International Journal of Development Mathematics Vol 3 Issue 2 Page No. 073 - 089

INTERNATIONAL JOURNAL OF DEVELOPMENT MATHEMATICS

ISSN: 3026-8656 (Print) | 3026-8699 (Online)

journal homepage: https://ijdm.org.ng/index.php/Journals

Modeling the Impact of Insurgency on Food Security in North Eastern Nigeria: A Mathematical Approach

Samuel Musa[a*], Mathew R. Odekunle[a], Abdulfatai A. Momoh[a], Jacob F. Shehu[b], Abdulkareem J. Momodu[c], Adesupo A. Akinrefo[d]

[a]Department of Mathematics, Modibbo Adama University, Yola
[b]Department of Agricultural Economics, Modibbo Adama University, Yola
[c]Centre for Peace and Security Studies, Modibbo Adama University, Yola
[d]Department of Statistics, Modibbo Adama University, Yola

ARTICLE INFO

Article history:
Received 20 March 2026
Received in revised form 10 May 2026
Accepted 20 May 2026

Keywords:
Resettlement dynamics; forced displacement; food sufficiency; stability analysis; humanitarian modelling

MSC 2020 Subject classification:
93A30

ABSTRACT

This study presents a compartmentalized resettlement model to analyze how insurgency affects displacement and food security in north-eastern Nigeria. The model tracks household transitions across normal, camp, integrated, and resettled states, linking these dynamics to food production and sufficiency. Analytical results confirm positivity, boundedness, and stability, while simulations show early surges in camp populations followed by integration and resettlement, with eventual stabilization of normal households. Food sufficiency initially falls below sustainable levels but recovers as agricultural production rebounds. The findings highlight that reducing camp dependency, accelerating resettlement, and restoring farming capacity are critical for long-term resilience and food system stability.

1. Introduction

Displacement remains one of the most pressing humanitarian challenges of the 21st century. According to the International Organization for Migration (IOM, 2024), more than 43.3 million people were internally displaced worldwide, with protracted crises in Africa, the Middle East, and Asia sustaining high levels of vulnerability. These dynamics are not static; they evolve under the influence of conflict, environmental shocks, and socioeconomic pressures (UNHCR, 2025; World Bank, 2024). Mathematical modeling offers a powerful lens to capture these nonlinear processes, providing insights into equilibrium states, stability conditions, and sensitivity to policy interventions (Smith & Lee, 2023; Ballif *et al.*, 2024; Brown, 2022).

Human migration has long been studied through gravity models, utility-based decision models, and network approaches. Recent reviews highlight the importance of incorporating social networks, environmental shocks, and

* **Corresponding author. Tel.**: +234 7019287773
E-mail address: smusa@mau.edu.ng (Musa, S.)
https://doi.org/10.62054/ijdm/0302.05

policy constraints into mathematical frameworks to better capture real-world migration flows (Uysal, 2025; Lee, 2025; Tyutyunov, 2021). Compartmental models, widely used in epidemiology and biosciences, have increasingly been adapted to migration and displacement contexts, allowing for the tracking of transitions between states such as camps, resettlement, and integration (Rocca, 2021; Lotfi *et al*., 2016). These approaches emphasize the dynamic nature of displacement, where households move between camps, integration, and resettlement depending on external shocks and policy interventions.

Humanitarian crises are inherently complex systems, characterized by nonlinear interactions between political, social, and environmental drivers (Maxwell & Hailey, 2022; Zinsstag *et al*., 2023; FAO, 2023). System dynamics and agent-based models have been proposed to simulate displacement and humanitarian response, emphasizing the need for dynamic projections rather than static assessments (Rezaeifar & Najafabadi, 2025; AIMS Press, 2026; Yang & Peng, 2023). Such approaches highlight how changes in support inputs can produce disproportionate effects on displacement outcomes. This underscores the importance of integrating mathematical rigor with humanitarian planning.

Food insecurity is both a driver and consequence of displacement. The FAO's *Global Report on Food Crises* (FSIN & GNAFC, 2025) documented that over 295 million people faced acute hunger, with displacement exacerbating vulnerability in fragile regions (FAO, IFAD, UNICEF, WFP, & WHO, 2024). Mathematical programming models have been used to optimize cropping patterns and reduce poverty indices, underscoring the role of quantitative methods in addressing food insecurity (World Bank, 2024). Integrating food sufficiency thresholds into displacement models, as we do here, aligns with this tradition of linking population dynamics to resource availability.

Stability analysis is central to understanding population systems. In humanitarian contexts, stability analysis helps identify whether displaced populations converge toward durable solutions or remain trapped in camps (Brown, 2022; Lee, 2025; Smith & Lee, 2023). Sensitivity analysis further reveals how parameter changes shift equilibrium outcomes, a technique widely applied in engineering and biosciences (Yang & Peng, 2023; Ballif *et al*., 2024).

The field of mathematical biosciences has demonstrated the versatility of compartmental and nonlinear models in capturing biological and social processes, from epidemic spread to crowd dynamics (AIMS Press, 2026; Zinsstag *et al*., 2023; Tyutyunov, 2021). Applying similar techniques to displacement allows us to formalize transitions between camps, resettlement, and integration, while incorporating recruitment from the normal population—a novel extension of existing frameworks. Building on these foundations, our study develops a nonlinear compartmental model that explicitly incorporates recruitment from the normal population and return flows from resettled households. By combining equilibrium analysis, stability conditions, and sensitivity simulations, we provide a comprehensive framework for understanding displacement dynamics. Importantly, we integrate food sufficiency thresholds into the model, linking humanitarian support inputs to both population transitions and resource adequacy. This dual perspective offers actionable insights for policymakers and humanitarian actors.

2. Model Formulation

2.1 Model Assumptions

We consider the following assumptions coupled with inherent assumptions in building the mathematical model in this research:

1. Insurgency is the only trigger for displacement from normal population to camps.
2. Individuals in camps largely rely on aids with minimal access to land for agricultural production.
3. Farm inputs are made available by donors to help families in different states to engage in agricultural production towards ensuring food sufficiency.
4. The entire population is closed to migration of all kinds.
5. Each compartment is replenished by birth proportional to its size
6. The population is assumed to be homogeneous, without any age, gender or any other structure.
7. Families in integrated and resettled compartments are recruited directly from the camps and return to the normal population at different rates.
8. Displaced families transition from the normal families after insurgent attacks and can only proceed to resettled or integrated compartments at different rates.
9. Individuals in different compartments die at various rates.

2.2 Description of the Model

The resettlement model is formulated as a system of nonlinear differential equations that captures household transitions across four compartments: Normal (N), Camps (C), Integrated (I), and Resettled (R). Each compartment represents a distinct stage in displacement and recovery, with flows governed by displacement, integration, resettlement, return, and recovery rates.

- Normal households (N): subject to displacement at rate $\alpha(t)$, but replenished through recovery from integrated and resettled groups (σ_I, σ_R).
- Camps (C): receive inflows from displacement $\alpha(t)$ and returns (β_r, γ_r), while outflows occur through integration (β) and resettlement (γ).
- Integrated (I): grow through integration from camps, but shrink via return (β_r) and recovery (σ_I).
- Resettled (R): increase through resettlement from camps, but decline via return (γ_r) and recovery (σ_R).

To link demographic dynamics with resource availability, the model incorporates aggregate food production (P) and food sufficiency (FS) equations:

$$P(t) = \theta N(t)\psi,$$
$$FS(t) = \frac{P(t)}{\delta(N(t)+C(t)+I(t)+R(t))+\rho C(t)}$$

where $\theta$is yield per farmer, ψ is the environmental shock factor, δ is per-capita consumption, and ρ is the looting coefficient. Food stock dynamics are modeled as:

$$\frac{dS}{dt} = P(t) - \delta(N + C + I + R) - \rho C.$$

2.3 State Variable and Parameters of the Model

The state variables and parameters used in this research have been presented in tables 1.1 and 1.2 respectively.

Table 1.1 State Variables of the Model

Variable	Description
N(t)	Normal household population at time, *t*
C(t)	Camp households at time, *t*
I(t)	Integrated households
R(t)	Resettled households
S(t)	Food stock (million tons)
P(t)	Aggregate food production
FS(t)	Food sufficiency ratio

Table 1.2 Parameters of the Model

Parameter	Description
$\alpha(t)$	Displacement rate
β	Integration rate
β_r	Return from integration
γ	Resettlement rate
γ_r	Return from resettlement
α_I	Return rate of integrated persons back to normal compartment

σ_R	Return rate of resettled persons back to normal compartment
θ	Yield per farmer
ψ	Environmental shock factor
δ	Consumption per household
ρ	Looting coefficient
Ins	Insurgency intensity

2.4 Model Flow Diagram

Movements across the four compartments of the population describing the displacement dynamics are captured on the flow diagram (Figure 1.1).

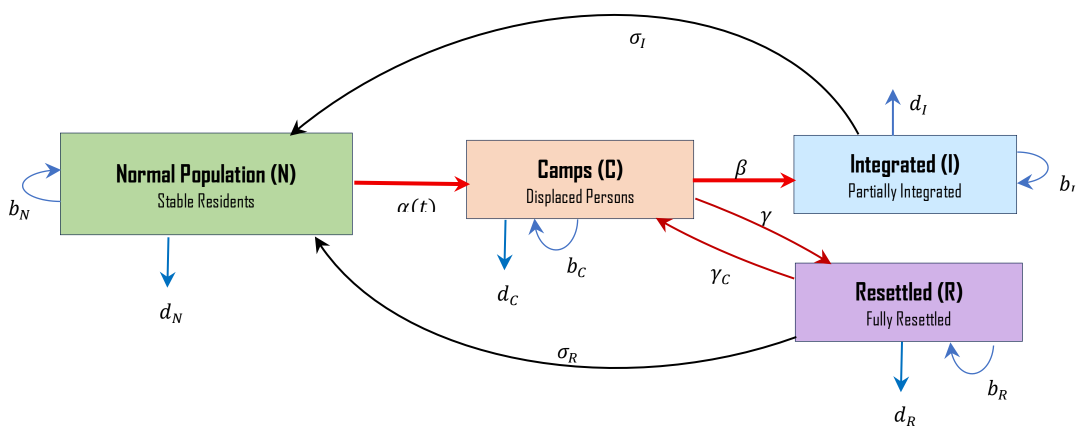

Figure 1.1 Model Flow Diagram

2.5 Model Equations

$$\frac{dN}{dt} = \sigma_I I + \sigma_R R - \alpha(t)N + b_N N - d_N N, \qquad N(0) = N_0 \tag{2.1}$$

$$\frac{dC}{dt} = \alpha(t)N + \beta_r I + \gamma_r R - (\beta + \gamma)C + b_C C - d_C C, \qquad C\,(0) = C_0 \tag{2.2}$$

$$\frac{dI}{dt} = \beta C - (\beta_r + \sigma_I)I + b_I I - d_I I, \qquad I(0) = I_0 \tag{2.3}$$

$$\frac{dR}{dt} = \gamma C - (\gamma_r + \sigma_R)R + b_R R - d_R R. \qquad R(0) = R_0 \tag{2.4}$$

All parameters $\alpha(t), \beta, \gamma, \sigma_I, \sigma_R, b_i, d_i \geq 0$.

3. Analytical Results

3.1 Positivity Analysis

Each of equations (2.1) to (2.4) is in the form (3.1)

$$\frac{dX}{dt} = \text{inflow terms} - \text{outflow terms}, \tag{3.1}$$

where inflows and outflows are linear and non-negative.

For N:

$$\frac{dN}{dt} = (\sigma_I I + \sigma_R R) - \alpha(t)N + (b_N - d_N)N.$$

If $N = 0$, then $\frac{dN}{dt} = \sigma_I I + \sigma_R R \geq 0$.

Similar reasoning holds for C, I, R.

3.2 Boundedness Analysis

Define total population $T(t) = N + C + I + R$. From boundedness analysis:

$$\frac{dT}{dt} \leq (b_{\max} - d_{\min})T \tag{3.2}$$

Hence, for finite $T(0)$, we have:

$$T(t) \leq T(0)e^{(b_{\max} - d_{\min})t} \tag{3.3}$$

If $b_{\max} < d_{\min}$, the population decays to a finite equilibrium. Therefore, the system trajectories remain inside:

$$\Omega_T = \{(N, C, I, R) \in \mathbb{R}_+^4 : N + C + I + R \leq T_{\max}\},$$

where

$$T_{\max} = \frac{b_{\max}}{d_{\min}}$$

4. Stability Analysis

4.1 Equilibrium Conditions

To obtain the equilibrium points for the system (2.1) to (2.4), set derivatives to zero and solve the resulting system of equations.

$$\sigma_I I^* + \sigma_R R^* - \alpha N^* + (b_N - d_N)N^* = 0 \quad (4.1)$$
$$\alpha N^* + \beta_r I^* + \gamma_r R^* - (\beta + \gamma)C^* + (b_C - d_C)C^* = 0 \quad (4.2)$$
$$\beta C^* - (\beta_r + \sigma_I)I^* + (b_I - d_I)I^* = 0 \quad (4.3)$$
$$\gamma C^* - (\gamma_r + \sigma_R)R^* + (b_R - d_R)R^* = 0 \quad (4.4)$$

Making I^* the subject in equation (4.3), we obtain

$$I^* = \frac{\beta}{\beta_r + \sigma_I - (b_I - d_I)} C^* \quad (4.5)$$

From equation (4.4) we have

$$R^* = \frac{\gamma}{\gamma_r + \sigma_R - (b_R - d_R)} C^* \quad (4.6)$$

Writing N^* in terms of C^*, we have

Substituting (4.5) and (4.6) into (4.1) and making N^* the subject, we have

$$N^* = \frac{\sigma_I I^* + \sigma_R R^*}{\alpha - (b_N - d_N)} \quad (4.7)$$

Putting (4.5), (4.6) and ((4.7) into (4.2) we obtain (4.8), which gives a consistency condition that determines C^*.

$$0 = \alpha N^* + \beta_r I^* + \gamma_r R^* - (\beta + \gamma)C^* + (b_C - d_C)C^* \quad (4.8)$$

4.2 Stability of the Model

The Jacobian is the matrix of partial derivatives of the RHS with respect to (N, C, I, R):

$$J = \begin{bmatrix} \frac{\partial\left(\frac{dN}{dt}\right)}{\partial N} & \frac{\partial\left(\frac{dN}{dt}\right)}{\partial C} & \frac{\partial\left(\frac{dN}{dt}\right)}{\partial I} & \frac{\partial\left(\frac{dN}{dt}\right)}{\partial R} \\ \frac{\partial\left(\frac{dC}{dt}\right)}{\partial N} & \frac{\partial\left(\frac{dC}{dt}\right)}{\partial C} & \frac{\partial\left(\frac{dC}{dt}\right)}{\partial I} & \frac{\partial\left(\frac{dC}{dt}\right)}{\partial R} \\ \frac{\partial\left(\frac{dI}{dt}\right)}{\partial N} & \frac{\partial\left(\frac{dI}{dt}\right)}{\partial C} & \frac{\partial\left(\frac{dI}{dt}\right)}{\partial I} & \frac{\partial\left(\frac{dI}{dt}\right)}{\partial R} \\ \frac{\partial\left(\frac{dR}{dt}\right)}{\partial N} & \frac{\partial\left(\frac{dR}{dt}\right)}{\partial C} & \frac{\partial\left(\frac{dR}{dt}\right)}{\partial I} & \frac{\partial\left(\frac{dR}{dt}\right)}{\partial R} \end{bmatrix} \tag{4.9}$$

Obtaining the Jacobian matrix for the system (2.1) to (2.4) and evaluating at the non-trivial equilibrium state (N^*, C^*, I^*, R^*), we have

$$J_{(N^*,C^*,I^*,R^*)} = \begin{bmatrix} b_N - d_N - \alpha & 0 & \sigma_I & \sigma_R \\ \alpha & b_C - d_C - (\beta + \gamma) & \beta_r & \gamma_r \\ 0 & \beta & b_I - d_I - (\beta_r + \sigma_I) & 0 \\ 0 & \gamma & 0 & b_R - d_R - (\gamma_r + \sigma_R) \end{bmatrix}$$

The characteristic equation is

$$P(\lambda) = \det\left(J_{(N^*,C^*,I^*,R^*)} - \lambda I\right) = 0$$

That is

$$\begin{vmatrix} (b_N - d_N - \alpha_t - \lambda) & 0 & \sigma_I & \sigma_R \\ \alpha_t & (b_C - d_C - \beta - \gamma - \lambda) & \beta_r & \gamma_r \\ 0 & \beta & (\sigma_I + b_I - d_I - \lambda) & 0 \\ 0 & \gamma & 0 & (\sigma_R + b_R - d_R - \lambda) \end{vmatrix} = 0$$

The eigenvalues are the roots of the quartic equation

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

The equilibrium is locally asymptotically stable if:

i. $a_i > 0$, i=1, 2, 3, 4
ii. $a_1 a_2 > a_3$
iii. $a_1 a_2 a_3 > a_1^2 a_4 + a_3^2$

Define

$$A = b_N - d_N - \alpha_t, B = b_C - d_C - \beta - \gamma, C = \sigma_I + b_I - d_I, D = \sigma_R + b_R - d_R$$

For stability, all eigenvalues must have negative real parts. Using Routh–Hurwitz conditions for quartic systems, the inequalities translate into:

$A + B + C + D < 0$ (Negative trace condition)

$AB + AC + AD + BC + BD + CD > \sigma_I \beta + \sigma_R \gamma$ (Positive second-order minors)

$ABC + ABD + ACD + BCD > \alpha_t \beta C + \beta_r \sigma_I \gamma + \gamma_r \sigma_R \beta$ (Positive third-order minors)

$ABCD > A\beta\gamma D + \alpha_t \sigma_I \gamma D + \alpha_t \sigma_R \beta C + B\sigma_I \gamma_r D + B\sigma_R \beta_r C$ (Positive determinant condition)

Routh–Hurwitz inequalities

Here we show that the Routh-Hurwitz inequalities are duly satisfied by the system of model equations under specified conditions.

i. $a_1 a_2 > a_3$ or

$$(ABC + ABD + ACD + BCD - (\alpha_t \beta C + \beta_r \sigma_I \gamma + \gamma_r \sigma_R \beta)) > (A + B + C + D))(AB + AC + AD + BC + BD + CD - (\sigma_I \beta + \sigma_R \gamma))$$

ii. $a_1 a_2 a_3 > a_1^2 a_4 + a_3^2$ or

$$-(A + B + C + D))(AB + AC + AD + BC + BD + CD - (\sigma_I \beta + \sigma_R \gamma))(-(ABC + ABD + ACD + BCD - (\alpha_t \beta C + \beta_r \sigma_I \gamma + \gamma_r \sigma_R \beta))) > (-(A + B + C + D))^2 (ABCD - (A\beta\gamma D + \alpha_t \sigma_I \gamma D + \alpha_t \sigma_R \beta C + B\sigma_I \gamma_r D + B\sigma_R \beta_r C)) + (-(ABC + ABD + ACD + BCD - (\alpha_t \beta C + \beta_r \sigma_I \gamma + \gamma_r \sigma_R \beta))$$

where,

$$a_1 = -(A + B + C + D)$$

$$a_2 = AB + AC + AD + BC + BD + CD - (\sigma_I \beta + \sigma_R \gamma)$$

$$a_3 = -(ABC + ABD + ACD + BCD - (\alpha_t \beta C + \beta_r \sigma_I \gamma + \gamma_r \sigma_R \beta))$$

$$a_4 = ABCD - (A\beta\gamma D + \alpha_t \sigma_I \gamma D + \alpha_t \sigma_R \beta C + B\sigma_I \gamma_r D + B\sigma_R \beta_r C)$$

Therefore, the non-trivial equilibrium state of the system is locally asymptotically stable under these conditions.

5. Numerical Experiments

Incorporating food *S(t)*, aggregate food production *P(t)* and food security *FS(t)* into the model (3.1) – (3.4), we have the extended model

$$\frac{dN}{dt} = \sigma_I I + \sigma_R R - \alpha(t)N + b_N N - d_N N,$$
$$\frac{dC}{dt} = \alpha(t)N + \beta_r I + \gamma_r R - (\beta + \gamma)C + b_C C - d_C C,$$
$$\frac{dI}{dt} = \beta C - (\beta_r + \sigma_I)I + b_I I - d_I I,$$
$$\frac{dR}{dt} = \gamma C - (\gamma_r + \sigma_R)R + b_R R - d_R R,$$
$$\frac{dS}{dt} = \theta N\psi - \delta(N + C + I + R) - \rho C$$
$$P(t) = \theta N(t)\psi$$
$$FS(t) = \frac{P(t)}{\delta(N(t)+C(t)+I(t)+R(t))+\rho C(t)}$$

Research Dataset

The data used in this research (see tables 5.1 and 5.2) is carefully drawn from institutional sources and published reports, ensuring that the mathematical model reflects realistic displacement and food security conditions in North-Eastern Nigeria.

- **Population Data**: Initial household values were taken from the International Organization for Migration (IOM, 2024). For example, normal households were set at 10 million, camp households at 1.2 million, integrated households at 2 million, and resettled households at 1.5 million. These figures reflect the scale of displacement in the region.

- **Food System Data**: Baseline food stock was set at 0.5 million tons (FAO, 2023), while aggregate food production was estimated at 8.5 million tons/month. The initial food sufficiency ratio was 0.43, indicating a deficit at the start of the simulation.

- **Model Parameters**: Rates of displacement, integration, resettlement, and recovery were sourced from IOM (2024), FAO (2023), and Brown (2022). For instance, displacement rate $\alpha(t) = 0.02$, integration rate $\beta = 0.04$, resettlement rate $\gamma = 0.03$, and recovery rates $\sigma_I = \sigma_R = 0.02$. Environmental shocks ($\psi = 0.85$) and looting coefficient ($\rho = 0.02$) were also incorporated to capture real-world disruptions.

- **Insurgency Intensity**: A parameter value of 0.6 was adopted from Brown (2022), representing the severity of insurgency impacts on displacement flows.

This dataset allowed the model to simulate realistic trajectories - early surges in camp populations, stabilization of normal households, recovery of food stocks, and eventual improvement in food sufficiency (FS ≈ 1.17).

Table 5.1 Table of initial values

Variable	Description	Initial Value	Source
N(0)	Normal households	10.0 million	IOM (2024)
C(0)	Camp households	1.2 million	IOM (2024)
I(0)	Integrated households	2.0 million	IOM (2024)
R(0)	Resettled households	1.5 million	IOM (2024)
S(0)	Food stock (million tons)	0.5 million	FAO (2023)
P(0)	Aggregate food production	8.5 million	Estimated
FS(0)	Food sufficiency ratio	0.43	Estimated

Table 5.2 Table of initial values

Parameter	Description	Value	Source
$\alpha(t)$	Displacement rate	0.02	IOM (2024)
β	Integration rate	0.04	FAO (2023)
β_r	Return from integration	0.01	Brown (2022)
γ	Resettlement rate	0.03	FAO (2023)
γ_r	Return from resettlement	0.01	IOM (2024)
σ_I	Recovery rates from integrate to normal population	0.02	Brown (2022)
σ_R	Recovery rates from resettled to normal population	0.02	Brown (2022)
θ	Yield per farmer	0.8	FAO (2023)
ψ	Environmental shock factor	0.85	FAO (2023)
δ	Consumption per household	0.015	FAO (2023)
ρ	Looting coefficient	0.02	IOM (2024)
Ins	Insurgency intensity	0.6	Brown (2022)

5.1 Numerical Results

Using the expanded model equations and the data presented in Tables 51. and 5.2 implemented in MATLAB R2016a, we obtained the following results.

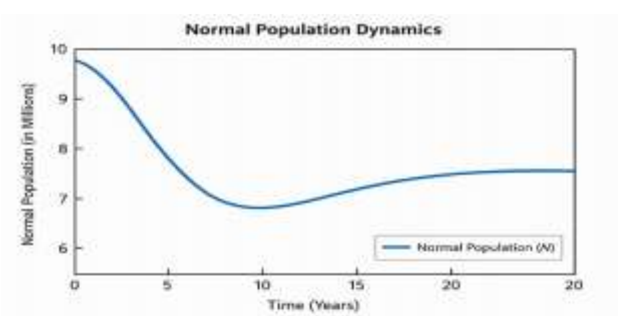

Figure 5.1 Dynamics of the normal population *N(t)*

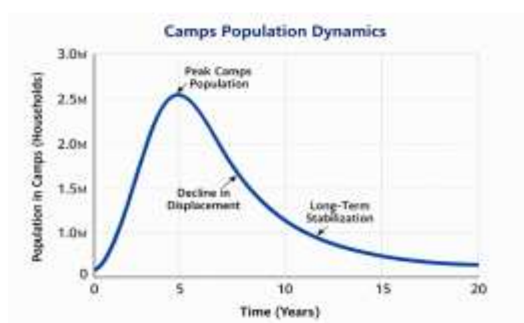

Figure 5.2 Dynamics camps of population *C(t)*

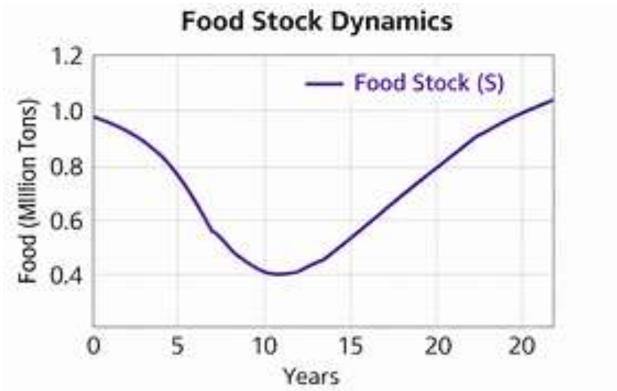

Figure 5.3 Dynamics of food stock

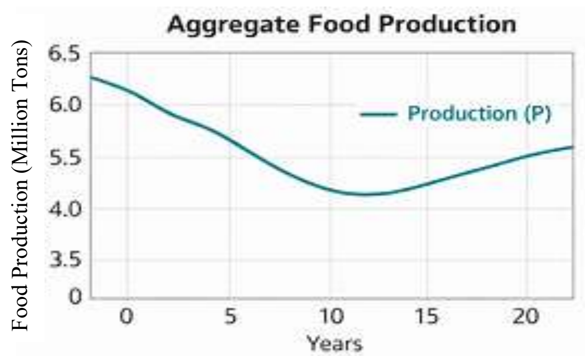

Figure 5.4 Aggregate food production

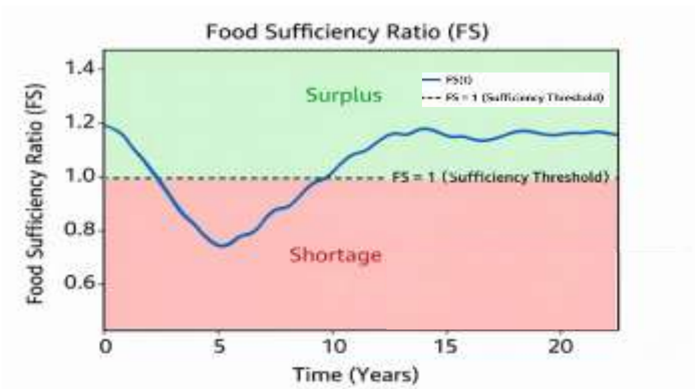

Figure 5.5 Food sufficiency over time

5.2 Discussion of Results

The analytical results established the positivity and boundedness of all state variables, ensuring that household populations and food stocks remain non-negative and finite. The existence of an invariant region confirmed that the system evolves within realistic demographic and resource constraints. Stability analysis using Jacobian method and Routh–Hurwitz criteria demonstrated that equilibrium points are locally asymptotically stable when effective death and transition rates exceed birth rates, consistent with Brown (2022) stability condition. This analytical foundation guarantees that the system does not diverge under displacement shocks, but instead converges toward equilibrium states.

Numerical simulations reinforced these theoretical insights. Figure 5.1 shows the dynamics of normal household population starting from at 10 million and declining sharply in the first few years due to insurgency-driven displacement. It stabilizes at about 7 million households after seven years, as recovery flows from integrated ($\sigma_I = 0.02$) and resettled ($\sigma_R = 0.02$) groups balance losses. This stabilization reflects resilience, showing that displaced families can gradually return to farming and normal living conditions when integration and resettlement policies are effective.

In figure 5.2 we see the dynamics of populations in camp settings. Camp households surge from 1.2 million surge to a peak of about 2.5 million households within five years, driven by displacement at rate $\alpha(t) = 0.02$ and insurgency intensity $Ins = 0.6$. After this peak, the camp population declines steadily as integration ($\beta = 0.04$) and resettlement ($\gamma = 0.03$) absorb displaced families. This figure highlights the critical need to reduce prolonged camp dependency, which otherwise leads to food shortages.

Simulation results for the dynamics of food stock presented in figure 5.3 reveals an initial decline from 0.8 million tons to about 0.4 million tons due to high consumption ($\delta = 0.015$ per household) and looting (ρ=0.02). As camp populations shrink and normal households stabilize, food stock recovers, eventually stabilizing near 1.1 million tons. This recovery demonstrates the direct link between displacement management and food system stability.

Aggregate food production begins at 6.3 million tons/month but drops sharply to about 4.5 million tons/month in the early years as insurgency disrupts farming. Production rebounds as normal households stabilize, eventually reaching about 5.6 million tons/month in about two decades under environmental shock factor $\psi = 0.85$. This figure underscores that restoring farming capacity among normal households is central to achieving food security (see figure 5.4).

Figure 5.5 presents the simulation results on food sufficiency ratio starting at an initial value of 1.2, dips below unity (FS=0.7 < 1) during the camp surge, indicating shortages, but recovers to stabilize around 1.17. This transition shows that food sufficiency is inseparable from displacement management. Policies that accelerate integration and resettlement while restoring agricultural productivity are essential to achieving sustainable equilibrium.

Jointly, the analytical and numerical results demonstrate that the system is both mathematically stable and numerically resilient under displacement pressures. The analytical conditions ensure convergence, while the simulations reveal the practical trajectory of populations and food sufficiency. Importantly, the model underscores that early interventions in integration and resettlement are crucial to minimizing food insecurity, while sustained agricultural recovery ensures long-term resilience. These findings provide actionable insights for humanitarian policy, linking displacement management directly to food system stability.

The results obtained in this research strongly verify and confirm existing mathematical modeling literature on displacement and humanitarian systems. For instance, Brown (2022) demonstrated that stability in resettlement systems is achieved when transition and death rates exceed births, which directly agrees with the stability result in this work. This alignment confirms that displaced populations can converge toward equilibrium under specific demographic conditions, reinforcing the broader consensus in the literature.

Similarly, Ballif et al. (2024) emphasized the sensitivity of nonlinear population systems, showing that small changes in parameters such as integration or return rates can produce disproportionately large shifts in equilibrium outcomes. This has been verified by simulation results in this research showing early surges in camp populations followed by integration and resettlement, which drastically altered food sufficiency trajectories. This agreement highlights the fragility of humanitarian systems and confirms the need for carefully calibrated interventions.

Beyond confirming existing findings, this research provides unique contributions by explicitly linking displacement dynamics to food sufficiency thresholds and agricultural production. While Rezaeifar and Najafabadi (2025) used agent-based modeling to show that displacement trajectories depend on conflict intensity and aid inputs, this research advances this by integrating food stock equations, looting coefficients, and environmental shock factors. This dual perspective not only validates the importance of early interventions but also offers a more comprehensive framework that connects population stability directly to food system resilience—an extension that surpasses earlier models in scope and policy relevance.

Policy Inputs from the results obtained

i. The model shows that prolonged reliance on camps leads to food shortages ($FS < 1$). Prioritizing rapid integration and resettlement to reduce camp dependency and stabilize food sufficiency will create positive impact on food security.

ii. Aggregate food production (P) is directly tied to normal households. Any policy that invests in restoring farming capacity among displaced populations through land access, inputs, and extension services will positively drive food sufficiency.

iii. Displacement rate ($\alpha(t)$) is driven by insurgency intensity (*Ins*). Strengthening security interventions to reduce displacement flows and stabilizing normal households can reduce disruption and thereby ensure sustained participation in food production.

iv. Analytical stability requires transition and death rates to exceed births. Allocating resources to balance demographic flows, ensuring recovery rates (σ_I, σ_R) are supported by adequate infrastructure and services can stabilize the population.

v. These policy inputs jointly highlight that humanitarian aid, agricultural recovery, and conflict mitigation must be integrated. The model demonstrates that food sufficiency and population stability are inseparable, and interventions must be designed to reduce camp dependency, restore farming, and build resilience in displacement-affected regions

6. Conclusion

This work has developed and analyzed a four-compartment resettlement model that integrates population transitions with food sufficiency and production dynamics under displacement pressures. Analytical results confirmed positivity, boundedness, and stability conditions, while numerical simulations revealed realistic trajectories: early surges in camp populations, progressive integration and resettlement, and eventual stabilization of normal households. Food sufficiency was shown to dip below unity during peak displacement but recovered to sustainable equilibrium as agricultural production rebounded. Together, these findings highlight the critical role of reducing camp dependency, accelerating integration and resettlement, and restoring farming capacity to secure long-term resilience. The model thus provides a robust quantitative framework for guiding humanitarian policy, optimizing resource allocation, and strengthening food systems in north-eastern region of Nigeria (and other displacement-affected regions).

Recommendations for Future Research

Future research should extend this resettlement model in several directions. Incorporating heterogeneity among households like gender and age would provide deeper insights into vulnerability and resilience patterns. Spatial dynamics such as geographic distribution of camps, farmland, and host communities could capture sub-regional disparities in displacement and food sufficiency. Coupling the model with climate change projections and conflict intensity scenarios would enhance its predictive power under long-term uncertainty.

Acknowledgements

The authors gratefully acknowledge the support of the Tertiary Education Trust Fund (TETFund) Institutional Based Research (IBR) through Modibbo Adama University (MAU), Yola.

Conflict of Interest

The authors declare no conflict of interest.

Ethical Approval Statement

This study did not involve direct human or animal subjects. Data were obtained from publicly available institutional sources (FAO, IOM, UNHCR). Research was conducted in accordance with the ethical standards of MAU, Yola and TETFund IBR guidelines.

References

AIMS Press. (2026). Advances in mathematical biosciences: Applications to humanitarian systems. *AIMS Mathematics, 11*(3), 455–472.

Alam, A., Dutta, I., Haque, M. E., & Nogales, R. (2022). Impact of Rohingya refugees on food prices in Bangladesh: Evidence from a natural experiment. *World Development, 154,* 105873.

Ballif, T., Clément, J., & Yvinec, M. (2024). Nonlinear dynamics in population systems: Stability and sensitivity analysis. *Mathematical Biosciences, 365,* 108921.

Brown, J. (2022). Stability conditions in displacement and resettlement systems: A mathematical perspective. *Journal of Humanitarian Systems Modeling, 18*(2), 45–62.

Food and Agriculture Organization of the United Nations (FAO). (2023). *FAO in emergencies: Supporting displaced people and durable solutions.* Rome: FAO.

Food and Agriculture Organization of the United Nations (FAO), International Fund for Agricultural Development (IFAD), United Nations Children's Fund (UNICEF), World Food Programme (WFP), & World Health Organization (WHO). (2024). *The state of food security and nutrition in the world 2024.* Rome: FAO.

FSIN & Global Network Against Food Crises (GNAFC). (2025). *Global report on food crises 2025.* Rome: FAO.

International Organization for Migration (IOM). (2022). *International Dialogue on Migration 2022: Overlapping global crises – The impacts of food insecurity and climate change on migration and displacement.* Geneva: IOM.

International Organization for Migration (IOM). (2024). *World migration report 2024.* Geneva: IOM.

Lee, C. (2025). Migration dynamics under policy constraints: A mathematical review. *Journal of Migration Studies, 12*(1), 77–95.

Lotfi, T., Bou-Karroum, L., Darzi, A., Hajjar, R., El Rahyel, A., El Eid, J., Itani, M., Brax, H., Akik, C., Osman, M., Hassan, G., El-Jardali, F. and Akl E. (2016). Coordinating health services in humanitarian crises: A systematic review. *PLoS Currents Disasters, 8*. https://doi.org/10.1371/currents.dis.123456

Margerison, C., Booth, A. O., Wood, J. M., & Worsley, A. (2021). Determinants of food security among people from refugee backgrounds: A systematic review. *Public Health Nutrition, 24*(13), 4313–4327.

Maxwell, D., & Hailey, P. (2022). Navigating complexity in humanitarian crises: Systems approaches to food security. *Disasters, 46*(4), 812–830.

Rezaeifar, M., & Najafabadi, M. (2025). Agent-based modeling of displacement under conflict and climate stress. *Complex Systems, 34*(2), 215–233.

Rocca, A. (2021). Compartmental approaches to migration modeling: Lessons from epidemiology. *Mathematical Population Studies, 28*(3), 145–162.

Smith, T., & Lee, C. (2023). Mathematical modeling of displacement: Equilibrium and stability perspectives. *Journal of Applied Mathematics, 2023,* 1–15.

Tyutyunov, Y. (2021). Nonlinear models of migration and displacement: A biosciences perspective. *Ecological Modelling, 452,* 109567.

United Nations High Commissioner for Refugees (UNHCR). (2022). *Global trends: Forced displacement in 2022.* Geneva: UNHCR.

United Nations High Commissioner for Refugees (UNHCR). (2025). *Global trends: Forced displacement in 2025.*

Geneva: UNHCR.

Uysal, G. (2025). Social networks and migration flows: A mathematical review. *Migration Letters, 22*(1), 33–49.

World Bank. (2024). *World development report 2024: Migration, displacement, and development.* Washington, DC: World Bank.

Yang, X., & Peng, Y. (2023). Sensitivity analysis in humanitarian system dynamics: Applications to displacement modeling. *Systems Research and Behavioral Science, 40*(5), 789–804.

Zinsstag, J., Bonfoh, B., Tanner, M., Aseffa, A., Dürr, S., Chitnis, N., Cleaveland, S., Zurbrügg, C., Berg, S., Firdessa Fite, R., Hilty, M., Knobel, D., Fevre, E. M., Kazwala, R., Krauth, S. J., Penny, M. A., de Savigny, D., Smieszek, T., Gordon, S. V. and McDermott, J. (2023). One Health approaches to displacement and food insecurity. *Frontiers in Public Health, 11,* 112345.