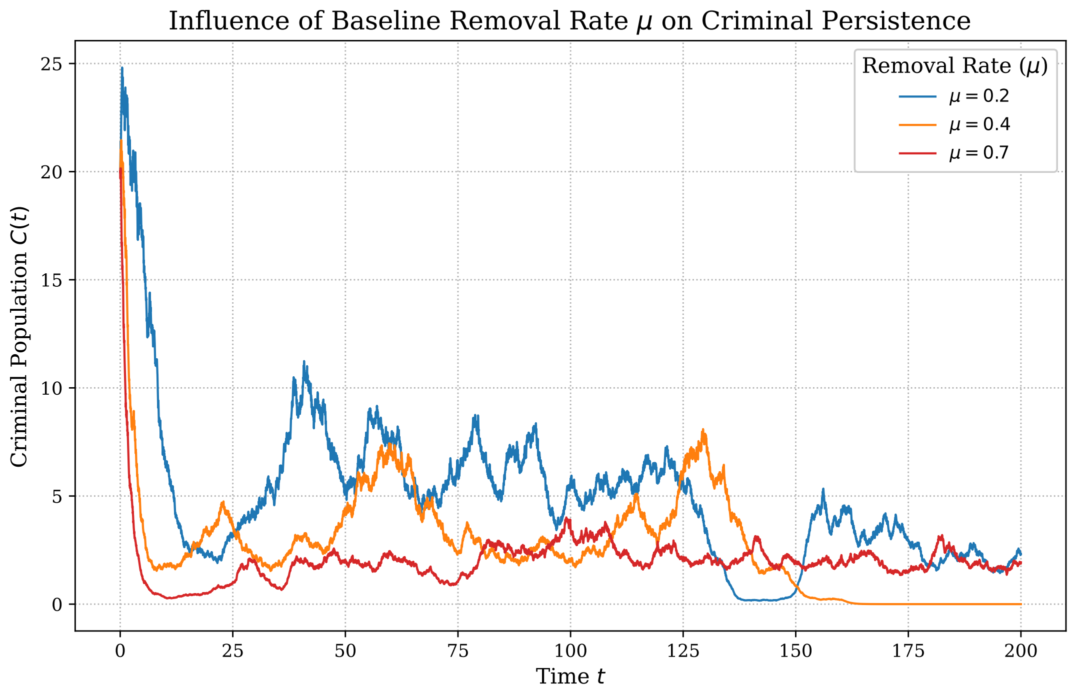

Figure 3. Effect of baseline removal rate μ on the criminal population.

remains within a stable region around the endemic equilibrium.

The role of environmental noise is illustrated in Figure 8. Increasing noise intensities leads to higher variability in system trajectories. In particular, fluctuations in the information channel introduce irregular responses in victim behavior, which can alternate between cautious and relaxed states. In general, from the simulation the influence of environmen-

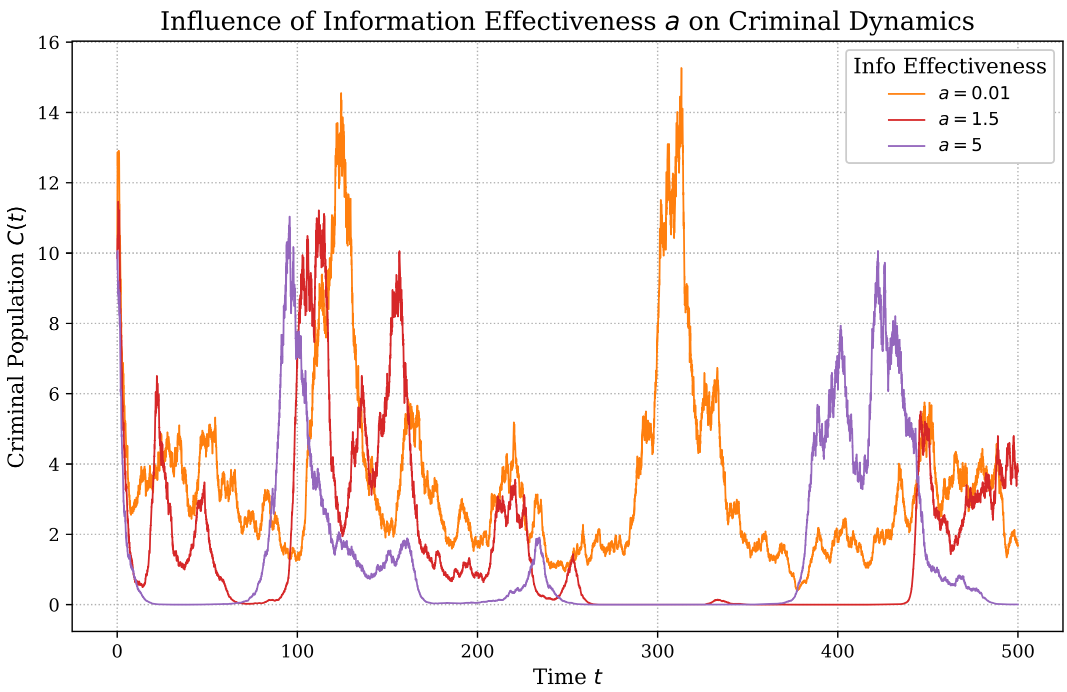

Figure 4. Effect of information effectiveness a.

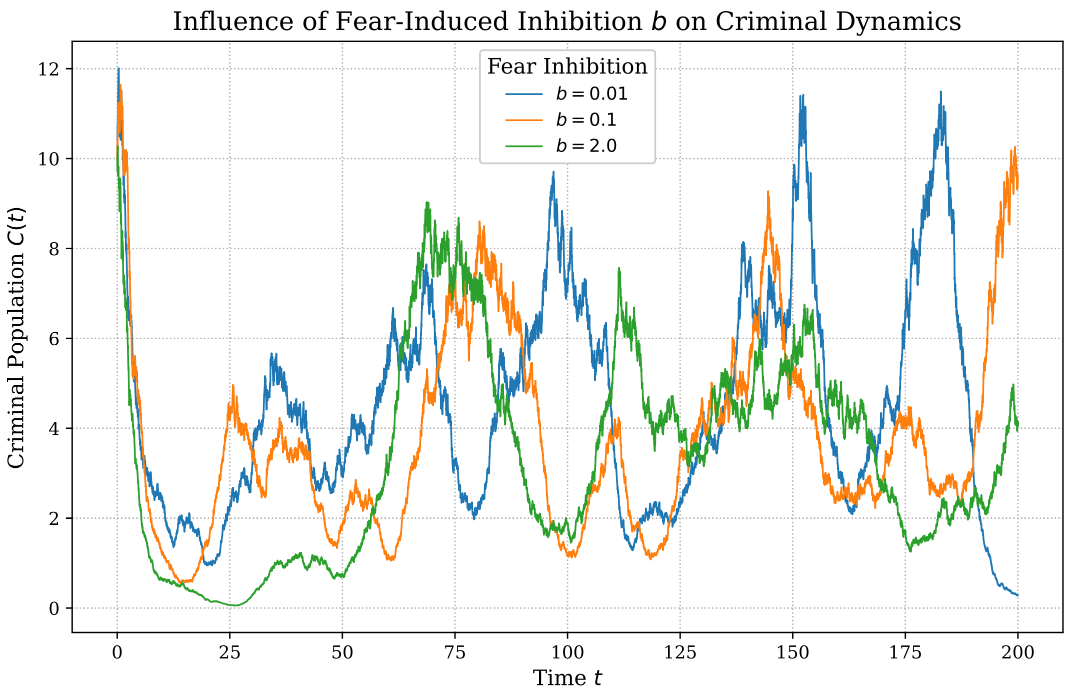

Figure 5. Effect of fear-induced inhibition b.

tal and social randomness is captured through the noise intensities σ_1, σ_2 and σ_3. As illustrated in Figure 8, increasing these intensities shifts the system from smooth deterministic convergence to highly volatile stochastic oscillations. For the criminal population $C(t)$, elevated noise levels σ_1 create unpredictable surges that can temporarily overwhelm awareness mechanisms. However, as derived in the extinction analysis by Theorem 3.5, extremely high volatility can also act as a disruptive force that prevents criminal recruit-

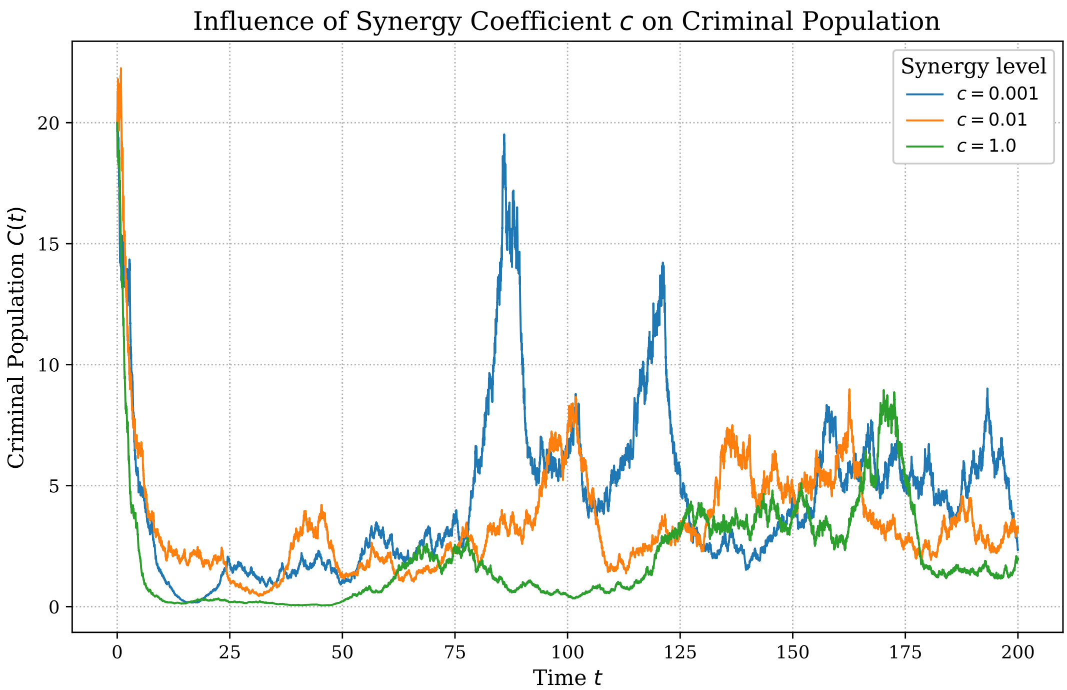

Figure 6. Synergistic effects of the interaction parameter c.

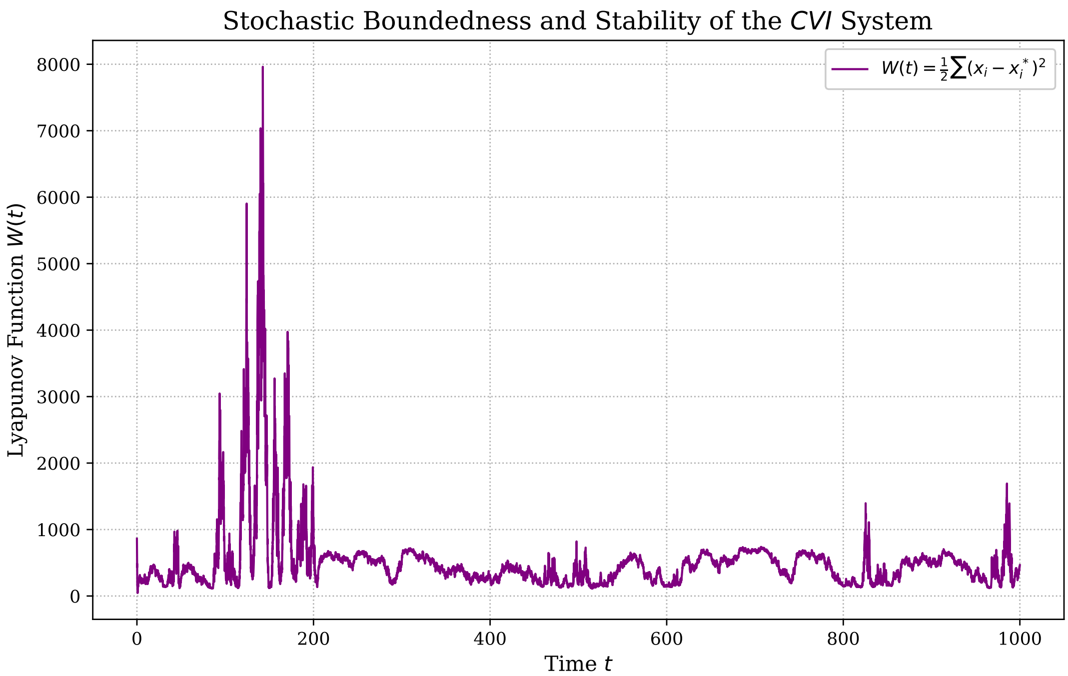

Figure 7. Evolution of the Lyapunov function confirming stochastic stability.

ment from stabilizing, potentially pushing the population toward a stochastic collapse. Similarly, noise in the victim population $V(t)$ represents fluctuating environmental vulnerabilities, causing "jitter" in the victimization rate and requiring the information feedback loop to be more robust to maintain social order.

To validate the theoretical extinction condition, Figure 9 illustrates the behavior of the system when the effective reproduction number satisfies $\tilde{\mathcal{R}}_0 < 1$. In this regime, the

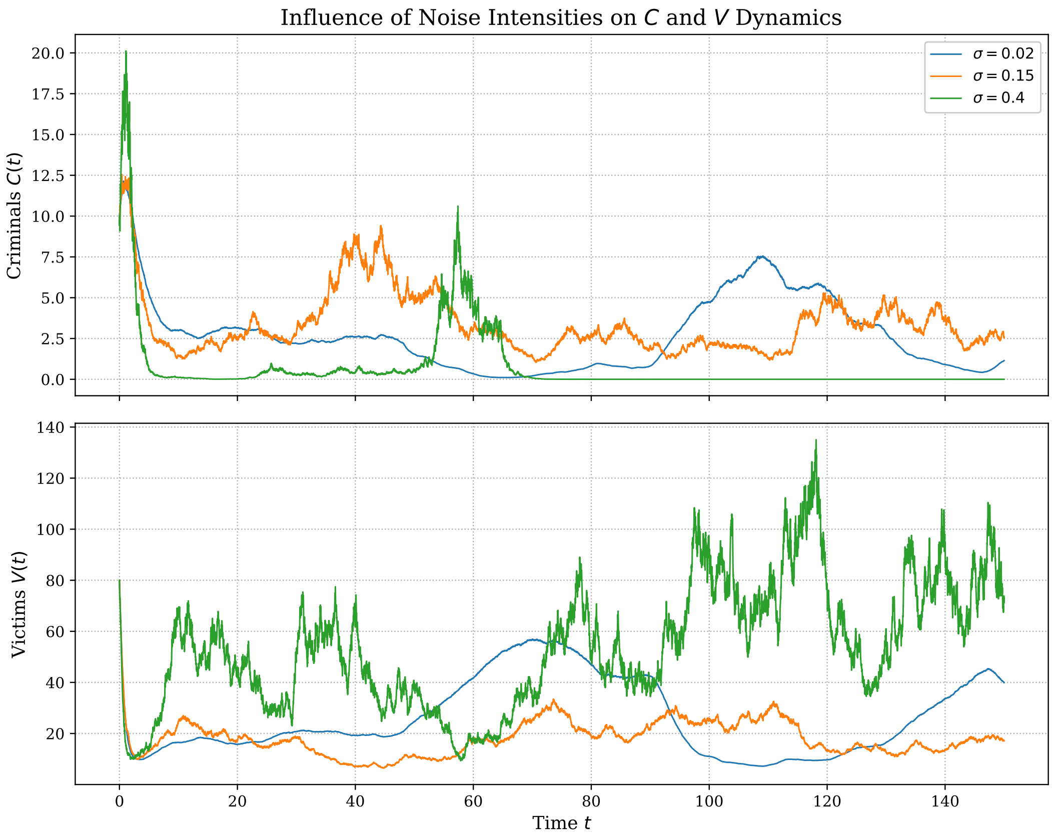

Figure 8. Influence of noise on criminal and victim dynamics.

criminal population $C(t)$ declines toward zero over time, confirming the occurrence of stochastic extinction as established in Theorem 3.5. The information level $I(t)$ also diminishes as criminal activity vanishes, while the victim population $V(t)$ stabilizes around its crime-free level.

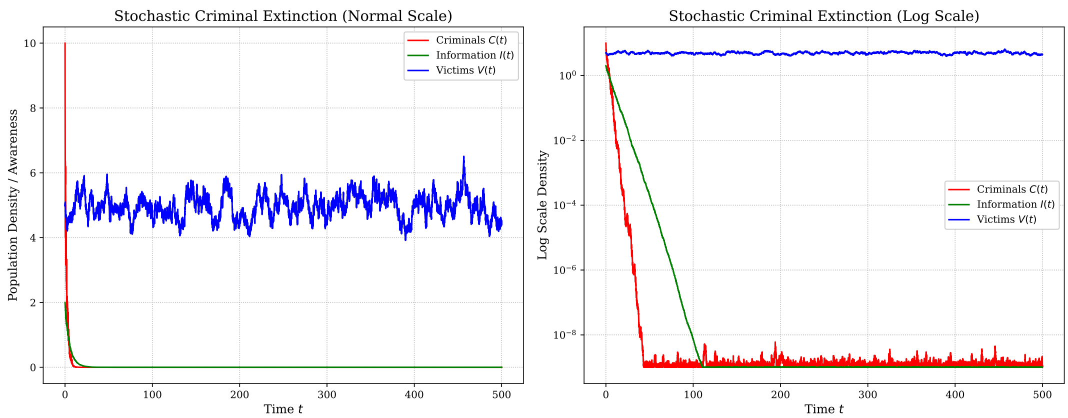

Figure 9. Stochastic extinction of criminal activity when $\tilde{\mathcal{R}}_0 < 1$.

To further validate the persistence condition, Figure 10 presents the system dynamics when $\tilde{\mathcal{R}}_0 > 1$. In this case, the criminal population does not vanish but instead fluctuates persistently around a positive equilibrium. This behavior confirms the existence of a stable endemic regime driven by the balance between criminal growth, victimization, and removal mechanisms under stochastic perturbations.

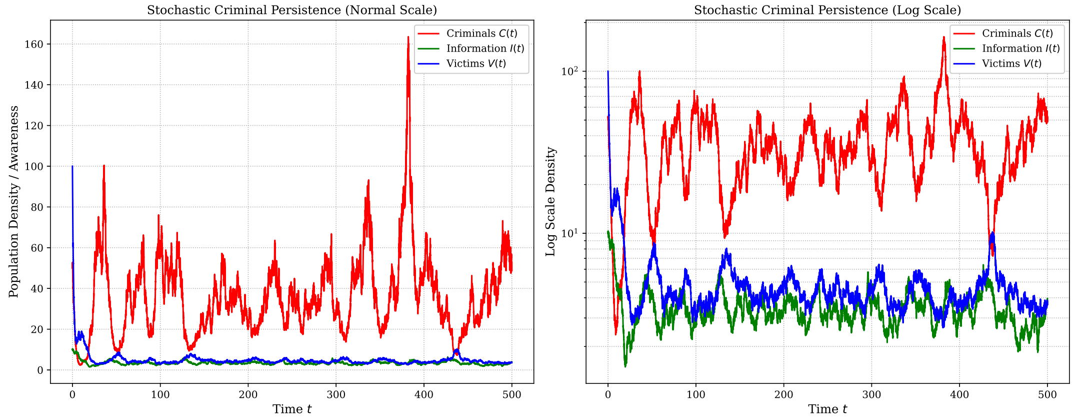

Figure 10. Persistent criminal dynamics when $\tilde{\mathcal{R}}_0 > 1$, showing fluctuations around a positive equilibrium.

Finally, the empirical distributions in Figure 11 confirm the existence of an ergodic stationary distribution. The peaks align with the deterministic endemic equilibrium, while their spread reflects the magnitude of stochastic fluctuations. Overall, the results show that while baseline enforcement (μ) is important, the combined effects of awareness saturation and information-driven removal (α) are equally critical for long-term crime control.

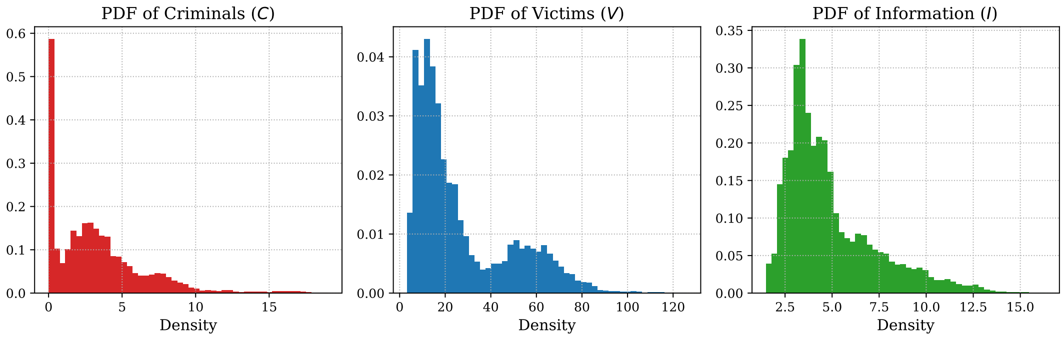

Figure 11. Empirical histograms of $C(t)$, $V(t)$, and $I(t)$ over a long simulation horizon, confirming convergence to an ergodic stationary distribution around the endemic equilibrium.

5 Conclusion

In this study, we developed and analyzed a stochastic mathematical model describing the interaction between criminal activity, victim susceptibility, and information dissemination under the influence of fear-driven behavioral responses. By extending classical predator–prey frameworks to include nonlinear inhibition mechanisms and stochastic perturbations, the model provides a more realistic representation of crime dynamics in uncertain social environments.

From a theoretical perspective, we established the well-posedness of the model by proving the existence, uniqueness, and positivity of solutions. These results ensure that the system is mathematically consistent and suitable for long-term analysis. Furthermore, conditions for extinction and persistence were identified, highlighting the critical role of system parameters in determining whether criminal activity dies out or remains endemic.

The numerical simulations complemented the analytical findings and provided deeper insight into the dynamic behavior of the system. In particular, the results demonstrated that information dissemination and fear-induced behavioral changes act as strong regulatory mechanisms that significantly reduce criminal activity. The interaction between these two factors was shown to be especially important, as their combined effect enhances the responsiveness of the system during periods of high crime. In contrast, formal control measures such as law enforcement were found to be decisive in driving the system toward extinction when sufficiently strong.

Another important observation is that stochastic effects introduce persistent fluctuations and temporal delays in the system. These features reflect real-world conditions where responses to crime are not instantaneous and where external uncertainties continuously influence social behavior. Despite these fluctuations, the system remains bounded and exhibits a stable long-term statistical structure, providing evidence for the existence of a stationary distribution.

Overall, the results of this study emphasize that effective crime control requires a combination of formal intervention and informal social responses. Public awareness campaigns, timely dissemination of information, and adaptive behavioral changes play a crucial role in limiting criminal opportunities, while law enforcement through criminal removal ensures long-term suppression of criminal activity.

Future work may consider extending the model to include spatial dynamics, time delays in information propagation, or optimal control strategies aimed at minimizing crime through cost-effective interventions. Such extensions would further enhance the applicability of the model to real-world policy design and decision-making.

Declarations

Conflict of Interest

The authors declare that there are no conflicts of interest.

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Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Author Contributions

D.N. contributed to conceptualisation, methodology, and software. A.O. contributed to conceptualisation, methodology, and editing. P.P. contributed to conceptualisation, formal analysis, and editing. G.O. contributed to methodology, and editing

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