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Mathematical Model of the Effect of Out-of-School Children on the Development of Northern Nigeria

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Abstract

Out-of-school children represent a significant developmental challenge in Northern Nigeria, where low enrollment and high dropout rates perpetuate cycles of poverty and underdevelopment. This paper presents a deterministic compartmental model that captures the dynamic relationship between educational attainment and socioeconomic development. The model stratifies the population into six compartments: school-age children not yet enrolled, in-school children, out-of-school children, skilled individuals, economic productivity index, and development index. The positivity and boundedness of solutions are established to ensure biological feasibility. We obtain the out-of-school-free equilibrium and endemic equilibrium, and calculate the basic reproduction number $\mathcal{R}_0$ using the

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next-generation matrix approach. Stability analysis reveals that the out-of-school-free equilibrium is locally asymptotically stable if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$. Sensitivity analysis identifies key parameters driving the persistence of out-of-school children. The model reveals critical policy insights, including threshold conditions for intervention effectiveness and development self-sustainability.

1 Introduction

Education is widely recognized as a fundamental driver of socioeconomic development, serving as a catalyst for poverty reduction, improved health outcomes, and sustainable economic growth. The United Nations Sustainable Development Goal 4 (SDG 4) explicitly aims to "ensure inclusive and equitable quality education and promote lifelong learning opportunities for all" (United Nations, 2015). Despite global progress, millions of children remain out of school, with Sub-Saharan Africa bearing the largest burden. Nigeria, particularly the northern regions, faces one of the highest rates of out-of-school children globally, with estimates suggesting over 10 million children are not enrolled in formal education (UNESCO, 2022; UNICEF, 2023).

The challenge of out-of-school children in Northern Nigeria is deeply intertwined with multiple socioeconomic factors. Poverty, child labor, insecurity, cultural norms, and inadequate school infrastructure collectively contribute to low enrollment and high dropout rates (Federal Ministry of Education, 2019; Human Rights Watch, 2022). The scale of this crisis is staggering: Nigeria currently has an estimated 18.3 million out-of-school children, the highest number globally, with one in every five out-of-school children worldwide being Nigerian (UNICEF, 2023). Of this alarming figure, between 70 and 80 percent are concentrated in Northern Nigeria, translating to over 12 million children (UNICEF, 2023). The North-West and North-East regions bear the heaviest burden, accounting for approximately 66 percent of the nation's out-of-school population, with states like Yobe (43%), Zamfara (41%), and Sokoto (37%) recording the highest proportions of children who never attend school (UNICEF, 2023). The Almajiri system remains the single largest structural contributor, with Almajiri children accounting for between 72 and 81 percent of Nigeria's total out-of-school population (Human Rights Watch, 2022).

The regional breakdown of the crisis reveals a deeply entrenched educational emergency across Northern Nigeria. In the **North-West**, which has the second highest out-of-school rate in Nigeria, the situation in Kano, Jigawa, and Katsina states is particularly alarming (UNICEF, 2025). According to the Multiple Indicator Cluster Survey (MICS 2021), 32 percent of primary school-going age children are out of school in Kano State alone (UNICEF, 2025). Across these three states, there are 1,863,207 out-of-school children at the primary level and 1,176,918 at the junior secondary level, with Kano accounting for close to one million out-of-school children (989,234), Katsina with 536,112, and Jigawa with 337,861 (UNICEF, 2025; Guardian, 2024). The crisis in the **North-East** is compounded by conflict and insurgency: 2.8 million children are in need of education-in-emergencies support across the three most conflict-affected states (Borno, Yobe, Adamawa), where at least 802 schools remain closed, 497 classrooms have been destroyed, and another 1,392 damaged but repairable (UNICEF, 2025). In the **North-Central**, states like Niger have an estimated 640,000 out-of-school children (Guardian, 2024). Nationally, primary school-age children who are out of school number 10.2 million, with 16 percent of them concentrated in just Kano, Jigawa, and Katsina states alone (UNICEF, 2025). More alarmingly, foundational learning rates stand at 11 percent or lower across these three states: only 2 percent of primary school learners in Jigawa have foundational reading skills and less than 1 percent have numeracy skills; while Kano's figures of 9.6 percent for reading proficiency and 11.2 percent for numeracy remain far below the national averages of 26 percent and 25 percent, respectively (UNICEF, 2025). These statistics reveal that even for children who are enrolled, the quality of education remains critically inadequate.

The consequences of this educational deficit extend far beyond the classroom, manifesting in a direct and self-reinforcing relationship with the escalating insecurity that has gripped Northern Nigeria. As Governor Muhammadu Inuwa Yahaya, Chairman of the Northern States Governors' Forum, warned: "In a country where 60 per cent of the population is below the age of 35, a hungry, jobless and illiterate youth population is nothing more than a cheap recruitment pool for bandits and Boko Haram" (Northern States Governors Forum, 2026). The Northern States Governors' Forum has explicitly identified widespread poverty, youth unemployment, and the growing number of out-of-school children as major factors fuelling insecurity across the region (Northern States Governors Forum, 2026). Nasarawa State Governor Abdullahi Sule similarly warned that many of the minors recruited into banditry and other criminal activities are products of the Almajiri system, stressing that urgent action is needed to break the cycle of poverty, illiteracy, and insecurity (Sule, 2026). The link is direct, vicious, and self-reinforcing: insecurity keeps children out of school; out-of-school children become easy recruits for bandits, insurgents, and criminal gangs; and the resulting violence closes more schools (Leadership, 2026).

The manifestations of this crisis are visible across the region in various forms of organized criminality. In Adamawa State, the notorious *Shila Boys* — violent street gangs consisting of boys and girls typically aged between 18 and 25 years — have terrorized residents, engaging in phone snatching, shop breaking, and armed robbery (Shila Boys, 2026). These youths, armed with cutlasses, knives, and other dangerous weapons, harass innocent citizens by forcibly collecting their belongings such as mobile phones, laptops, and other valuables (Shila Boys, 2026). The phenomenon is so widespread that Adamawa State ranks third in substance abuse in North-Eastern Nigeria, with 370,000 drug users, further fueling the criminal activities (Yan Shila Study, 2026). The underlying factors driving this crisis are clear: lack of adequate western education among the vast population of youths, combined with unemployment rates of 54.89% in Adamawa State, has created a generation with no productive means of livelihood, making crime an inevitable recourse (Yan Shila Study, 2026). Studies confirm that it is largely the non-educated population that currently constitutes the greatest threat to the vast majority of the state (Yan Shila Study, 2026). Similarly, in the Federal Capital Territory, the *one chance* kidnapping and robbery syndicates have exploited a transportation gap to terrorize residents, particularly women. In one recent operation, a syndicate abducted a female victim and collected

N8.8 million in ransom before releasing her (Punch One Chance, 2026); in another, a six-member gang operating along the Kubwa Expressway and Maitama Federal Housing Road kidnapped and robbed five female victims, collecting a total of N12.8 million in ransom (Newswatch One Chance, 2026). These criminals deliberately target women because, as one suspect admitted, "they are not struggling" (Punch One Chance, 2026). The victims are often beaten, tied up, forced to transfer money from their accounts, and in some cases, thrown out of moving vehicles (Vanguard One Chance, 2026).

Government efforts to address this crisis have been characterized by a mix of well-intentioned but poorly implemented policies, inconsistent funding, and competing priorities that have largely failed to stem the tide. While the Federal Government has adopted the Kaduna State household mapping model to replace estimate-based planning with verified data for identifying out-of-school children across the 36 states (see News Agency of Nigeria, 2026, for household mapping initiative), and Northern governors have called for legislation to abolish the Almajiri system and establish a Northern Nigeria Security Trust Fund to protect schools from insecurity (Radio Nigeria, 2026), these initiatives have been undermined by governance failures. Reports indicate that in some states, billions are allocated to pilgrimage subsidies and mass wedding programs while education remains chronically underfunded, with security votes of up to N56 billion withdrawn across northern states in a single nine-month period without public accountability (Punch, 2026). The legacy of abandoned Almajiri schools from previous administrations serves as a cautionary tale of policy announcements without sustained implementation (The Cable, 2025). The recent suspension of all social media activities by the Nigerian Government in August 2026 and the declaration of a state of emergency in 10 northern states (News Agency of Nigeria, 2026) reflect the severity of the crisis, yet such measures often prioritize military intervention over addressing the root causes of the educational crisis. Furthermore, the declaration of a state of emergency in education in some states (Radio Nigeria, 2026) has yet to translate into tangible improvements in enrollment and retention rates. These factors create a vicious cycle where low educational attainment perpetuates poverty, which in turn reinforces barriers to education. The consequences extend beyond individual life chances, affecting community development, economic productivity, and social stability (Psacharopoulos and Patrinos, 2018; World Bank, 2020).

Mathematical modeling has emerged as a powerful tool for understanding complex social and epidemiological dynamics, evaluating intervention strategies, and informing policy decisions (Hethcote, 2000; Brauer and Castillo-Chavez, 2012). The application of compartmental modeling approaches, originally developed for infectious disease epidemiology, has been extended to educational and developmental contexts (Acemoglu et al., 2005; Lutz et al., 2008). These models allow researchers to capture the dynamic interactions between population subgroups, identify critical thresholds, and assess the impact of various interventions.

Several studies have employed mathematical models to examine educational dynamics. Birachi et al. (2019) developed a model for primary school dropout in Tanzania, identifying key factors influencing dropout rates. Similarly, Okuonghae and Okuonghae (2018) proposed a mathematical model for the dynamics of out-of-school children in Nigeria, incorporating socio-economic factors. However, these models did not explicitly link educational outcomes to broader development indices or consider the feedback mechanisms between development and educational participation.

Recent advances in mathematical epidemiology have provided frameworks for analyzing disease transmission dynamics, with concepts such as the basic reproduction number $\mathcal{R}_0$ proving invaluable for understanding threshold behavior (Van den Driessche and Watmough, 2002). These concepts have been adapted to educational contexts, where $\mathcal{R}_0$ represents the average number of new out-of-school children generated by a single out-of-school child (Mushayabasa, 2017). This framing enables the identification of conditions under which out-of-school children persist or are eliminated from the population.

The current study addresses gaps in existing literature by developing a comprehensive mathematical model that explicitly links educational dynamics with economic productivity and development. The model incorporates state-dependent transition rates, capturing how development influences enrollment and how out-of-school children impact development. Specific objectives are to: (1) formulate a six-compartment model capturing the dynamics of school-age children, in-school children, out-of-school children, skilled individuals, economic productivity, and development; (2) establish qualitative properties including positivity and boundedness; (3) determine equilibrium points and compute the basic reproduction number; (4) conduct stability analysis; (5) perform sensitivity analysis; and (6) derive policy-relevant threshold conditions.

This paper is organized as follows. Section 2 presents the mathematical model formulation, including variables, parameters, and governing equations. Section 3 establishes the basic properties of the model, including positivity and boundedness of solutions. Section 4 presents the theoretical analysis, including equilibrium points, reproduction numbers, and stability results. Section 5 discusses policy implications and threshold conditions. Section 6 provides conclusions and recommendations.

2 Model Formulation

The proposed model stratifies the population into six mutually-exclusive compartments to describe the dynamics of out-of-school children and their impact on development. These compartments are: school-age children not yet enrolled (S), in-school children (I), out-of-school children (O), skilled individuals (H), economic productivity index (E), and development index (D). Individuals enter the school-age population at a constant rate Λ. School-age children may enroll in school at rate $\alpha(D)$, becoming in-school children, or drop out at rate $\theta(S, I, O, D)$, becoming out-of-school children. Natural exit occurs at rate μ in all compartments, and aging out without enrollment occurs at rate γ for school-age children. In-school children graduate at rate σ and enter the skilled population (H). Out-of-school children may re-enroll at rate $\rho(G, A)$, returning to the in-school population, or exit through natural mortality at rate μ, or be removed through targeted interventions at rate ν. Skilled individuals exit the workforce at rate δ. Economic productivity improves through skilled labor at rate ϕ and declines due to the economic burden of out-of-school children at rate ψ and natural depreciation at rate ξ. Development increases through skilled human capital at rate η and economic productivity at rate η_1, and decreases through the negative impact of out-of-school children at rate ω and natural depreciation at rate κ. The state variables and parameters are presented in Tables 1 and 2 respectively. The equations for the out-of-school children model is given by the following deterministic system of nonlinear differential equations in (1).

$$\frac{dS}{dt} = \Lambda - \alpha(D)S - \theta(S, I, O, D)S - \mu S - \gamma S \tag{1}$$
$$\frac{dI}{dt} = \alpha(D)S + \rho(G, A)O - \sigma I - \mu I \tag{2}$$
$$\frac{dO}{dt} = \theta(S, I, O, D)S - \rho(G, A)O - \mu O - \nu O \tag{3}$$
$$\frac{dH}{dt} = \sigma I - \delta H \tag{4}$$
$$\frac{dE}{dt} = \phi H - \psi O - \xi E \tag{5}$$
$$\frac{dD}{dt} = \eta H + \eta_1 E - \omega O - \kappa D \tag{6}$$

with initial conditions:

$$S(0) = S_0 \geq 0, \qquad I(0) = I_0 \geq 0, \qquad O(0) = O_0 \geq 0,$$
$$H(0) = H_0 \geq 0, \qquad E(0) = E_0 \geq 0, \qquad D(0) = D_0 \geq 0.$$

Equation (1) describes the dynamics of school-age children not yet enrolled: school-age children enter at rate Λ and leave through enrollment ($\alpha(D)S$), becoming out-of-school ($\theta(S, I, O, D)S$), natural exit (μS), or aging out without enrollment (γS). Equation (2) describes the in-school children: they increase through enrollment from school-age children and re-enrollment of out-of-school children ($\rho(G, A)O$), and decrease through graduation (σI) and natural exit (μI). Equation (3) governs the out-of-school children population: they increase through dropout ($\theta(S, I, O, D)S$) and decrease through re-enrollment ($\rho(G, A)O$), natural exit (μO), and targeted interventions (νO). Equation (4) models the skilled population: skilled individuals increase through graduation (σI) and decrease through workforce exit (δH). Equation (5) describes the economic productivity index: it improves through skilled labor (ϕH) and declines due to the economic burden of out-of-school children (ψO) and natural depreciation (ξE). Equation (6) models the development index: it increases through skilled human capital (ηH) and economic productivity ($\eta_1 E$), and decreases through the negative impact of out-of-school children (ωO) and natural depreciation (κD).

Recognizing that transition rates depend on socioeconomic factors, we define the enrollment rate $\alpha(D) = \alpha_0 + \alpha_1 D(t)$, where α_0 is the baseline enrollment rate and α_1 captures how improved development enhances enrollment through better school infrastructure, teacher availability, and community awareness; the dropout rate $\theta(S, I, O, D) = \theta_0 + \theta_1(O(t)/N(t)) + \theta_2(1 - D(t))$, where θ_0 is the baseline dropout rate, θ_1 captures the peer influence effect (higher concentration of out-of-school children increases dropout), and θ_2 captures the development deprivation effect (lower development increases dropout due to poverty, child labor, and insecurity); and the re-enrollment rate $\rho(G, A) = \rho_0 + \rho_1 G(t) + \rho_2 A(t)$, where ρ_0 is the baseline re-enrollment rate, ρ_1 is the effectiveness of government intervention programs (conditional cash transfers, school feeding), ρ_2 is the effectiveness of awareness campaigns, $G(t)$ is the government intervention intensity ($0 \leq G \leq 1$), and $A(t)$ is the awareness campaign intensity ($0 \leq A \leq 1$).

Table 1. Description of Model State Variables.

Variable	**Description**
$S(t)$	Number of school-age children not yet enrolled at time t
$I(t)$	Number of in-school children at time t
$O(t)$	Number of out-of-school children at time t
$H(t)$	Number of skilled individuals (graduates) at time t
$E(t)$	Economic productivity index at time t
$D(t)$	Development index at time t
$N(t)$	Total population at time t, where $N = S + I + O + H$

Table 2. Description of Model Parameters.

Parameter	**Description**
Λ	Recruitment rate into school-age population
α_0	Baseline enrollment rate
α_1	Development-induced enrollment enhancement
θ_0	Baseline dropout rate
θ_1	Peer influence on dropout
θ_2	Development deprivation effect on dropout
ρ_0	Baseline re-enrollment rate
ρ_1	Government intervention effectiveness
ρ_2	Awareness campaign effectiveness
σ	Graduation rate
μ	Natural exit/mortality rate
γ	Aging out rate (never enrolled)
δ	Skilled workforce exit rate
η	Contribution of skilled individuals to development
η_1	Economic productivity contribution coefficient
ω	Negative impact of out-of-school children
κ	Development depreciation rate
ϕ	Economic productivity from skilled labor
ψ	Economic loss from out-of-school children
ξ	Economic productivity depreciation rate
ν	Targeted intervention rate

3 *Basic Properties of the Model*

All associated parameters and state variables of the model system (1)-(6) are non-negative for all $t \geq 0$ since it tracks the temporal dynamics of the populations of school-age children, in-school children, out-of-school children, skilled individuals, economic productivity, and development. We assert that the model has the following non-negativity result:

3.1 Existence and Uniqueness of Solutions

The existence and uniqueness of the solution of model system (1)-(6) will be established using the Lipschitz criteria. First, we rewrite the system in vector form. Let the state vector $\mathbf{Y} \in \mathbb{R}^6_+$ be defined as:

$$\mathbf{Y} = (y_1, y_2, y_3, y_4, y_5, y_6)^T$$

where:

$$y_1 = S, \quad y_2 = I, \quad y_3 = O, \quad y_4 = H, \quad y_5 = E, \quad y_6 = D.$$

Then the model system (1)-(6) can be written in the compact vector form:

$$\frac{d\mathbf{Y}}{dt} = \boldsymbol{\chi}(\mathbf{Y}), \qquad \mathbf{Y}(0) = \mathbf{Y}_0$$

where $\boldsymbol{\chi} : \mathbb{R}^6_+ \to \mathbb{R}^6$ is given componentwise by:

$$\begin{aligned}
\chi_1 &= \Lambda - \alpha(D)S - \theta(S, I, O, D)S - \mu S - \gamma S, \\
\chi_2 &= \alpha(D)S + \rho(G, A)O - \sigma I - \mu I, \\
\chi_3 &= \theta(S, I, O, D)S - \rho(G, A)O - \mu O - \nu O, \\
\chi_4 &= \sigma I - \delta H, \\
\chi_5 &= \phi H - \psi O - \xi E, \\
\chi_6 &= \eta H + \eta_1 E - \omega O - \kappa D,
\end{aligned} \tag{7}$$

with $\alpha(D)$, $\theta(S, I, O, D)$, and $\rho(G, A)$ as defined in Section 2.

Theorem 3.1. *In the region* $0 \leq \mathbf{Y} \leq V \in \Omega$*, the model system* (1)-(6) *has a unique solution provided that*

$$\frac{\partial \chi_i}{\partial y_j} \quad (i, j = 1, 2, \ldots, 6)$$

is bounded and continuous in Ω*.*

Proof. We demonstrate that $\boldsymbol{\chi}$ satisfies the Lipschitz condition on the feasible region Ω. By the Picard-Lindelöf theorem, if all partial derivatives $\partial\chi_i/\partial y_j$ are continuous and bounded on Ω, then $\boldsymbol{\chi}$ is Lipschitz continuous, guaranteeing the existence and uniqueness of a local solution. We compute the partial derivatives of χ_1 with respect to each state variable:

$$\begin{aligned}
&\left|\frac{\partial \chi_1}{\partial S}\right| = |-(\alpha(D)+\theta(S,I,O,D)+\mu+\gamma)| = \alpha(D)+\theta(S,I,O,D)+\mu+\gamma < \infty, \\
&\left|\frac{\partial \chi_1}{\partial I}\right| = \left|-\frac{\partial \theta}{\partial I}S\right| = \left|-\theta_1\frac{S}{N}\right| \leq \theta_1 < \infty, \\
&\left|\frac{\partial \chi_1}{\partial O}\right| = \left|-\frac{\partial \theta}{\partial O}S\right| = \left|-\theta_1\frac{S}{N}\right| \leq \theta_1 < \infty, \\
&\left|\frac{\partial \chi_1}{\partial H}\right| = 0 < \infty, \quad \left|\frac{\partial \chi_1}{\partial E}\right| = 0 < \infty, \quad \left|\frac{\partial \chi_1}{\partial D}\right| = |-(\alpha_1+\theta_2)S| = (\alpha_1+\theta_2)S < \infty.
\end{aligned} \tag{8}$$

A similar computation is carried out for $\chi_2, \chi_3, \chi_4, \chi_5$, and χ_6. Since every partial derivative $\partial\chi_i/\partial y_j$ is bounded and continuous on the compact region Ω, there exists a Lipschitz constant $L > 0$ such that:

$$\|\boldsymbol{\chi}(\mathbf{Y}) - \boldsymbol{\chi}(\tilde{\mathbf{Y}})\| \leq L\|\mathbf{Y} - \tilde{\mathbf{Y}}\|$$

for all $\mathbf{Y}, \tilde{\mathbf{Y}} \in \Omega$. Therefore, by the Picard-Lindelöf theorem, there exists a unique local solution to the initial value problem. Furthermore, from Theorem 3.2 (the boundedness and positive invariance result), the solution remains bounded and positively invariant in Ω, guaranteeing global existence of a unique solution. □

3.2 Positivity and Boundedness of Solutions

Theorem 3.2. *For every time $t > 0$, all solutions of the out-of-school children model system (1)-(6) with non-negative starting data stay non-negative.*

Proof. Let

$$t_1 = \sup\{t > 0 : S > 0, I \geq 0, O \geq 0, H \geq 0, E \geq 0, D \geq 0 \in [0,t]\}.$$

Thus, $t_1 > 0$. The model system's first equation (1) indicates that:

$$\frac{dS}{dt} = \Lambda - \alpha(D)S - \theta(S,I,O,D)S - \mu S - \gamma S \geq -[\alpha(D)+\theta(S,I,O,D)+\mu+\gamma]S$$

which can be written as:

$$\frac{d}{dt}\left[S(t)\exp\left\{\int_0^t \alpha(D(u))du + \int_0^t \theta(u)du + (\mu+\gamma)t\right\}\right] \geq 0.$$

Hence,

$$S(t_1) \geq S(0)\exp\left\{-\int_0^{t_1} \alpha(D(u))du - \int_0^{t_1} \theta(u)du - (\mu+\gamma)t_1\right\} > 0.$$

Similarly, it can be demonstrated that for $t \geq 0$, all of the model system's remaining state variables (2)-(6) are non-negative (for all non-negative beginning conditions). As a result, for all times $t \geq 0$, all solutions of the model system (1)-(6) with non-negative beginning conditions remain non-negative. $\square$

Theorem 3.3. *Consider the closed sets*

$$\begin{aligned} \Omega_S &= \left\{ (S, I, O, H) \in \mathbb{R}^4_+ : N(t) \leq \frac{\Lambda}{\mu} \right\}, \quad \Omega_E = \left\{ E \in \mathbb{R}_+ : E \leq \frac{\phi\Lambda}{\xi\mu} \right\}, \\ \Omega_D &= \left\{ D \in \mathbb{R}_+ : D \leq \frac{(\eta + \eta_1\phi)\Lambda}{\kappa\mu} \right\}, \end{aligned} \tag{9}$$

the region $\Omega = \Omega_S \times \Omega_E \times \Omega_D$ *is attractive and positively invariant in relation to the model system* (1)-(6).

Proof. Adding the human sub-populations S, I, O, and H from model system (1)-(6) gives:

$$\dot{N}(t) = \Lambda - \mu(S + I + O + H) - \gamma S - \nu O. \tag{10}$$

Given that the model system's parameters (1)-(6) are all non-negative, it follows from (10) that:

$$\dot{N}(t) \leq \Lambda - \mu N. \tag{11}$$

Hence, $N(t) \leq \frac{\Lambda}{\mu}$. Consequently, it follows from equation (11) using the comparison theorem (?), that:

$$N(t) \leq N(0)e^{-\mu t} + \frac{\Lambda}{\mu}\left(1 - e^{-\mu t}\right).$$

Since $N \leq \frac{\Lambda}{\mu}$, it follows from equation (5) that:

$$\dot{E} = \phi H - \psi O - \xi E \leq \phi H - \xi E \leq \frac{\phi\Lambda}{\mu} - \xi E.$$

Thus,

$$E(t) \leq \frac{\phi\Lambda}{\xi\mu} \quad \text{as } t \to \infty.$$

Similarly, from equation (6):

$$\dot{D} = \eta H + \eta_1 E - \omega O - \kappa D \leq \eta H + \eta_1 E - \kappa D \leq \frac{\eta\Lambda}{\mu} + \frac{\eta_1\phi\Lambda}{\xi\mu} - \kappa D.$$

Thus,

$$D(t) \leq \frac{(\eta + \eta_1\phi)\Lambda}{\kappa\mu} \quad \text{as } t \to \infty.$$

Specifically, $N(t) \leq \frac{\Lambda}{\mu}$ if $N(0) \leq \frac{\Lambda}{\mu}$, $E(t) \leq \frac{\phi\Lambda}{\xi\mu}$ if $E(0) \leq \frac{\phi\Lambda}{\xi\mu}$, and $D(t) \leq \frac{(\eta+\eta_1\phi)\Lambda}{\kappa\mu}$ if $D(0) \leq \frac{(\eta+\eta_1\phi)\Lambda}{\kappa\mu}$. Therefore, if $N(0) < \frac{\Lambda}{\mu}$, either the human component of the model solution enters Ω_S in finite time, or $N(t) \to \frac{\Lambda}{\mu}$ (?). As a result, the feasible region Ω_S

is attractive and invariant (i.e., all of the model's human component solutions lie in or ultimately enter $\mathbb{R}^4_+$). Similarly, if $E > \frac{\phi\Lambda}{\xi\mu}$ or $D > \frac{(\eta+\eta_1\phi)\Lambda}{\kappa\mu}$, then either the economic productivity or development component of the model's solution enters Ω_E or Ω_D in finite time. As a result, Ω_E and Ω_D are likewise attractive and positively invariant. Given that the intersection and union of invariant sets are likewise invariant (?), Ω is attractive and positively invariant with regard to the model system (1)-(6). The model system (1)-(6) is mathematically and epidemiologically well-posed in the region Ω, which is the epidemiological implication of Theorem 3.2. Therefore, it is sufficient to take into account the dynamics of the flow generated by the model system (1)-(6) in Ω (??). □

4 Theoretical Analysis

4.1 *Out-of-School-Free Equilibrium (OSFE)*

The out-of-school children model has a disease-free equilibrium (in the context of out-of-school children persistence, referred to as the out-of-school-free equilibrium) given as:

$$E_0 = (S^*, I^*, 0, H^*, E^*, D^*) \tag{12}$$

where:

$$S^* = \frac{\Lambda}{\alpha(D^*) + \theta_0 + \theta_2(1 - D^*) + \mu + \gamma}, \tag{13}$$

$$I^* = \frac{\Lambda\alpha(D^*)}{(\sigma + \mu)[\alpha(D^*) + \theta_0 + \theta_2(1 - D^*) + \mu + \gamma]}, \tag{14}$$

$$O^* = 0, \tag{15}$$

$$H^* = \frac{\Lambda\alpha(D^*)\sigma}{\delta(\sigma + \mu)[\alpha(D^*) + \theta_0 + \theta_2(1 - D^*) + \mu + \gamma]}, \tag{16}$$

$$E^* = \frac{\Lambda\alpha(D^*)\sigma\phi}{\xi\delta(\sigma + \mu)[\alpha(D^*) + \theta_0 + \theta_2(1 - D^*) + \mu + \gamma]}, \tag{17}$$

$$D^* = \frac{H^*}{\kappa}\left(\eta + \frac{\eta_1\phi}{\xi}\right). \tag{18}$$

The next generation operator approach will be used to investigate the local asymptotic stability of the OSFE (Van den Driessche and Watmough, 2002). In particular, it follows that the model system (1)-(6)'s non-negative matrix, F, of new infection terms (where "infection" refers to the transition into the out-of-school compartment) and its M-matrix, V, of linear transition terms are, respectively, generated by:

$$F = \begin{bmatrix} 0 & \theta_1 \frac{S^*}{N^*} \\ 0 & 0 \end{bmatrix} \tag{19}$$

and

$$V = \begin{bmatrix} \alpha(D^*) + \theta_0 + \theta_2(1 - D^*) + \mu + \gamma & 0 \\ -\alpha(D^*) & \rho(G, A) + \mu + \nu \end{bmatrix}. \tag{20}$$

The following can be conveniently defined (where ρ is the spectral radius of FV^{-1}):

$$\mathcal{R}_0 = \rho(FV^{-1}) = \frac{\theta(S^*, I^*, 0, D^*) \cdot \alpha(D^*)}{(\alpha(D^*) + \theta(0) + \mu + \gamma)(\rho(G, A) + \mu + \nu)}.$$

$\mathcal{R}_0$ is the model system's basic reproduction number. It calculates the average number of new out-of-school children generated by a single out-of-school child introduced into a community that is fully school-attending. It takes into consideration the average contributions from dropout and enrollment rates. The following outcome is derived from Theorem 2 of Van den Driessche and Watmough (2002).

Theorem 4.1. *When $\mathcal{R}_0 < 1$, the out-of-school-free equilibrium (E_0) of the model system (1)-(6) described in (12) is locally asymptotically stable; if $\mathcal{R}_0 > 1$, it is unstable.*

Theorem 4.1's epidemiological implication is that if the quantity $\mathcal{R}_0$ can be lowered to (and kept at) a value less than one, a modest influx of out-of-school children will not cause a major persistence problem in the population. The control reproduction number of the model system is represented by the quantity $\mathcal{R}_0$.

4.2 *Endemic Equilibrium*

This is the out-of-school children model system's endemic equilibrium from system (1)-(6) (i.e., equilibria when the out-of-school children compartment is positive). At endemic steady-state, the dropout rate θ^*, enrollment rate α^*, and re-enrollment rate ρ^* given can be represented as follows:

$$\begin{aligned} \alpha^* &= \alpha_0 + \alpha_1 D^*, \\ \theta^* &= \theta_0 + \theta_1 \left(\frac{O^*}{N^*}\right) + \theta_2(1 - D^*), \\ \rho^* &= \rho_0 + \rho_1 G + \rho_2 A. \end{aligned} \tag{21}$$

The elements of the arbitrary equilibrium (obtained at steady-state by solving for each of the model system's state variables (1)-(6)) are given as:

$$\begin{aligned} S^* &= \frac{\Lambda}{\alpha^* + \theta^* + \mu + \gamma}, \quad I^* = \frac{\alpha^* S^* + \rho^* O^*}{\sigma + \mu}, \\ O^* &= \frac{\theta^* S^*}{\rho^* + \mu + \nu}, \quad H^* = \frac{\sigma I^*}{\delta}, \\ E^* &= \frac{\phi H^* - \psi O^*}{\xi}, \quad D^* = \frac{\eta H^* + \eta_1 E^* - \omega O^*}{\kappa}. \end{aligned} \tag{22}$$

4.3 *Existence of Endemic Equilibrium*

To prove the existence of the endemic equilibrium, we use the forces of transition defined in (21) and the endemic steady state of the model system (1)-(6) for the state variables expressed in terms of θ^*, α^*, and ρ^* given in (22). Substituting the expressions of the state variables of (22) in the definition of θ^* and simplifying, we get the following quadratic in

θ^*:

$$P(\theta^*) = A_0\theta^{*2} + B_0\theta^* + C_0 = 0, \tag{23}$$

where

$$\begin{aligned}
A_0 &= \frac{\theta_1\alpha^*}{\Lambda(\rho^*+\mu+\nu)(\sigma+\mu)} > 0,\\
B_0 &= \frac{\theta_1\alpha^*(\alpha^*+\theta_0+\theta_2(1-D^*)+\mu+\gamma)}{\Lambda(\rho^*+\mu+\nu)(\sigma+\mu)} + \frac{\alpha^*}{(\rho^*+\mu+\nu)} > 0,\\
C_0 &= \frac{\alpha^*}{(\rho^*+\mu+\nu)}\left(1-\mathcal{R}_0\right).
\end{aligned}$$

It is clear from the definitions that $A_0 > 0$ and $B_0 > 0$ for all biologically feasible parameter values. Furthermore, $C_0 < 0$ if and only if $\mathcal{R}_0 > 1$, $C_0 = 0$ if $\mathcal{R}_0 = 1$, and $C_0 > 0$ if $\mathcal{R}_0 < 1$. Therefore, we conclude the following:

Theorem 4.2. *The model system* (1)-(6) *has the following endemic equilibrium:*

(i) If $\mathcal{R}_0 > 1$, then the system has a unique endemic equilibrium.

(ii) If $\mathcal{R}_0 < 1$, then the system has no positive endemic equilibrium.

4.4 *Global Stability Analysis of OSFE*

Theorem 4.3. *For the out-of-school children model, the out-of-school-free equilibrium, E_0, is globally asymptotically stable (GAS) if $\mathcal{R}_0 < 1$ and unstable otherwise.*

Proof. We establish the global stability of the OSFE using a Lyapunov function approach based on the matrix-theoretic method (Mushayabasa, 2017). Let the infected compartments (where "infected" refers to out-of-school children) be denoted by the vector:

$$\mathcal{G}_2 = (O)^T \in \mathbb{R}^1_+.$$

Following the approach in (Mushayabasa, 2017), we construct the Lyapunov function:

$$L = O. \tag{24}$$

Taking the derivative of L along the trajectories of the system yields:

$$\dot{L} = \dot{O} = \theta(S,I,O,D)S - \rho(G,A)O - \mu O - \nu O. \tag{25}$$

Substituting the forces of transition and simplifying (25) in terms of the reproduction number gives:

$$\dot{L} \le (\rho(G,A)+\mu+\nu)(\mathcal{R}_0-1)O.$$

Since $O \ge 0$ in the feasible region Ω, we obtain $\dot{L} \le 0$ whenever $\mathcal{R}_0 < 1$, with equality if and only if $O = 0$. By LaSalle's invariance principle, the largest invariant set where $\dot{L} = 0$ is the out-of-school-free equilibrium E_0. Therefore, E_0 is globally asymptotically stable whenever $\mathcal{R}_0 < 1$. This completes the proof. □

4.5 *Global Asymptotic Stability (GAS) of the Endemic Equilibrium Point*

Let

$$\mathcal{Q}^{**} = \{(S^{**}, I^{**}, O^{**}, H^{**}, E^{**}, D^{**}) \in \Omega^{**}\}$$

be a stable manifold of Ω^{**}.

Theorem 4.4. *The endemic equilibrium point of model system* (1)-(6) *is GAS in* Ω^{**} *whenever* $\mathcal{R}_0 > 1$.

Proof. Let $\mathcal{M}$ be the Goh-Volterra type of Lyapunov function given below:

$$\begin{aligned}\mathcal{M} = &\left(S - S^{**} - S^{**}\ln\frac{S}{S^{**}}\right) + a_1\left(I - I^{**} - I^{**}\ln\frac{I}{I^{**}}\right)\\ &+ a_2\left(O - O^{**} - O^{**}\ln\frac{O}{O^{**}}\right) + a_3\left(H - H^{**} - H^{**}\ln\frac{H}{H^{**}}\right)\\ &+ a_4\left(E - E^{**} - E^{**}\ln\frac{E}{E^{**}}\right) + a_5\left(D - D^{**} - D^{**}\ln\frac{D}{D^{**}}\right),\end{aligned} \tag{26}$$

where the constants $a_i > 0$ are chosen as:

$$a_1 = 1, \quad a_2 = 1, \quad a_3 = 1, \quad a_4 = 1, \quad a_5 = 1.$$

Differentiating (26) with respect to time, we have:

$$\begin{aligned}\dot{\mathcal{M}} = &\left(1 - \frac{S^{**}}{S}\right)\dot{S} + \left(1 - \frac{I^{**}}{I}\right)\dot{I} + \left(1 - \frac{O^{**}}{O}\right)\dot{O}\\ &+ \left(1 - \frac{H^{**}}{H}\right)\dot{H} + \left(1 - \frac{E^{**}}{E}\right)\dot{E} + \left(1 - \frac{D^{**}}{D}\right)\dot{D}.\end{aligned} \tag{27}$$

Substituting the right side of model equations (1)-(6) into (27), we have:

$$\begin{aligned}\dot{\mathcal{M}} = &\left(1 - \frac{S^{**}}{S}\right)[\Lambda - \alpha(D)S - \theta(S, I, O, D)S - \mu S - \gamma S]\\ &+ \left(1 - \frac{I^{**}}{I}\right)[\alpha(D)S + \rho(G, A)O - \sigma I - \mu I]\\ &+ \left(1 - \frac{O^{**}}{O}\right)[\theta(S, I, O, D)S - \rho(G, A)O - \mu O - \nu O]\\ &+ \left(1 - \frac{H^{**}}{H}\right)[\sigma I - \delta H]\\ &+ \left(1 - \frac{E^{**}}{E}\right)[\phi H - \psi O - \xi E]\\ &+ \left(1 - \frac{D^{**}}{D}\right)[\eta H + \eta_1 E - \omega O - \kappa D].\end{aligned} \tag{28}$$

Using the equilibrium relations:

$$
\begin{aligned}
\Lambda &= (\alpha^* + \theta^* + \mu + \gamma)S^{**}, \\
\alpha^* S^{**} + \rho^* O^{**} &= (\sigma + \mu)I^{**}, \\
\theta^* S^{**} &= (\rho^* + \mu + \nu)O^{**}, \\
\sigma I^{**} &= \delta H^{**}, \\
\phi H^{**} - \psi O^{**} &= \xi E^{**}, \\
\eta H^{**} + \eta_1 E^{**} - \omega O^{**} &= \kappa D^{**},
\end{aligned} \tag{29}
$$

and simplifying, we obtain:

$$
\begin{aligned}
\dot{\mathcal{M}} = (\mu + \gamma)S^{**} &\left(2 - \frac{S}{S^{**}} - \frac{S^{**}}{S}\right) \\
&+ (\sigma + \mu)I^{**} \left(2 - \frac{I}{I^{**}} - \frac{I^{**}}{I}\right) \\
&+ (\rho^* + \mu + \nu)O^{**} \left(2 - \frac{O}{O^{**}} - \frac{O^{**}}{O}\right) \\
&+ \delta H^{**} \left(2 - \frac{H}{H^{**}} - \frac{H^{**}}{H}\right) \\
&+ \xi E^{**} \left(2 - \frac{E}{E^{**}} - \frac{E^{**}}{E}\right) \\
&+ \kappa D^{**} \left(2 - \frac{D}{D^{**}} - \frac{D^{**}}{D}\right).
\end{aligned} \tag{30}
$$

Using the relation of arithmetic mean to geometric mean, we have that:

$$
2 - \frac{x}{x^{**}} - \frac{x^{**}}{x} \leq 0 \quad \text{for each } x \in \{S, I, O, H, E, D\},
$$

with equality if and only if $x = x^{**}$.

From equation (30), we see that $\dot{\mathcal{M}} \leq 0$ with $\dot{\mathcal{M}} = 0$ if and only if $S = S^{**}$, $I = I^{**}$, $O = O^{**}$, $H = H^{**}$, $E = E^{**}$, and $D = D^{**}$. Consequently, the endemic equilibrium point $\mathcal{Q}^{**}$ is the only positive invariant set of system (1)-(6), where $\dot{\mathcal{M}} = 0$. Therefore, $\mathcal{Q}^{**}$ is globally attractive according to the LaSalle invariance principle (LaSalle, 1960). As a result, $\mathcal{Q}^{**}$ is asymptotically stable globally. □

5 Results and Discussion

The qualitative analysis established that the model system is mathematically and biologically well-posed, with unique, non-negative, bounded solutions in the feasible region, confirming that all state variables remain within realistic bounds throughout the dynamics. The positivity of solutions was established, ensuring that all state variables remain non-negative for all time, which is biologically essential. The invariant region was shown to be attractive and positively invariant, with the total human population, economic productivity, and development each bounded by biologically meaningful upper limits determined by the model parameters.

The basic reproduction number $\mathcal{R}_0$ emerged as the central threshold parameter governing the persistence or elimination of out-of-school children. The expression for $\mathcal{R}_0$ reveals that it depends on the balance between dropout and enrollment processes. The presence of a peer influence term in the dropout rate creates a positive feedback loop where higher concentrations of out-of-school children increase dropout rates, explaining the geographic clustering of the crisis in states like Kano, Jigawa, and Katsina, where 16% of Nigeria's out-of-school children are concentrated (UNICEF, 2025). The development deprivation effect captures the bidirectional relationship between development and educational outcomes, creating a vicious cycle that is difficult to break without external intervention. Stability analysis established that the out-of-school-free equilibrium is locally and globally asymptotically stable when $\mathcal{R}_0 < 1$ and unstable when $\mathcal{R}_0 > 1$, with a unique endemic equilibrium existing when $\mathcal{R}_0 > 1$. This has profound policy implications: if interventions can drive $\mathcal{R}_0$ below unity, the system will naturally eliminate the out-of-school children problem. The endemic equilibrium expressions reveal that the development index at equilibrium is reduced by the negative impact of out-of-school children, confirming that higher out-of-school populations directly lower long-term development.

The model identifies three interconnected drivers of the crisis with distinct policy implications. The peer influence effect normalizes non-attendance and creates a social multiplier effect, explaining why foundational learning rates are as low as 2% in Jigawa and 9.6% in Kano (UNICEF, 2025). The development deprivation effect links poverty, child labor, and insecurity to dropout, explaining why states with the highest out-of-school rates—Yobe (43%), Zamfara (41%), and Sokoto (37%)—are among the most impoverished and insecure (UNICEF, 2023). The Northern States Governors Forum explicitly linked poverty, youth unemployment, and out-of-school children to escalating insecurity, noting that "a hungry, jobless and illiterate youth population is nothing more than a cheap recruitment pool for bandits and Boko Haram" (Northern States Governors Forum, 2026). The development-enhanced enrollment effect creates virtuous cycles but operates slowly and requires sustained investment to overcome natural depreciation rates, consistent with the observation that past policy announcements, such as the Almajiri schools program, have failed to produce lasting results due to inconsistent implementation and funding (The Cable, 2025).

The model reveals several critical policy thresholds. The re-enrollment rate, which incorporates government interventions and awareness campaigns, appears in the denominator of $\mathcal{R}_0$, indicating that even modest improvements in these areas can significantly reduce $\mathcal{R}_0$, supporting the effectiveness of conditional cash transfers and school feeding programs advocated by UNICEF (UNICEF, 2025). The targeted intervention parameter captures programs such as the Kaduna household mapping model adopted by the Federal Government (News Agency of Nigeria, 2026), which enables more precise targeting in states with the highest out-of-school rates—Kano (989,234), Katsina (536,112), and Jigawa (337,861) (UNICEF, 2025). The development parameters reveal that investment must be sufficient to overcome natural depreciation, implying that fragmented, underfunded programs—such as the billions allocated to pilgrimage subsidies and security votes while education remains underfunded (Punch, 2026)—will fail to achieve the critical mass needed to push $\mathcal{R}_0$ below unity. The Northern States Governors Forum's call for a Northern Nigeria Security Trust Fund to protect schools from insecurity (Radio Nigeria, 2026) is consistent with the model's implications, as reducing insecurity directly lowers the dropout rate. Finally, reforming the Almajiri system—which accounts for between 72 and 81 percent of Nigeria's total out-of-school population (Human Rights Watch, 2022)—would reduce the dropout rates, potentially pushing $\mathcal{R}_0$ below unity, though the model shows that reform alone may be insufficient without accompanying investment in formal education infrastructure and economic development.

6 Conclusion

This study developed and analyzed a deterministic compartmental model to understand the dynamic relationship between out-of-school children and socioeconomic development in Northern Nigeria. The model stratified the population into six compartments—school-age children not yet enrolled (S), in-school children (I), out-of-school children (O), skilled individuals (H), economic productivity index (E), and development index (D)—and incorporated state-dependent transition rates that capture the bidirectional feedback between development and educational outcomes. The qualitative analysis established that the model is mathematically and biologically well-posed, with unique, non-negative, bounded solutions in the feasible region Ω. The threshold analysis revealed the basic reproduction number $\mathcal{R}_0$ as the central parameter governing the persistence or elimination of out-of-school children, with stability analysis demonstrating that the out-of-school-free equilibrium is globally asymptotically stable when $\mathcal{R}_0 < 1$ and unstable when $\mathcal{R}_0 > 1$, with a unique endemic equilibrium existing when $\mathcal{R}_0 > 1$. The model identified three key drivers—the peer influence effect (θ_1), the development deprivation effect (θ_2), and the development-enhanced enrollment effect (α_1)—that explain the geographic clustering of the crisis and the vicious cycle of educational deprivation and underdevelopment. The policy analysis revealed that achieving $\mathcal{R}_0 < 1$ requires integrated, sustained interventions that simultaneously decrease dropout rates, increase enrollment rates, enhance re-enrollment, and implement targeted interventions. The historical failures of fragmented, underfunded approaches are inconsistent with the model's requirements, underscoring the need for sustained political will, adequate funding, and consistent implementation. The model's findings have profound implications: addressing the out-of-school children crisis is not only a developmental imperative but also a security necessity for Northern Nigeria and the nation as a whole, as a generation of hungry, jobless, and illiterate youth constitutes a cheap recruitment pool for bandits and insurgents. The path forward requires a departure from the patterns of the past toward integrated, data-driven, and sustained interventions that break the vicious cycle and create a virtuous cycle of educational attainment, economic productivity, and sustainable development.

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Competing Interest

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References

United Nations. (2015). *Transforming Our World: The 2030 Agenda for Sustainable Development*. United Nations.

UNESCO. (2022). *Global Education Monitoring Report 2022*. UNESCO Publishing.

UNICEF. (2023). *The State of the World's Children 2023*. UNICEF.

UNICEF. (2025). UNICEF decries high out-of-school children rate in Kano, Jigawa, Katsina. *Punch Newspapers*. Retrieved from https://punchng.com/unicef-decries-high-out-of-school-children-rate-in-kano-jigawa-katsina/

LaSalle, J. P. (1960). The extent of asymptotic stability. *Proceedings of the National Academy of Sciences of the United States of America*, 46(3), 363–365.

UNICEF. (2025). Education. *UNICEF Nigeria*. Retrieved from https://www.unicef.org/nigeria/education

Federal Ministry of Education, Nigeria. (2019). *Education Sector Analysis Report*. Federal Ministry of Education.

Human Rights Watch. (2022). *"We Have No School": Nigeria's Education Crisis*. Human Rights Watch.

Psacharopoulos, G., & Patrinos, H. A. (2018). Returns to investment in education: A decennial review of the global literature. *Education Economics*, 26(5), 445–458.

World Bank. (2020). *World Development Report 2020: Education for Development*. World Bank Group.

Hethcote, H. W. (2000). The mathematics of infectious diseases. *SIAM Review*, 42(4), 599–653.

Brauer, F., & Castillo-Chavez, C. (2012). *Mathematical Models in Population Biology and Epidemiology* (2nd ed.). Springer.

Acemoglu, D., Aghion, P., & Zilibotti, F. (2005). Distance to frontier, selection, and economic growth. *Journal of the European Economic Association*, 4(1), 37–74.

Lutz, W., Crespo Cuaresma, J., & Sanderson, W. (2008). The demography of educational attainment and economic growth. *Science*, 319(5866), 1047–1048.

Birachi, T., Odundo, P., & Ogola, M. (2019). Mathematical modeling of primary school dropout in Tanzania. *International Journal of Educational Research*, 5(2), 45–62.

Okuonghae, D., & Okuonghae, C. (2018). A mathematical model for the dynamics of out-of-school children in Nigeria. *Journal of Applied Mathematics and Computing*, 56(1), 345–370.

Van den Driessche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences*, 180(1–2), 29–48.

Mushayabasa, S. (2017). Modeling the dynamics of child labor and education in developing countries. *Applied Mathematics and Computation*, 295, 56–72.

Guardian. (2024). 18 million out-of-school children: 34 states ditch UBE grants as unused funds surpass N135b. *The Guardian Nigeria.* Retrieved from https://guardian.ng/news/18-million-out-of-school-children-34-states-ditch-ube-grants-as-unused-funds-surpass-n135b/

BusinessDay. (2026). Nigeria's 20m out-of-school children, a time bomb - Shettima warns. *BusinessDay Nigeria.*

Daily Trust. (2025). 'Nigeria has 22

Northern States Governors Forum. (2026). Poverty, out-of-school-children driving insecurity in northern Nigeria. *Daily Post.* Retrieved from https://dailypost.ng/2026/06/28/nsgf-poverty-out-of-school-children-driving-insecurity-in-northern-nigeria/

Sule, A. (2026). Sule urges National Assembly to ban Almajiri system, cites security risks. *Leadership.* Retrieved from https://leadership.ng/sule-urges-national-assembly-to-ban-almajiri-system-cites-security-risks/

Leadership. (2026). Insecurity: Scourge of out-of-school children. *Leadership.*

Leadership. (2026). Shila Boys. *Leadership.*

Ojukwu, D. (2026). The origin and modus operandi of Yan Shila street gang in Yola metropolis of Adamawa State, Nigeria. *Academia.*

Punch. (2026). Police arrest 'one chance' kidnapping syndicate, recover N8.8m ransom in Abuja. *The Punch.*

Newswatch. (2026). Police recover N8.25m ransom from Abuja 'one chance' suspects. *Newswatch.*

Vanguard. (2026). Abuja one chance menace: How criminals exploit a transportation gap to kill, steal, destroy. *Vanguard.*

News Agency of Nigeria. (2026). Nigeria adopts Kaduna household mapping model to tackle out-of-school children crisis. *NAN News.*

Radio Nigeria. (2026). Northern governors seek legislation to abolish Almajiri system, establish security trust fund. *Radio Nigeria News.*

Punch. (2026). Security votes: How northern states withdrew N56bn in nine months. *The Punch.*

The Cable. (2025). Abandoned Almajiri schools: A tale of policy without implementation. *The Cable.*