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Behaviour of Hollow Reinforced Concrete (RC) Beam Elements under Earthquake Force

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ABSTRACT

The behavior of hollow reinforced concrete beam elements under earthquake force was studied and analyzed. The governing partial differential equation of the motion is the Euler-Bernoulli beam equation subjected to the simply supported boundary conditions. Aboodh transform method was used to solve the partial differential equation of the fourth order. The solution of the governing equation of hollow reinforced concrete beam elements under earthquake force was obtained and the behaviour of the beam elements under flexure and ductility were analyzed. The finding of the study revealed that the hollow reinforced concrete beams under flexure and ductility form a uniform support in order to reduce the impact of earthquake on structures which established the stability of the solution.

1 Introduction

Over the years, concrete has been used widely around the world as a key construction material due to many reasons, including the availability of its raw materials, good durability and less required maintenance after construction, the reasonable construction cost, and high mechanical properties in terms of compressive strength in comparing with other materials, e.g., steel and timber. However, concrete has a lower resistance against tensile forces and is heavy. Therefore, it can be used in combination with steel bars to resist external forces based on its own high compressive strength and the high tensile strengths of the latter (Al-Balhawi & Abbass, 2018). This study focuses primarily on hollow reinforced concrete beam.

Reinforced concrete (RC) beam elements are structural elements that designed to carry transverse external loads. The loads cause bending moment, shear forces and in some cases torsion across their length. Moreover, concrete is strong in compression and very weak in tension. Thus, steel reinforcement used to take up tensile stresses in reinforced concrete beam elements. Furthermore, beam elements support the loads from slabs, other beams, walls, and columns. They transfer the loads safely to the columns supporting them. Additionally, beam elements can be simply supported, continuous, or cantilevered and they can be designed as rectangular, square, T-shaped, and L-shaped sections. Beam elements can be singly reinforced or doubly reinforced.

Hollow RC beam elements are those RC beam elements having a longitudinal hole, cavity, or space within. Hollow (RC) beam elements become an innovative development and advantage for economic purposes and modern constructions of light-weight members without largely affecting their resistance against the applied loads. This type of openings can also be used for maintenance purposes and usage space of communication lines, pipelines, etc. Obviously, the provision of openings in the beam elements will result in the reduction of self-weight of beam elements.

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The reinforced concrete beam elements are building materials with relatively higher stiffness, resistant to fire, and possess lower construction costs. In the reinforced concrete structures, pipes are often made conduit or holes are in the beam and column structure for plumbing, mechanical, and electrical installations. In column structure elements, the existence of a hole causes a reduction in cross-sectional area, and this certainly affects its mechanical properties. The study of the effect of holes on columns and beam elements was carried out by (Hoshikuma & Priestley (2000), Kim (2012), Ranzo & Priestley (2000), Zacoeb (2006). The results of the showed that the greater the hole size, the higher the axial load, flexural and column ductility, with less strength, and stiffness. The study also revealed that, it is necessary to limit the size of holes in the column to ensure the structure's safety. While (Abhay (2014), Gaikwad and Kannan (2017) in their investigations, established that the use of hollow column elements in the building can reduce its total weight, prevent earthquake loads, and produce an economic foundation size

Other researchers that carried out investigations on the behaviour of hollow reinforced concrete beam elements include; Al-Nuaimi and Bhatt (2005) who experimentally and numerically investigated the behaviour of hollow RC beams subjected to combined bending, shear and torsion and stated that the proposed numerical model had a good ability to predict the experimental results.

Altun *et al.*, (2006) who experimentally investigated the flexural performance of classic steel fibre reinforced concrete (SFRC) box beams subjected to bending and explored the effects of reduced dead weight, varied thicknesses, and different ratios of the wall thickness to the beam height on the ultimate load capacity of the beams. They stated that a reduction of 44% in the beam weight could lead to the decrease of the ultimate load carrying capacity by 29%. Al-Nuaimi *et al.*, (2008) who experimentally explored solid and hollow reinforced concrete beams and stated that solid beams cracked initially at higher loads than the hollow beams. The hollow beams failed close to the designated loads, while the solid beams failed at higher loading levels. This indicates that the core space can influence the ultimate load resistance of the section in terms of bending, shear, and torsion.

Alshimmeri and Al-Maliki (2014) who experimentally investigated the behaviors of solid and hollow reinforced concrete beams subjected to partial uniformly distributed load and explored the influence of vertical shear steel reinforcement and the size and direction of the openings on the flexural behaviour of the beams in terms of the ultimate load capacity, deflection, and ductility. They stated that as the opening size in the beams increased, the ultimate load capacity decreased from 37.14% to 58.33% for the increased cross-section opening ratio of the beams from 7.4% to 14.8%, respectively, while the corresponding deflection increased from 71.6% to 75.5%. On the other hand, the increase in shear reinforcement ratio led to the enhancement of the ultimate load capacity. The ductility of hollow reinforced concrete beams increases when the opening ratio decreases and the shear steel reinforcement increases.

Al-Gasham (2015) investigated the effect of installing a PVC pipe inside a reinforced concrete beam on the behaviour of the reinforced concrete deep beams and indicated that the pipe diameter below 1/3 of the beam width had limited effect on the capacity and rigidity of the beams. Hauhnar *et al.*, (2017) experimentally and numerically investigated the effect of circular openings in the flexural area on the behaviour of reinforced concrete beams strengthened with steel pipes and indicated that the differences in the experimental and numerical results were only below 10%. Similarly, Murugesan and Narayanan (2018) experimentally examined the effect of longitudinal circular hole on the flexural performance of reinforced concrete beams subjected to four-point bending. The studied parameters were the sizes and positions of holes, while other parameters remained unchanged, e.g., shear span to depth ratio, longitudinal and transverse steel reinforcement ratios, concrete grade, and cross-sectional area. They stated that the failure mode was flexural failure for all hollow reinforced concrete beams. Moreover, they developed a theoretical equation for predicting the flexural strength and first cracking load of hollow reinforced concrete beams. In the following study, Murugesan and Narayanan (2018) evaluated the behaviour of the previously studied beams in terms of the maximum deflection at different loading stages before and after initial cracks and stated that the developed theoretical formulas would be able to reasonably predict the maximum deflection at different stages.

Abbass *et al.*, (2019) investigated the flexural behaviors of solid and hollow reinforced concrete beams subjected to four-point bending, and the studied parameters included the addition of steel fibres, the percentage reduction in the void size, the longitudinal reinforcement ratio and the use of lateral stirrups. They found that the hollow reinforced concrete beams with 1% steel fibre content and a size reduction by 45% due to the opening could be used to replace solid beams, without huge declines in the strength, ductility, and toughness of the beams. Hassan *et al.*, (2020)

experimentally explored the torsional behaviour of hollow reinforced concrete beams strengthened with different types of fibres subjected to pure torsional loading and indicated that the used types and lengths of fibres improved the initial cracking load and ultimate load capacity of the beams.

Balaji and Vetturayasudharsanan (2020) experimentally investigated the flexural performance of hollow reinforced concrete beams subjected to four-point bending by utilizing two types of pipes to create a longitudinal opening in the beams. The investigated parameters included crack pattern, deflection, initial cracking load and ultimate load. Hassan *et al.*, (2020) experimentally investigated the behaviour of hollow reinforced concrete beams with different locations of openings subjected to two concentrated loads and indicated that the opening locations highly affected both the ultimate failure loads and stiffnesses. The tested hollow beams possessed reduced stiffnesses and increased deflections. Moreover, these beams sustained reduced load carrying capacities and ductility index values. El-Kassas *et al.*, (2020) experimentally examined the effect of longitudinal openings on the structural behaviors of reinforced concrete deep beams by varying the location, size, and shape of openings. In general, the load capacity of the beams decreased due to the direct effects of openings. The shapes of openings had insignificant effects on the behaviour of the beams, while the position of the openings had an effective role in governing the structural behaviour of the beams.

Dinesh, Kanna and Arun (2021) in their study conducted on the effects of longitudinal and transverse direction opening in reinforced concrete beam concluded that it is desirable to work on the performance of different types of concrete, investigation of different strengthening materials and techniques to reduce the deflection, or behaviour of hollow reinforced concrete elements under earthquake force. Vijayakumar and Madhavi (2021) experimentally studied the behaviour of hollow reinforced concrete beams strengthened with different types of fibres and indicated that with a specified content of the used fibres, the loading capacity of the hollow beams was significantly enhanced under both compression and flexure.

Elamary *et al.*, (2021) experimentally and numerically investigated the structural performance of hollow reinforced concrete beams with varied areas and locations of hollow cores subjected to three-point loading. They indicated that core areas below 10% of the cross-section areas had a negligible effect on the failure loads of these beams, while the openings had an effect of worsening the crack patterns including crack sizes and heights.

It is noted that most of the studies mentioned have experimentally and numerically investigated the structural behaviors of hollow reinforced concrete beams. Therefore, the effect of hollow reinforced concrete beam elements on buildings needs to be assessed adequately because the response of hollow beam elements structures due to earthquake loads may differ significantly.

2 Materials and Method

2.1 Problem Formulation

The Euler-Bernoulli beam equation is a partial differential equation of the fourth order and is given by (Fryba, 1972)

$$EI \frac{\partial^4 U(x,t)}{\partial x^4} + \rho A \frac{\partial^2 U(x,t)}{\partial t^2} = P(x,t) \quad (1)$$

Where x is the spatial co-ordinate, t is the time interval, E is the elasticity modulus of the beam material, I is the moment of inertia of the beam cross section, ρ is the mass density of the beam, A the area of the beam cross section, $U(x,t)$ is the transverse displacement, $P(x,t)$ is the lateral distributed dynamic load per beam unit length.

In Figure 1, the Euler-Bernoulli beam is shown with $P(x,t)$ as the lateral distributed dynamic load per beam unit length and $U(x,t)$ the transverse displacement.

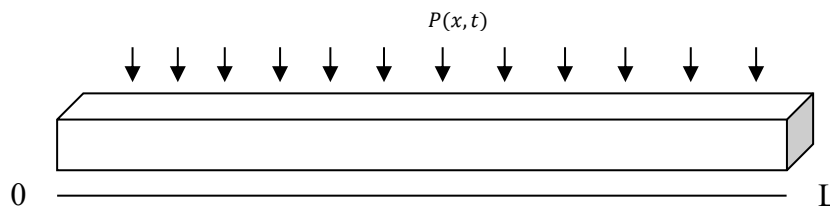


Figure 1: Euler-Bernoulli beam with $P(x,t)$ as the lateral distributed dynamic load per beam

For the problem under consideration, the lateral distributed dynamic load, $P(x, t)$ is the Earthquake force. The earthquake force $P(x, t)$ acts horizontally to the hollow reinforced concrete Euler-Bernoulli beam and is represented in fig. 2

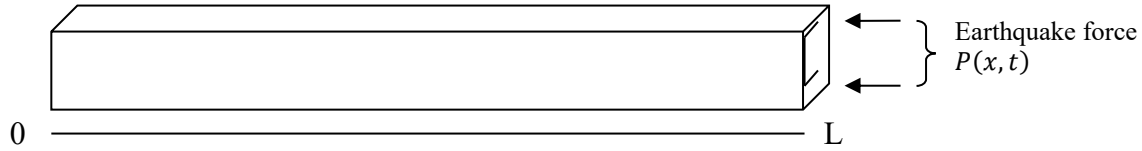


Figure 2: Hollow RC Euler-Bernoulli beam element under earthquake force $P(x, t)$

The expression for the earthquake force acting on the hollow reinforced concrete beam element, obtained using the following diagram.

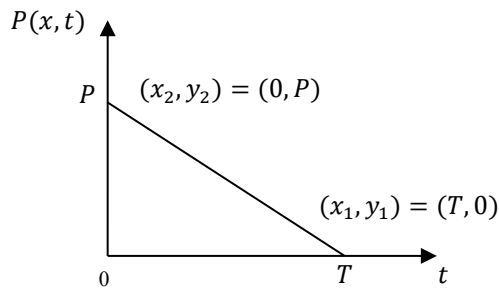


Figure 3: Earthquake loading $P(x, t)$

Where the earthquake loading $P(x, t)$ is idealized as a triangular pulse having a peak force P and positive phase duration T . $P(x, t)$ can be derived from fig. 3 as follows;

Using the equation of a straight line passing through two points,

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \quad (2)$$

from figure (3), it is seen that,

$$\left. \begin{array}{l} y_1 = 0 \\ y_2 = P \\ x_1 = T \\ x_2 = 0 \end{array} \right\} \quad (3)$$

Using (3), one obtains,

$$\frac{P(x,t)-0}{P-0} = \frac{t-T}{0-T} \quad (4)$$

then,

$$P(x, t) = \frac{-P(t-T)}{T} \quad (5)$$

or,

$$P(x, t) = \frac{P(T-t)}{T} \quad (6)$$

$$\text{Hence, the forcing function is given as } P(x, t) = P \left(1 - \frac{t}{T} \right) \quad (7)$$

With the forcing function in (7), equation (1) can be written as;

$$EI \frac{\partial^4 U(x,t)}{\partial x^4} + \rho A \frac{\partial^2 U(x,t)}{\partial t^2} = P \left(1 - \frac{t}{T} \right) \quad (8)$$

The simply supported boundary conditions are,

$$U(0, t) = 0 = U(L, t) \quad (9)$$

$$\text{and } \frac{\partial^2 U(0,t)}{\partial x^2} = 0 = \frac{\partial^2 U(L,t)}{\partial x^2} \quad (10)$$

The initial conditions, without loss of generality is given by

$$U(x, 0) = U_t(x, 0) = 0 \quad (11)$$

Equation (8) is the governing equation of motion of the hollow reinforced concrete beam elements under earthquake force.

2.2 Method

In this section, the Aboodh transformation method is employed to transform equation (8) into an ordinary differential equation. The ordinary differential equation is then tackled using the homotopy perturbation method which is a semi-analytic technique for solving nonlinear differential equations.

The Aboodh transformation method was introduced by Khalid Aboodh to facilitate the process of solving ordinary and partial differential equations in the time domain. It is similar to the Laplace transform and other integral transforms that are defined in the time domain $t \geq 0$, such as the Sumudu transform, the Natural transform and Elzaki transform, respectively.

The Aboodh method is a transformation belonging to a class A,

where,

$$A = \{u(t): \exists M, k_1, k_2 > 0 \text{ s.t. } |u(t)| < Me^{-vt}, t \in (-1)^i \times [0, \infty)\}, \quad (\text{Aboodh, 2013})$$

and,

$$u(t) \text{ is denoted by } A[u(t)] = U(v),$$

$$A\{u(t)\} = \frac{1}{v} \int_0^\infty e^{-vt} u(t) dt = U(v) \quad v \in (k_1, k_2) \quad (12)$$

Some properties of Aboodh transform are given as,

$$\left. \begin{aligned} \text{(i)} \quad & A\{1\} = \frac{1}{v^2} \\ \text{(ii)} \quad & A\{t^n\} = \frac{n!}{v^{n+2}}, n \geq 0 \\ \text{(iii)} \quad & A\{e^{at}\} = \frac{1}{v^2 - av} \\ \text{(iv)} \quad & A\{\sin(at)\} = \frac{a}{v(v^2 + a^2)} \\ \text{(v)} \quad & A\{\cos(at)\} = \frac{1}{(v^2 + a^2)} \\ \text{(vi)} \quad & A\{u^n(t)\} = v^n U(v) - \sum_{k=0}^{n-1} \frac{u^k(0)}{v^{2-n+k}} \end{aligned} \right\} \quad (13)$$

To present the Aboodh transform method, a more general non homogeneous nonlinear partial differential equation is considered;

$$Lu(x, t) + Ru(x, t) + Nu(x, t) = h(x, t) \quad (14)$$

With the initial conditions

$$u(x, 0) = f(x), \quad u_t(x, 0) = g(x) \quad (15)$$

Where L is the second order linear differential operator with an inverse operator, if it exists, as $L^{-1}(\cdot) = \int_0^t \int_0^t (\cdot) dt dt$. R is the remaining linear operator less than L , N is the nonlinear operator and $h(x, t)$ is the nonhomogeneous term.

Now, on taking the Aboodh transform of equation (14) in ' t ' one gets

$$A\{Lu(x, t)\} + A\{Ru(x, t)\} + A\{Nu(x, t)\} = A\{h(x, t)\} \quad (16)$$

From the property of Aboodh transform, equation (3.17) becomes

$$v^2 A\{Lu(x, t)\} - u(x, 0) - \frac{1}{v} u_t(x, 0) + A\{Ru(x, t)\} + A\{Nu(x, t)\} = A\{h(x, t)\}$$

(17)

Which can be simplified as

$$A\{Lu(x, t)\} - \frac{1}{v^2} u(x, 0) - \frac{1}{v^3} u_t(x, 0) + \frac{1}{v^2} A\{Ru(x, t)\} + \frac{1}{v^2} A\{Nu(x, t)\} = \frac{1}{v^2} A\{h(x, t)\} \quad (18)$$

Now, the system's vibration response $u(x, t)$ is characterized by its natural frequency, modal damping factor, and mode shape.

Hence, $u(x, t)$ can be written as an infinite series of u_m 's given by

$$u(x, t) = \sum_{m=0}^{\infty} u_m(x, t) \quad (19)$$

Where $u_m(x, t)$ are the m th normal mode of vibration.

And the nonlinear term N by an infinite series of the Adomian polynomial A_m 's of $u_0, u_1, u_2, \dots, u_m$, given by

$$Nu(x, t) = \sum_{m=0}^{\infty} A_m(u_1, u_2, u_3, \dots) \quad m = 0, 1, 2, \dots \quad (20)$$

Where,

$$A_m = \frac{1}{m!} \frac{d^m}{d\lambda^m} \left[F \left(\sum_{i=0}^{\infty} \lambda^i u_i \right) \right] \quad m = 0, 1, 2, \dots \quad (\text{Adomian, 1995}) \quad (21)$$

Substituting equations (19) and (20) into equation (18) one obtains

$$A \left\{ \sum_{m=0}^{\infty} Lu_m(x, t) \right\} - \frac{1}{v^2} u(x, 0) - \frac{1}{v^3} u_t(x, 0) + \frac{1}{v^2} \left(A \left\{ \sum_{m=0}^{\infty} Ru_m(x, t) \right\} + A \left\{ \sum_{m=0}^{\infty} A_m u_m(x, t) \right\} \right) = \frac{1}{v^2} A\{h(x, t)\} \quad (22)$$

Also, substituting the initial conditions given in equation (15) results to

$$A\{\sum_{m=0}^{\infty} Lu_m(x, t)\} = \frac{1}{v^2} u(x, 0) + \frac{1}{v^3} u_1(x, 0) + \frac{1}{v^2} A\{h(x, t)\} - \frac{1}{v^2} (A\{\sum_{m=0}^{\infty} Ru_m(x, t)\} + A\{\sum_{m=0}^{\infty} A_m u_m(x, t)\}) \quad (23)$$

Finally, comparing coefficients of $u_m(x, t)$ on both sides of equation (23) and, thereafter taking the inverse Aboodh transform, one obtains the general solution given recursively by

$$\begin{cases} Lu_m(x, t) = f(x) + tg(x) + A^{-1} \left\{ \frac{1}{v^2} A\{h(x, t)\} \right\}, m = 0 \\ Lu_{m+1}(x, t) = A^{-1} \left\{ \frac{1}{v^2} A\{Ru_m(x, t) + A_m u_m(x, t)\} \right\}, m \geq 0 \end{cases} \quad (24)$$

Solution of the Problem

The Earthquake force is a simple harmonic motion, hence, the forcing function $F(t)$ for the problem under consideration can be represented as

$$F(t) = F_0(1 - \cos \omega t) \quad (25)$$

where,

$$\begin{aligned} P \left(1 - \frac{t}{T} \right) &= F_0 \left(\cos 0 - \cos 2\pi \frac{t}{T} \right); \quad P = F_0 \\ &= F_0 \left(\cos 0 - \cos \frac{2\pi}{T} t \right) \\ &= F_0 (\cos 0 - \cos \omega t); \quad \omega = 2\pi f, \quad f = \frac{1}{T} \\ &= F_0 (1 - \cos \omega t) \end{aligned}$$

and ω represents the frequency of the earthquake, F_0 the amplitude of the earthquake oscillation.

By Newton's law, earthquake force is thus;

$$F_{\text{extenal}} = ma = m\ddot{x}(t)$$

from (25), we have,

$$F_{\text{extenal}} = ma = m\ddot{x}(t) = -m\omega^2 F_0 \cos(\omega t); \quad (26)$$

where,

$$a = \ddot{x}(t) = -\omega^2 F_0 \cos(\omega t), \text{ the second derivative of (26)}$$

In view of (26), equation (8) becomes

$$EI \frac{\partial^4 U(x, t)}{\partial x^4} + \rho A \frac{\partial^2 U(x, t)}{\partial t^2} = -m\omega^2 F_0 \cos(\omega t) \quad (27)$$

The initial conditions are,

$$U(x, 0) = 0, \quad \frac{\partial U(x, 0)}{\partial t} = 0 \quad (28)$$

And the boundary conditions are,

$$\begin{cases} U(0, t) = 0 = U(L, t) \\ \frac{\partial^2 U(0, t)}{\partial x^2} = 0 = \frac{\partial^2 U(L, t)}{\partial x^2} \end{cases} \quad (29)$$

Now, taking the Aboodh transformation on both sides of equation (27), yields,

$$\begin{aligned} A \left[a_1 \frac{\partial^2 U(x, t)}{\partial t^2} + a_2 \frac{\partial^4 U(x, t)}{\partial x^4} \right] &= A \left[-m \left(\frac{x}{m} \right) F_0 \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] \\ \Rightarrow A \left[a_1 \frac{\partial^2 U(x, t)}{\partial t^2} + a_2 \frac{\partial^4 U(x, t)}{\partial x^4} \right] &= A \left[a_3 x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] \end{aligned} \quad (30)$$

where,

$$a_1 = \rho A, a_2 = EI, a_3 = -F_0, \omega = 2\pi f, f = \frac{1}{2\pi} \sqrt{\frac{x}{m}}, \omega^2 = \frac{x}{m}, \omega = \sqrt{\frac{x}{m}} = \left(\frac{x}{m}\right)^{\frac{1}{2}}$$

$$\text{But } A \left[\frac{\partial^2 f(x,t)}{\partial t^2} \right] = v^2 A [f(x,t)] - f(x,0) - \frac{1}{v} \frac{\partial f(x,0)}{\partial t}$$

$$\text{Thus, } A \left[\frac{\partial^2 U(x,t)}{\partial t^2} \right] = v^2 A [U(x,t)] - U(x,0) - \frac{1}{v} \frac{\partial U(x,0)}{\partial t}$$

Equation (30) now becomes

$$a_1 \left\{ v^2 A [U(x,t)] - U(x,0) - \frac{1}{v} \frac{\partial U(x,0)}{\partial t} \right\} + a_2 \left\{ A \left[\frac{\partial^4 U(x,t)}{\partial x^4} \right] \right\} = a_3 \left\{ A \left[x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] \right\}$$

(31)

Applying the initial condition in (28)

$$\begin{aligned} a_1 \left\{ v^2 A [U(x,t)] - 0 - \frac{1}{v} (0) \right\} + a_2 \left\{ A \left[\frac{\partial^4 U(x,t)}{\partial x^4} \right] \right\} &= a_3 \left\{ A \left[x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] \right\} \\ \Rightarrow a_1 \{ v^2 A [U(x,t)] \} &= a_3 \left\{ A \left[x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] \right\} - a_2 \left\{ A \left[\frac{\partial^4 U(x,t)}{\partial x^4} \right] \right\} \\ \Rightarrow A [U(x,t)] &= \frac{a_3}{a_1} \left\{ \frac{1}{v^2} A \left[x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] \right\} - \frac{a_2}{a_1} \left\{ \frac{1}{v^2} A \left[\frac{\partial^4 U(x,t)}{\partial x^4} \right] \right\} \end{aligned} \quad (32)$$

Taking the Aboodh inverse on both side of equation (32)

$$\begin{aligned} \Rightarrow A^{-1} A [U(x,t)] &= \frac{a_3}{a_1} A^{-1} \left\{ \frac{1}{v^2} A \left[x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] \right\} - \frac{a_2}{a_1} A^{-1} \left\{ \frac{1}{v^2} A \left[\frac{\partial^4 U(x,t)}{\partial x^4} \right] \right\} \\ \Rightarrow U(x,t) &= \frac{a_3}{a_1} A^{-1} \left\{ \frac{1}{v^2} A \left[x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] \right\} - \frac{a_2}{a_1} A^{-1} \left\{ \frac{1}{v^2} A \left[\frac{\partial^4 U(x,t)}{\partial x^4} \right] \right\} \end{aligned} \quad (33)$$

Applying the Homotopy perturbation method

$$U(x,t) = \sum_{m=0}^{\infty} p^n u_m(x,t) \quad (34)$$

Equation (33) now becomes

$$\sum_{m=0}^{\infty} p^n u_m(x,t) = \frac{a_3}{a_1} x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) - p A^{-1} \left\{ \frac{a_2}{a_1} \frac{1}{v^2} A \left[\frac{\partial^4 U(x,t)}{\partial x^4} \right] \right\} \quad (35)$$

Collecting the coefficient of the like powers of p , we have

$$p^0: U_0(x,t) = \frac{a_3}{a_1} x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \quad (36)$$

$$p^1: U_1(x,t) = -\frac{a_2}{a_1} A^{-1} \left\{ \frac{1}{v^2} A \left[\frac{\partial^4 U_0(x,t)}{\partial x^4} \right] \right\} \quad (37)$$

$$p^2: U_2(x,t) = -\frac{a_2}{a_1} A^{-1} \left\{ \frac{1}{v^2} A \left[\frac{\partial^4 U_1(x,t)}{\partial x^4} \right] \right\} \quad (38)$$

$$p^3: U_3(x,t) = -\frac{a_2}{a_1} A^{-1} \left\{ \frac{1}{v^2} A \left[\frac{\partial^4 U_2(x,t)}{\partial x^4} \right] \right\} \quad (39)$$

$$p^4: U_4(x,t) = -\frac{a_2}{a_1} A^{-1} \left\{ \frac{1}{v^2} A \left[\frac{\partial^4 U_3(x,t)}{\partial x^4} \right] \right\} \quad (40)$$

Now, let $U = x \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right)$

$$\frac{\partial U}{\partial x} = \frac{-\left(m^{-\frac{1}{2}}\right)}{2} t \left[x^{\frac{1}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] + \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \quad (41)$$

$$\begin{aligned} \frac{\partial^2 U}{\partial x^2} &= \frac{-\left(m^{-\frac{1}{2}}\right)}{2} t \left[x^{\frac{1}{2}} \left(\frac{-1}{2m^{\frac{1}{2}}} t \right) \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) + \frac{1}{2} x^{-\frac{1}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] - \frac{x^{-\frac{1}{2}}}{2m^{\frac{1}{2}}} t \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \\ &= \frac{-t^2}{4m} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) - \frac{(xm)^{-\frac{1}{2}}}{4} t \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) - \frac{x^{-\frac{1}{2}}}{2m^{\frac{1}{2}}} t \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \end{aligned} \quad (42)$$

$$\begin{aligned} \frac{\partial^3 U}{\partial x^3} &= \frac{-t^2}{4m} \left[-\frac{x^{-\frac{1}{2}}}{2m^{\frac{1}{2}}} t \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right] - \frac{t}{4m^{\frac{1}{2}}} \left[x^{\frac{1}{2}} \cdot \frac{-1}{2} \frac{x^{-\frac{1}{2}}}{2m^{\frac{1}{2}}} t \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) + \left(-\frac{1}{2} x^{-\frac{3}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right) \right] - \\ &\frac{t}{2m^{\frac{1}{2}}} \left[x^{\frac{1}{2}} \cdot \frac{-1}{2} \frac{x^{-\frac{1}{2}}}{2m^{\frac{1}{2}}} t \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) + \left(-\frac{1}{2} x^{-\frac{3}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right) \right] \\ &= \frac{t^3 x^{-\frac{1}{2}}}{8m^{\frac{1}{2}}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) - \frac{t^2 x^{-1}}{8m} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) + \frac{tx^{-\frac{3}{2}}}{8m^{\frac{1}{2}}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) - \frac{t^2 x^{-1}}{4m} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) + \frac{tx^{-\frac{1}{2}}}{4m^{\frac{1}{2}}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \quad (43) \\ \frac{\partial^4 U}{\partial x^4} &= \frac{t^3}{8m^{\frac{1}{2}}} \left[x^{\frac{1}{2}} \cdot \frac{-1}{2} \frac{x^{-\frac{1}{2}}}{2m^{\frac{1}{2}}} t \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) + \left(-\frac{1}{2} x^{-\frac{3}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right) \right] + \frac{t^2}{8m} \left[x^{-1} \left(-\frac{1}{2} \frac{x^{-\frac{3}{2}}}{m^{\frac{1}{2}}} t \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right) + \right. \\ &\left(-x^{-2} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right) \right] + \frac{t}{8m^{\frac{1}{2}}} \left[x^{\frac{1}{2}} \cdot \frac{-1}{2} \frac{x^{-\frac{1}{2}}}{2m^{\frac{1}{2}}} t \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) + \left(-\frac{3}{2} x^{-\frac{5}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right) \right] - \frac{t^2}{4m} \left[x^{-1} - \right. \\ &\left. \frac{1}{2} \cdot \frac{x^{-\frac{1}{2}}}{m^{\frac{1}{2}}} t \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) + \left(-x^{-2} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right) \right] + \frac{t}{4m^{\frac{1}{2}}} \left[x^{\frac{1}{2}} \cdot \frac{-1}{2} \frac{x^{-\frac{1}{2}}}{2m^{\frac{1}{2}}} t \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) + \left(-\frac{3}{2} x^{-\frac{5}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \right) \right] \quad (44) \end{aligned}$$

Substituting equations (41), (42), (43) and (44) into (37) and taking the Aboodh inverse, one have

$$\begin{aligned} U_1(x, t) &= -\frac{a_2}{a_1} \left[\frac{-3m}{8} x^{-\frac{5}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^3}{3!} - \frac{-3m^{\frac{1}{2}}}{4} x^{-\frac{5}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^3}{3!} + \frac{m^{\frac{3}{2}}}{8} x^{-2} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^4}{4!} + \right. \\ &\frac{m^{\frac{1}{2}}}{8} x^{-2} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^4}{4!} + \frac{m}{4} x^{-2} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^4}{4!} + \frac{m}{2} x^{-2} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^4}{4!} - \frac{m^{\frac{1}{2}}}{16} x^{-\frac{3}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^5}{5!} + \\ &\left. \frac{m^2}{8} x^{-\frac{3}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^5}{5!} - \frac{m^{\frac{3}{2}}}{4} x^{-\frac{3}{2}} \sin \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^5}{5!} + \frac{m^{\frac{5}{6}}}{16} x^{-1} \cos \left(\left(\frac{x}{m} \right)^{\frac{1}{2}} t \right) \frac{t^6}{6!} \right] \quad (45) \end{aligned}$$

where,

$\frac{a_2}{a_1} = \frac{EI}{\rho A}$ represent the flexural rigidity/behaviour of the beam. The flexural rigidity (EI) is a measure of the beam's ability to resist bending, and ρA is the mass density.

and,

$\frac{a_3}{a_1} = \frac{P}{\rho A}$ representing ductility. This expression represents the stress in the beam, which is a measure of the loading force per unit area. By using this expression, one can model the ductility of the beam by analyzing how the stress changes as the beam deforms.

Therefore, combining (36) and (45) the solution is

$$U(x, t) = U_0(x, t) + U_1(x, t)$$

$$\begin{aligned}
 U(x, t) = & \frac{a_3}{a_1} x \cos\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) - \frac{a_2}{a_1} \left[\frac{-3m}{8} x^{\frac{-5}{2}} \sin\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^3}{3!} - \frac{-3m^{\frac{-5}{2}}}{4} x^{\frac{-5}{2}} \sin\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^3}{3!} + \frac{m^{\frac{3}{2}}}{8} x^{-2} \cos\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^4}{4!} + \right. \\
 & \frac{m^{\frac{1}{2}}}{8} x^{-2} \cos\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^4}{4!} + \frac{m}{4} x^{-2} \cos\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^4}{4!} + \frac{m}{2} x^{-2} \cos\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^4}{4!} - \frac{m^{\frac{1}{2}}}{16} x^{\frac{-3}{2}} \sin\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^5}{5!} + \\
 & \left. \frac{m^2}{8} x^{\frac{-3}{2}} \sin\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^5}{5!} - \frac{m^{\frac{3}{2}}}{4} x^{\frac{-3}{2}} \sin\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^5}{5!} + \frac{m^{\frac{5}{2}}}{16} x^{-1} \cos\left(\left(\frac{x}{m}\right)^{\frac{1}{2}} t\right) \frac{t^6}{6!} \right] \quad (46)
 \end{aligned}$$

Equation (46) is the semi-analytic solution to the governing equation of the hollow reinforced concrete beam elements under earthquake force. It is an infinite series representing the vibration modes the system.

3 Results and Discussion

Behaviour of hollow reinforced concrete beam elements under earthquake force related to the beam's flexure and ductility has been studied. The default values for the parameters were picked from previous literature and inputted into the MATLAB Program (Analysis of Hollow Reinforced Concrete Beam Elements under Earthquake loading, 09-06-2024). As elucidated by El-Amoury et al. (2020), the ductility value is 2.0-4.0 and for this study, the ductility value of 4.0 was considered. This indicates high level ductility, meaning the beam element can undergo significant deformation before failing. Also, in the context of science and engineering, flexure is the ability of a material to withstand bending forces without failing. El-Amoury et al. (2020) considered a flexure of 10.0-12.0 MPa, but since this study is on earthquake force, a flexure of 12.0 MPa was considered meaning the material can withstand some bending forces.

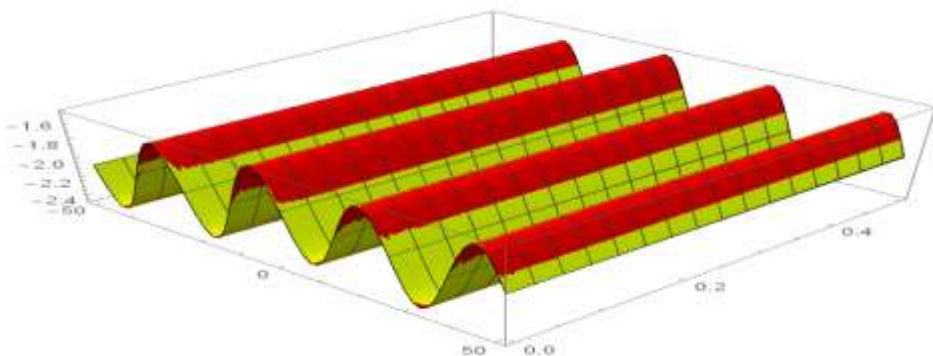
Other parameters are taken from, Thamrin et al. (2022), as shown in the Table 1.

Table 1: Hollow Reinforced Concrete Beam Element Specification

Symbol (Parameter)	Value (Unit)
ρ (density)	7.85kg/m ³
E (young modulus)	210,000 N/mm ²
A (area of cross section)	750m ²
I (moment of inertia)	685,900mm ⁴
ω (frequency)	4.5Hz
t (time)	1.5 sec
P (weight)	10N
M (flexure)	12.0 MPa
μ (ductility)	4

The behaviour of hollow reinforced concrete beam elements under earthquake force when the various parameters in Table 1 are inputted into the MATLAB program, is shown in figure 4.

$U(x, t)$



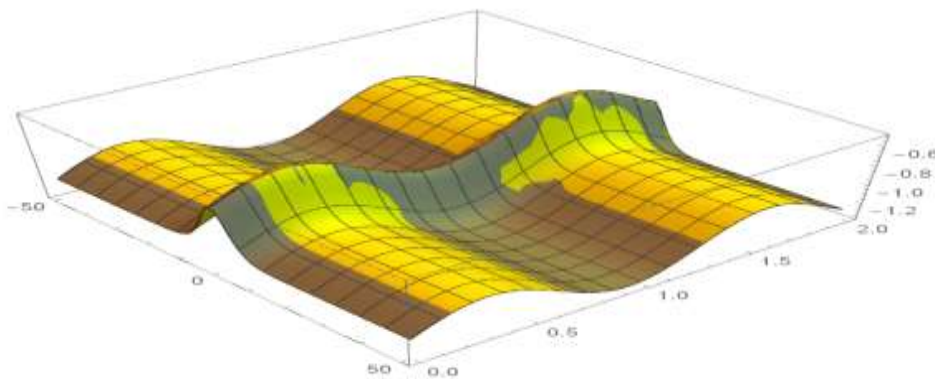
Position x

Figure 4: Behaviour of hollow reinforced concrete beam elements under earthquake force.

It can be seen from the graph the beam elements are uniformly arranged which reduces the damage impact of the earthquake force on building structures.

when $\omega = 4.5\text{Hz}$, $t = 1.5$ sec, flexure = 12.0 MPa and ductility = 4., a plot of the behaviour of hollow reinforced concrete beam element under earthquake force at different displacements $U_0(x, t)$ and $U_1(x, t)$, is depicted in figure 5.

$U(x, t)$



Position x

Figure 5: Behaviour of hollow reinforced concrete beam elements under earthquake force at different displacements, $U_0(x, t)$ and $U_1(x, t)$.

It is seen that as the earthquake force occurs, the reinforced concrete beam elements under ductility and flexure form a uniform support in order to reduce the impact of the earthquake on the building.

when $\omega = 4.5\text{Hz}$, $t = 1.5$ sec, flexure = 12.0 MPa, ductility = 4.0, a plot of the stability of the solution at different time interval (t), is clearly seen in figure 6.

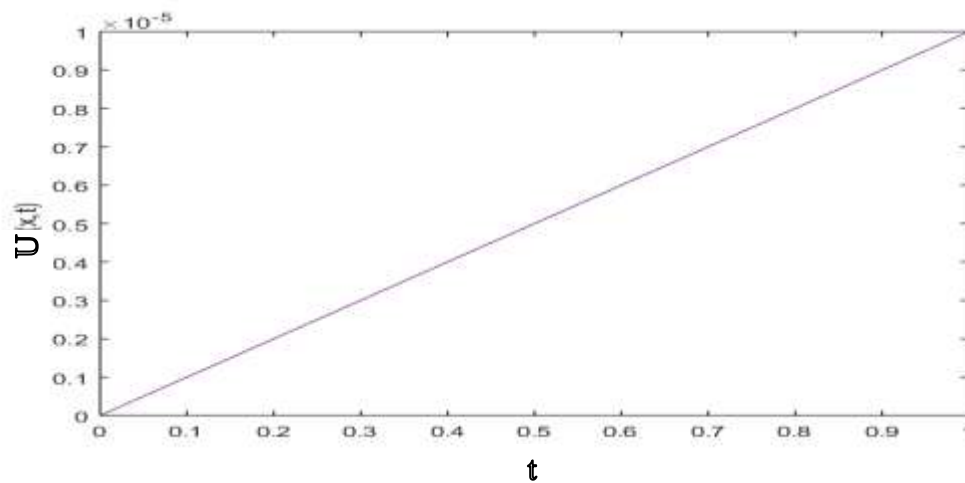


Figure 6: Stability of solution at different time interval

The graph clearly shows a straight line which demonstrates the stability and convergence of the solution. The stability of the solution was verified through the displacement – time graph (Figure 4.3). The graph demonstrates straight line convergence of displacement values over time, Absence of exponential growth or divergence in displacement and predictable vibration response under earthquake excitation. This indicates that the hollow reinforced concrete beam maintains bounded oscillations and does not experience uncontrolled deflection, which is a hallmark of stability in dynamic structural analysis. (Murugesan & Narayanan, (2018); Balaji & Vetturayasudharsanan (2020); Whittaker et al. (1987); Hassan et al. (2020)).

The stability analysis demonstrates that hollow reinforced concrete beams subjected to earthquake forces exhibit bounded displacement, predictable vibration response, and enhanced ductility. The semi-analytical solution converges consistently, and the graphical evidence supports the claim that these beams remain structurally stable under seismic excitation. This stability provides confidence in their application for earthquake-resistant construction.

4 Conclusion

The effects of hollow opening in reinforced concrete beam elements have been studied by various authors as reported in the related literature. Based on the vigorous effort made in this area and the finding of this study, the following conclusions are drawn:

- (i) It could be concluded that the usage of circular-shaped openings is more effective than other shapes of openings since it avoids sharp edges which are subjected to high-stress concentration while loading.
- (ii) The hollow reinforced concrete beams under flexure and ductility forms a uniform support in order to reduce the impact of the earthquake to the building.
- (iii) The solution is stable.

Therefore, based on the findings of this dissertation, the following are recommended:

- (i) Solutions can be obtained up to $U_2(x, t)$, $U_3(x, t)$,...
- (ii) Consider experimental investigation for the purpose of validation.
- (iii) Consider hollow slabs under similar loading conditions.

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