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A Mathematical Model for Banditry Transmission Dynamics with Informant Networks and Optimal Control Interventions

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Abstract

Banditry has become a significant security challenge in Nigeria, particularly in the Northwestern and North-central regions, where armed groups engage in kidnapping, armed robbery, cattle rustling, and attacks on communities and military installations. This study develops a mathematical model for banditry transmission dynamics that incorporates informant networks and special military force interventions. The model, built as a system of nonlinear differential equations, captures the recruitment of susceptible individuals into informancy and banditry, the progression of informants to bandits, and the impact of military interventions. The basic reproduction number $\mathcal{R}_c$ is derived, and it is proven that the banditry-controlled free equilibrium is globally asymptotically stable when $\mathcal{R}_c < 1$. Sensitivity analysis

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highlights parameters most influencing transmission, including the progression rate of informants to bandits (χ), rehabilitation rates (γ_1, γ_2), and the deployment of special military forces (τ). An optimal control problem is formulated with three time-dependent control measures: education and enlightenment campaigns (u_1), rehabilitation programs (u_2), and special military force deployment (u_3). Using Pontryagin's Maximum Principle, the optimal controls are characterized and numerically solved. Numerical simulations demonstrate that combined intervention strategies significantly reduce informant and bandit populations, achieving up to 90% reduction in bandits and 78.9% reduction in informants. The study provides a rigorous framework for understanding banditry dynamics and offers practical insights for improving security interventions and public policy in conflict-prone regions.

1 Introduction

A bandit is a robber or armed thief who engages in banditry by attacking others while travelling in quest of money and occasionally other valuable objects (Walker, 2012). Banditry refers to criminal activities carried out to accumulate wealth through armed robbery, kidnapping, cattle rustling, and village raids with impunity, among other atrocities (Abdullahi, 2022). In Nigeria's case, banditry has reached alarming heights to the extent of setting up fortified enclaves in the hinterlands and on the frontiers, from where they plot and carry out their operations (Bildirici and Gokmenoglu, 2020; Chukwuma et al., 2022). Many bandit groups exist in different hide-outs across Northwestern and North-central Nigeria (Barnett et al., 2022). Though there is no significant evidence suggesting either collaboration or the existence of any central authority, there exist similarities in terms of their operations and recruitment approaches (Ojewale, 2021). The victims of the banditry attack were initially communities with material valuables, but recently, they have had the effrontery to attack military bases, schools, and even a moving train (Dami, 2021; Saidu, 2022; Sanchi et al., 2022). The banditry activities have led to the loss of lives, goods and services, leading to astronomical increases in prices of local products (Ghosh, 2019; Ladan and Matawalli, 2020).

Bandits primarily target the residents in rural areas, but they sometimes ambush motorists on highways, killing anyone who tries to resist kidnapping or refuses to pay a ransom. A bandit, according to the English Learners dictionary, a criminal is who attacks and steals from travelers and is frequently a part of a gang of thieves. The occurrence or prevalence of armed robbery or violent crime is known as banditry. It entails the use of force or the threat of force to frighten a person in order to rob, rape, or kill them. People in northern Nigerian states such as Sokoto, Kaduna, Katsina, Kebbi, Niger and Zamfara have been afflicted by banditry violence. The acts of banditry have over 21 million people residing in these states.

The existence and actions of armed Bandits in the northwest have been exacerbated by the discovery of gold mines and the operations of illicit miners competing for control of gold reserves. More than 210,000 people will have been domestically displaced by March 2020. By the beginning of March 2020, more than 35,000 refugees had crossed community borders to Maradi, the Niger Republic. Though there are variations in these groups' mode of destructions from the beginning, their system of operation now is almost identical (co-menace). There are six main causes cited for why people (youths) join the deadly gang (Banditry): unemployment and poverty, challenging child rearing, high levels of illiteracy, excessive military responses, and widespread corruption. The reasons why young people join terrorism (Banditry) include peer pressure, family and ethnic pressure, marriage, dissatisfaction with the government, a desire for status and prestige, a lack of education, a desire to belong to a group, a desire to make money, a desire to be respected, and other factors (Nadama, 2019; Onuoha, 2014).

Boko Haram and the Bandits are primarily made up of young people. There are a variety of reasons why youths join gangs in huge numbers at various times. Ignorance of religious teachings opposing violence makes youth more vulnerable to recruitment, unemployment, and poverty, which make youth more vulnerable to radicalisation. Children with difficult upbringings are more vulnerable to extremist views, high levels of illiteracy are linked to youth radicalisation and extremism, and responses to security forces' excesses were mixed (Onuoha, 2014). According to Ewi and Salifu (2017), youths join Boko Haram and Bandits for a variety of reasons, including a desire to make money, a desire to belong to a specific group, family and ethnic pressure, a desire to be respected, a lack of education, a desire to be feared, peer pressure, and the fact that they are unemployed and see Boko Haram and Banditry as a job. As a result, both Boko Haram and the Bandits' activities in Northern Nigeria continue unabated. This means that the majority of the homicides, destructions, armed robberies, thefts, kidnappings, and other criminal actions are the result of the radical and criminal activities of the strong and agile youngsters from both groups. Radicalization is a term used to describe the process through which people adopt progressively extreme political or religious ideas. Radicalization has become a major national security problem in recent years because it can lead to violent extremism (Mccluskey and Fei Xu, 2017).

In Nigeria, banditry violence is not a new problem. Banditry in Nigeria dates back to the pre-civil war period, when government in some sections of the ancient Western region collapsed, resulting in political bloodshed, criminality, and organized insurgency (Odinkala, 2018). Local robbers were reportedly snatching domestic animals during the civilian reign (Nadama, 2019). Bandit activity has recently been particularly concerning in Nigeria's northwest, particularly in the states of Zamfara, Sokoto, Katsina, Niger, Kaduna, and Kebbi. Kidnapping, murder, robbery, rape, livestock rustling, and other crimes are among the actions of these bandits. Their method of operation includes maiming and murdering their victims when they are least expecting it. Momoh et al. (2023) did a study "Mathematical Modeling and Optimal Control of Intervention Strategies for A Banditry Model" where they modified model and sub-divides the population at time t into five sub-population, these are susceptible $S(t)$, Bandits $B(t)$, detention Centre for Bandits $D(t)$, rehabilitated Bandits $R(t)$, and Military forces $M(t)$. The positivity and uniqueness of the model equations' solutions were established, and it was found that they do indeed exist and are distinct.

In this work, after careful study of the above work, two subdivided populations have been incorporated, which are informant $I(t)$ and Special Military $M_S(t)$. The proposed control measures in this research are as follows: eradicating illiteracy and providing an enabling environment for employment, to campaign against the recruitment of informants and Bandits, the use of special military forces against the two groups and rehabilitating the detained members of the groups. Mathematical models have proven to be indispensable tools in understanding the transmission dynamics of crime and violence, providing quantitative frameworks for evaluating intervention strategies (Chukwuma et al., 2022; Momoh et al., 2023). The application of optimal control theory to such models allows for the determination of time-dependent intervention strategies that minimize both the criminal population and the cost of implementing control measures (Lenhart and Workman, 2007; Okosun et al., 2013). This approach has been successfully applied to various epidemiological and criminal dynamics problems, demonstrating its effectiveness in identifying cost-effective control policies (Bildirici and Gokmenoglu, 2020; Ghosh, 2019).

The presence of informants within bandit networks represents a critical yet understudied aspect of banditry dynamics. Informants facilitate bandit operations through intelligence gathering, early warning systems, and logistical support, thereby sustaining criminal activities (Barnett et al., 2022; Ojewale, 2021). Understanding the recruitment and progression of informants into banditry is essential for developing targeted interventions. Furthermore, the role of special military forces in countering banditry has gained prominence, with deployments aimed at disrupting criminal networks and restoring state authority in affected regions (Dami, 2021; Saidu, 2022). However, the optimal allocation of these military resources remains a significant challenge for policymakers.

Although optimal control theory has proven to be a powerful tool for identifying cost-effective, time-dependent intervention strategies for crime and violence (Lenhart and Workman, 2007; Momoh et al., 2023), existing models that incorporate informant dynamics and military intervention often rely on simplified linear transmission terms and static control parameters. This confluence of factors—the informant reservoir, the need for dynamic military deployment, and the necessity of time-dependent controls—creates a significant gap in the literature. There is a pressing need for a modeling framework that integrates a more biologically and socially plausible nonlinear force of infection with the distinct criminal classes of informants and bandits to derive robust, optimal control strategies that can guide security policy.

In this study, we address this gap by developing and analyzing a novel optimal control model for banditry transmission dynamics. Our model explicitly stratifies the criminal population into informant and bandit classes and incorporates a nonlinear force of infection to capture the complex dynamics of recruitment and progression. We formulate an optimal control problem with time-dependent intervention strategies, including education and enlightenment campaigns to reduce recruitment (u_1), rehabilitation programs for detained criminals (u_2), and deployment of special military forces (u_3), with the dual objective of minimizing the total number of informants and bandits and the cost of implementing these interventions. Using Pontryagin's Maximum Principle, we characterize the optimal controls and numerically solve the system to assess the impact of nonlinear incidence on optimal strategies. By providing a more realistic mathematical framework, this work offers critical insights for designing next-generation, adaptive crime control programs that are both effective and operationally feasible, with the potential to accelerate progress towards reducing banditry in Nigeria.

2 Model Description

The schematic diagram of the banditry model as explained with assumptions above is given in Figure 1 and the corresponding model equations which are generated from Figure 1 are as given in (1). The variables and parameter interpretations are as given in Table 1.

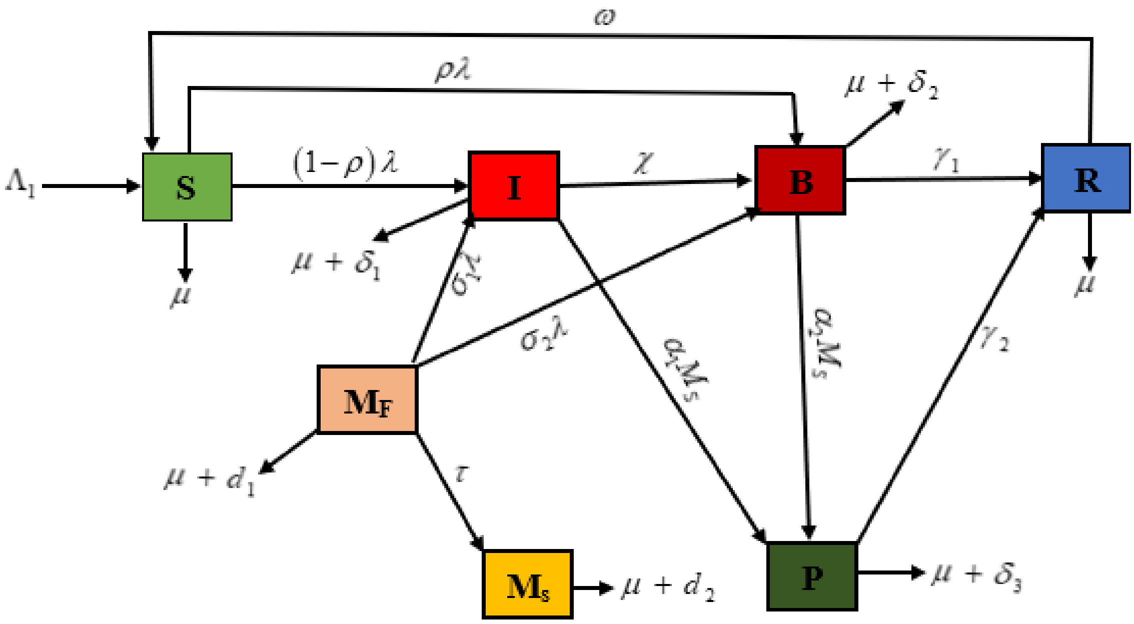


Figure 1. Schematic diagram of the model (1). The arrows indicate transitions, and expressions next to arrows show the *per capita* flow rate between compartments.

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda_1 + \omega R - \rho\lambda S - (1-\rho)\lambda S - \mu S,\\
\frac{dI}{dt} &= (1-\rho)\lambda S + \sigma\lambda M_F I - \alpha_1 M_S I - (\chi + \mu + \delta_1)I,\\
\frac{dB}{dt} &= \rho\lambda S + \chi I + \sigma_2\lambda B M_F - \alpha_2 M_S B - (\gamma_1 + \mu + \delta_2)B,\\
\frac{dP}{dt} &= \alpha_1 M_S I + \alpha_2 M_S B - (\gamma_2 + \mu + \delta_3)P,\\
\frac{dR}{dt} &= \gamma_1 B + \gamma_2 P - (\omega + \mu)R,\\
\frac{dM_F}{dt} &= \Lambda_2 - \sigma_1\lambda I M_F - \sigma_2\lambda B M_F - (\tau + \mu + d_1)M_F,\\
\frac{dM_S}{dt} &= \tau M_F - (\mu + d_2)M_S,
\end{aligned}
\tag{1}
$$

where

$$
\lambda = \frac{(1-\epsilon\eta)\psi(B + \theta I)}{N} \tag{2}
$$

Table 1. Interpretation of the state variables and parameters used in the model (1).

Variable	Description
N	Total population
S	Susceptible individuals
I	Informant
B	Bandit
R	Rehabilitation center
P	Prison
M_F	Military force
M_S	Special military
Parameter	**Description**
Λ_1, Λ_2	Recruitment rate of susceptible and military
μ	Natural mortality rate
$\delta_1, \delta_2, \delta_3$	Death due to Banditry
d_1, d_2	Death of Military due to engagement with Bandit
ρ	Proportion of susceptible move to Banditry
ω	Waning rate of Rehabilitated Bandit into susceptible
γ_1, γ_2	Rehabilitation rates
α_1, α_2	Imprisonment rates
χ	Progression rate of informant to banditry
σ_1, σ_2	Recruitment rates of informant and banditry from the military
τ	Progression rate of M_F into M_S
ψ	Effective contact rate
ϵ	Education rate
η	Financial independence
θ	Reduction rate of informant due to fear of identity

2.1 Modified Model with Optimal Control

To effectively control banditry transmission, we incorporate three time-dependent control measures into the banditry model. The controls target the education rate (ϵ), the rehabilitation rates (γ_1, γ_2), and the imprisonment rates (α_1, α_2) along with special military deployment (τ). The control measures are defined as follows:

$u_1(t)$: Education and enlightenment campaign effort — This control reduces the recruitment of susceptible individuals into informancy and banditry, and decreases the effective contact rate. It targets the education rate parameter (ϵ) and the proportion of susceptibles recruited (ρ).

$u_2(t)$: Rehabilitation and reintegration effort — This control enhances the rehabilitation rates of detained bandits and informants, facilitating their successful reintegration into society. It targets the rehabilitation parameters (γ_1, γ_2).

$u_3(t)$: Special military force deployment and operational intensity — This control increases the imprisonment rates of informants and bandits, and accelerates the progression of regular military forces to special military forces. It targets the imprisonment parameters (α_1, α_2) and the progression rate (τ).

The modified model with controls is given by the following system of nonlinear differential equations:

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda_1 + \omega R - (1-u_1)\rho\lambda S - (1-u_1)(1-\rho)\lambda S - \mu S,\\
\frac{dI}{dt} &= (1-u_1)(1-\rho)\lambda S + \sigma_1\lambda M_F I - (1+u_3)\alpha_1 M_S I - (\chi+\mu+\delta_1)I,\\
\frac{dB}{dt} &= (1-u_1)\rho\lambda S + \chi I + \sigma_2\lambda B M_F - (1+u_3)\alpha_2 M_S B - (\gamma_1 u_2+\mu+\delta_2)B,\\
\frac{dP}{dt} &= (1+u_3)\alpha_1 M_S I + (1+u_3)\alpha_2 M_S B - (\gamma_2 u_2+\mu+\delta_3)P,\\
\frac{dR}{dt} &= \gamma_1 u_2 B + \gamma_2 u_2 P - (\omega+\mu)R,\\
\frac{dM_F}{dt} &= \Lambda_2 - \sigma_1\lambda I M_F - \sigma_2\lambda B M_F - (\tau(1+u_3)+\mu+d_1)M_F,\\
\frac{dM_S}{dt} &= \tau(1+u_3)M_F - (\mu+d_2)M_S,
\end{aligned}
\tag{3}
$$

where the modified force of infection is given by:

$$
\lambda = \frac{(1-u_1)(1-\epsilon\eta)\psi(B+\theta I)}{N} \tag{4}
$$

and the total population is given by

$$
N = S + I + B + P + R + M_F + M_S. \tag{5}
$$

3 Basic Properties of the Model

3.1 Boundedness and Positivity

In this subsection, we add up the model equations in the controlled system (3) to obtain:

$$
\begin{aligned}
\frac{d\left(S+I+B+P+R+M_F+M_S\right)}{dt} &= \Lambda_1 + \Lambda_2 - \mu\left(S+I+B+P+R+M_F+M_S\right)\\
&\quad - \delta_1 I - \delta_2 B - \delta_3 P - d_1 M_F - d_2 M_S,
\end{aligned}
\tag{6}
$$

Since $N = S + I + B + P + R + M_F + M_S$, equation (6) becomes:

$$
\frac{dN}{dt} = \Lambda_1 + \Lambda_2 - \mu N - \delta_1 I - \delta_2 B - \delta_3 P - d_1 M_F - d_2 M_S. \tag{7}
$$

Since all parameters are non-negative, we have:

$$
\frac{dN}{dt} \le \Lambda_1 + \Lambda_2 - \mu N. \tag{8}
$$

Solving this linear differential inequality using the integrating factor $e^{\mu t}$, we obtain:

$$
N(t) \le N(0)e^{-\mu t} + \frac{(\Lambda_1+\Lambda_2)}{\mu}\left(1 - e^{-\mu t}\right). \tag{9}
$$

Taking the limit as $t \to \infty$:

$$\limsup_{t\to\infty} N(t) \leq \frac{(\Lambda_1 + \Lambda_2)}{\mu}. \tag{10}$$

Therefore, the feasible region is given by:

$$\mathcal{D} = \left\{ (S, I, B, P, R, M_F, M_S) \in \mathbb{R}^7_+ : 0 \leq N \leq \frac{(\Lambda_1 + \Lambda_2)}{\mu} \right\}. \tag{11}$$

Hence, $\mathcal{D}$ is positively invariant and attracting. Thus, all solutions of the controlled model (3) remain bounded in the feasible region $\mathcal{D}$ for all $t \geq 0$.

3.2 Positivity

Lemma 3.1. *Let the initial data of the controlled model be $S(t) > 0, I(t) > 0, B(t) > 0, P(t) > 0, R(t) > 0, M_F(t) > 0$, and $M_S(t) > 0$. Then the solution of $S(t), I(t), B(t), P(t)$, $R(t), M_F(t)$, and $M_S(t)$ with positive initial data will remain positive for all time $t > 0$.*

Proof. Let $t_1 = \sup[t > 0 : S(t) > 0, I(t) > 0, B(t) > 0, P(t) > 0, R(t) > 0, M_F(t) > 0, \text{ and } M_S(t) > 0]$. For the susceptible compartment from the controlled system (3):

$$\frac{dS}{dt} = \Lambda_1 + \omega R - (1 - u_1)\lambda S - \mu S. \tag{12}$$

For simplicity, we assume that the waning rate (ω) is non-negative and equal to zero:

$$\frac{dS}{dt} \geq -(\lambda + \mu)S. \tag{13}$$

Solving this differential inequality, we obtain:

$$S(t) \geq S(0) \exp\left[-\int_0^t (\lambda(\tau) + \mu) d\tau \right] > 0, \quad \forall t > 0. \tag{14}$$

Following a similar approach, the remaining state variables I, B, P, R, M_F, M_S can be shown to remain non-negative. Since all the model equations in (3) are proven to be positive, the proof follows. □

3.3 Local Stability and Basic Reproduction Number

The banditry-controlled free equilibrium point of the controlled model (3) is obtained by setting the infectious compartments to zero. The infected compartments are the informants (I) and bandits (B), while the prison compartment (P) is considered as a secondary compartment. At the banditry-free equilibrium, we have $I = B = 0$, and the equilibrium point is given by:

$$E_0 = \left(\frac{\Lambda_1}{\mu}, 0, 0, 0, 0, \frac{\Lambda_2}{\tau(1 + u_3) + \mu + d_1}, \frac{\tau(1 + u_3)\Lambda_2}{(\mu + d_2)(\tau(1 + u_3) + \mu + d_1)} \right). \tag{15}$$

We now compute the basic reproduction number $\mathcal{R}_c$ for the controlled model using the next-generation matrix method (van den Driessche and Watmough, 2002). The matrices

F and V are given by:

$$F = \begin{bmatrix} \phi_1 & \phi_2 & 0 \\ \phi_3 & \phi_4 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} a_1 & 0 & 0 \\ -\chi & a_2 & 0 \\ -a_3 & -a_4 & k_3 \end{bmatrix}, \tag{16}$$

where the elements are defined in terms of the controlled model parameters:

$$\begin{aligned} \phi_1 &= \frac{(1-\rho)(1-\epsilon\eta)\psi\theta\Lambda_1}{\Lambda_1+\Lambda_2} + \frac{\sigma_1(1-\epsilon\eta)\psi\theta\Lambda_2\mu}{(\Lambda_1+\Lambda_2)k_5}, \\ \phi_2 &= \frac{(1-\rho)(1-\epsilon\eta)\psi\Lambda_1}{\Lambda_1+\Lambda_2} + \frac{\sigma_1(1-\epsilon\eta)\psi\Lambda_2\mu}{(\Lambda_1+\Lambda_2)k_5}, \\ \phi_3 &= \frac{\rho(1-\epsilon\eta)\psi\theta\Lambda_1}{\Lambda_1+\Lambda_2} + \frac{\sigma_2(1-\epsilon\eta)\psi\theta\Lambda_2\mu}{(\Lambda_1+\Lambda_2)k_5}, \\ \phi_4 &= \frac{\rho(1-\epsilon\eta)\psi\Lambda_1}{\Lambda_1+\Lambda_2} + \frac{\sigma_2(1-\epsilon\eta)\psi\Lambda_2\mu}{(\Lambda_1+\Lambda_2)k_5}, \end{aligned} \tag{17}$$

and

$$\begin{aligned} a_1 &= \frac{(1+u_3)\alpha_1\tau\Lambda_2 + k_1k_5k_6}{k_5k_6}, & a_2 &= \frac{(1+u_3)\alpha_2\tau\Lambda_2 + k_2k_5k_6}{k_5k_6}, \\ a_3 &= \frac{(1+u_3)\alpha_1\tau\Lambda_2}{k_5k_6}, & a_4 &= \frac{(1+u_3)\alpha_2\tau\Lambda_2}{k_5k_6}, \end{aligned} \tag{18}$$

with

$$k_1 = \chi+\mu+\delta_1, \quad k_2 = \gamma_1u_2+\mu+\delta_2, \quad k_3 = \gamma_2u_2+\mu+\delta_3, \quad k_5 = \tau(1+u_3)+\mu+d_1, \quad k_6 = \mu+d_2.$$

The inverse of V is:

$$V^{-1} = \begin{bmatrix} \frac{1}{a_1} & 0 & 0 \\ \frac{\chi}{a_1a_2} & \frac{1}{a_2} & 0 \\ \frac{\chi a_4+a_2a_3}{a_1a_2k_3} & \frac{a_4}{a_2k_3} & \frac{1}{k_3} \end{bmatrix}. \tag{19}$$

The next-generation matrix FV^{-1} is:

$$FV^{-1} = \begin{bmatrix} \frac{\phi_1}{a_1} + \frac{\phi_2\chi}{a_1a_2} & \frac{\phi_2}{a_2} & 0 \\ \frac{\phi_3}{a_1} + \frac{\phi_4\chi}{a_1a_2} & \frac{\phi_4}{a_2} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{20}$$

The eigenvalues of FV^{-1} are the roots of the characteristic equation. The spectral radius $\rho(FV^{-1})$ is the largest eigenvalue, and the basic reproduction number $\mathcal{R}_c$ is given by:

$$\mathcal{R}_c = \frac{\phi_4 a_1 + \phi_1 a_2 + \phi_2\chi + \sqrt{\phi_2^2\chi^2 + ((2\phi_4\chi + 4\phi_3 a_2)a_1 + 2\phi_1 a_2\chi)\phi_2 + (\phi_4 a_1 - \phi_1 a_2)^2}}{2a_1a_2}. \tag{21}$$

3.4 Interpretation of the Control Effects on $\mathcal{R}_c$

The basic reproduction number $\mathcal{R}_c$ is a function of the control parameters u_1, u_2, and u_3. Specifically:

- The education control u_1 appears in the force of infection λ through the factor $(1 - u_1)$, reducing the effective contact rate and recruitment into informancy and banditry. This decreases $\phi_1, \phi_2, \phi_3, \phi_4$.
- The rehabilitation control u_2 appears in a_2 through $k_2 = \gamma_1 u_2 + \mu + \delta_2$ and in $k_3 = \gamma_2 u_2 + \mu + \delta_3$, which increases the denominators and thus reduces $\mathcal{R}_c$.
- The special military control u_3 appears in a_1, a_2, a_3, a_4 through the factors $(1 + u_3)$, increasing imprisonment rates and thus reducing $\mathcal{R}_c$.

Therefore, we can write $\mathcal{R}_c = \mathcal{R}_c(u_1, u_2, u_3)$, and the goal of optimal control is to find controls that drive $\mathcal{R}_c < 1$, ensuring the eradication of banditry from the population.

Theorem 3.2. *The banditry-controlled free equilibrium point E_0 of the controlled model (3) is locally asymptotically stable if $\mathcal{R}_c < 1$ and unstable if $\mathcal{R}_c > 1$.*

Proof. The proof follows from the standard next-generation matrix approach (van den Driessche and Watmough, 2002). The controlled model (3) has a banditry-free equilibrium E_0 as given above. The Jacobian matrix evaluated at E_0 has eigenvalues with negative real parts if and only if $\mathcal{R}_c < 1$. The characteristic polynomial can be reduced to a quadratic whose coefficients satisfy the Routh-Hurwitz criteria when $\mathcal{R}_c < 1$. □

3.5 Global Stability of the Banditry-Controlled Free Equilibrium

For the controlled model (3), the global asymptotic stability (GAS) of the banditry-controlled free equilibrium is examined using the methodology in (?).

Lemma 3.3. *Suppose the controlled system (3) is of the form*

$$\frac{dX}{dt} = F(X, Z), \qquad \frac{dZ}{dt} = G(X, Z), \quad G(X, 0) = 0, \tag{22}$$

where $X = (S, R, M_F, M_S)$ denotes the populations that are not infected $X \in \mathbb{R}^4$, while $Z = (I, B, P)$ represents the populations that are affected $Z \in \mathbb{R}^3$. We define the banditry-controlled free state as $E_0 = (X^0, 0)$. To guarantee global asymptotic stability, the following two conditions, H_1 and H_2, must be met:

(H1) $\frac{dX}{dt} = F(X^0, 0)$, X^0 *is globally asymptotically stable* $(G.A.S)$,

(H2) $G(X, Z) = CZ - \hat{G}(X, Z)$, *where* $\hat{G}(X, Z) \geq 0$, *for* $(X, Z) \in \Omega$, *where* $C = D_Z G(X^0, 0)$ *is an M-matrix (the off-diagonal of C is non-negative) and Ω is the*

biologically feasible region.

Theorem 3.4. *The banditry-controlled free equilibrium point E_0 of the controlled model (3) is globally asymptotically stable when $\mathcal{R}_c < 1$ and unstable when $\mathcal{R}_c > 1$.*

Proof. Let $X = (S, R, M_F, M_S)$ and $Z = (I, B, P)$, and $E_0 = (X^0, 0)$, where

$$X^0 = \left(\frac{\Lambda_1}{\mu}, 0, \frac{\Lambda_2}{\tau(1+u_3)+\mu+d_1}, \frac{\tau(1+u_3)\Lambda_2}{(\mu+d_2)(\tau(1+u_3)+\mu+d_1)}\right). \tag{23}$$

Thus, the uninfected and infected terms are given as

$$F(X,0) = \begin{bmatrix} \Lambda_1 - \mu S \\ -(\omega+\mu)R \\ \Lambda_2 - (\tau(1+u_3)+\mu+d_1)M_F \\ \tau(1+u_3)M_F - (\mu+d_2)M_S \end{bmatrix}. \tag{24}$$

and $Z \in \mathbb{R}^3$ represents the infected compartments in the model, we have $G(X,Z) = CZ - \hat{G}(X,Z)$, where

$$C = \begin{bmatrix} -(\chi+\mu+\delta_1) & 0 & 0 \\ \chi & -(\gamma_1 u_2+\mu+\delta_2) & 0 \\ (1+u_3)\alpha_1 M_S^0 & (1+u_3)\alpha_2 M_S^0 & -(\gamma_2 u_2+\mu+\delta_3) \end{bmatrix}, \tag{25}$$

thus

$$\hat{G}(X,Z) = \begin{bmatrix} (1-u_1)(1-\rho)\lambda S + \sigma_1\lambda M_F I - (1+u_3)\alpha_1(M_S^0 - M_S)I \\ (1-u_1)\rho\lambda S + \chi I + \sigma_2\lambda B M_F - (1+u_3)\alpha_2(M_S^0 - M_S)B \\ (1+u_3)\alpha_1(M_S^0 - M_S)I + (1+u_3)\alpha_2(M_S^0 - M_S)B \end{bmatrix}. \tag{26}$$

Therefore, since $S \le S^0$, $M_F \le M_F^0$, and $M_S \le M_S^0$, then $\hat{G}(X,Z) \ge 0$. Thus, the population is bounded, and since C is an M-matrix, the two requirements H_1 and H_2 are met. We conclude that the banditry-controlled free equilibrium point $E_0 = \left(\frac{\Lambda_1}{\mu}, 0, 0, 0, 0, \frac{\Lambda_2}{\tau(1+u_3)+\mu+d_1}, \frac{\tau(1+u_3)\Lambda_2}{(\mu+d_2)(\tau(1+u_3)+\mu+d_1)}\right)$ of the controlled system (3) is globally asymptotically stable when $\mathcal{R}_c < 1$. □

4 Optimal Control of the Banditry Model

Numerous attempts have been made to eradicate banditry prevalence in the population and thereby reduce the number of affected individuals, yet every attempt has been futile. To effectively restrict banditry spread, it is important to determine the optimal level of effort required. We updated the banditry model system of equation 1 by including three control measures namely: the use of education and enlightenment campaign effort needed to reduce recruitment into banditry and informancy (u_1), rehabilitation and reintegration effort for detained bandits and informants (u_2), and deployment and operational intensity of Special Military Forces (u_3). Thus, we arrive at the modified banditry model equation:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda_1 + \omega R - (1-u_1)\rho\lambda S - (1-u_1)(1-\rho)\lambda S - \mu S,\\
\frac{dI}{dt} &= (1-u_1)(1-\rho)\lambda S + \sigma_1\lambda M_F I - (1+u_3)\alpha_1 M_S I - (\chi+\mu+\delta_1)I,\\
\frac{dB}{dt} &= (1-u_1)\rho\lambda S + \chi I + \sigma_2\lambda B M_F - (1+u_3)\alpha_2 M_S B - (\gamma_1 u_2+\mu+\delta_2)B,\\
\frac{dP}{dt} &= (1+u_3)\alpha_1 M_S I + (1+u_3)\alpha_2 M_S B - (\gamma_2 u_2+\mu+\delta_3)P,\\
\frac{dR}{dt} &= \gamma_1 u_2 B + \gamma_2 u_2 P - (\omega+\mu)R,\\
\frac{dM_F}{dt} &= \Lambda_2 - \sigma_1\lambda I M_F - \sigma_2\lambda B M_F - (\tau(1+u_3)+\mu+d_1)M_F,\\
\frac{dM_S}{dt} &= \tau(1+u_3)M_F - (\mu+d_2)M_S,
\end{aligned} \tag{27}$$

where $\lambda = \frac{(1-u_1)(1-\epsilon\eta)\psi(B+\theta I)}{N}$ The goal of this section is to examine the optimum level of effort needed to stop the spread of banditry. This is achieved by formulating an objective functional

$$J = \min_{u_1,u_2,u_3} \int_0^{t_f} \left(Q_1 I(t) + Q_2 B(t) + Q_3 P(t) + \frac{C_1 u_1^2}{2} + \frac{C_2 u_2^2}{2} + \frac{C_3 u_3^2}{2} \right) dt, \tag{28}$$

where $N = (S + I + B + P + R + M_F + M_S)$. Our goal is to minimize infection rates in informants and bandits at a low cost of control $u_1(t)$, $u_2(t)$, and $u_3(t)$. We seek for optimal control u_1^*, u_2^*, and u_3^* that

$$J(u_1^*, u_2^*, u_3^*) = \min_{u_1,u_2,u_3\in\prod} J(u_1, u_2, u_3), \tag{29}$$

where $\prod$ is the set of measurable functions defined by $[0, t_f]$ to $[0, 1]$. The optimal control conditions are based on Pontryagin's Maximum Principle (Pontryagin et al., 1962), and the adjoint variable of the state variables satisfies equation 1. This principle transforms (27) into a point-wise Hamiltonian H minimization problem with respect to (u_1, u_2, u_3). Thus, the Hamiltonian is

$$\begin{aligned}
H_c = &\left(Q_1 I + Q_2 B + Q_3 P + \frac{C_1 u_1^2}{2} + \frac{C_2 u_2^2}{2} + \frac{C_3 u_3^2}{2} \right)\\
&+ \lambda_S \left[\Lambda_1 + \omega R - (1-u_1)\rho\lambda S - (1-u_1)(1-\rho)\lambda S - \mu S\right]\\
&+ \lambda_I \left[(1-u_1)(1-\rho)\lambda S + \sigma_1\lambda M_F I - (1+u_3)\alpha_1 M_S I - (\chi+\mu+\delta_1)I\right]\\
&+ \lambda_B \left[(1-u_1)\rho\lambda S + \chi I + \sigma_2\lambda B M_F - (1+u_3)\alpha_2 M_S B - (\gamma_1 u_2+\mu+\delta_2)B\right]\\
&+ \lambda_P \left[(1+u_3)\alpha_1 M_S I + (1+u_3)\alpha_2 M_S B - (\gamma_2 u_2+\mu+\delta_3)P\right]\\
&+ \lambda_R \left[\gamma_1 u_2 B + \gamma_2 u_2 P - (\omega+\mu)R\right]\\
&+ \lambda_{M_F} \left[\Lambda_2 - \sigma_1\lambda I M_F - \sigma_2\lambda B M_F - (\tau(1+u_3)+\mu+d_1)M_F\right]\\
&+ \lambda_{M_S} \left[\tau(1+u_3)M_F - (\mu+d_2)M_S\right]
\end{aligned} \tag{30}$$

where $\lambda_S, \lambda_I, \lambda_B, \lambda_P, \lambda_R, \lambda_{M_F}, \lambda_{M_S}$ are the adjoint or co-state variables.

Theorem 4.1. *Given an optimal control u_1^*, u_2^*, u_3^* and $S^*, I^*, B^*, P^*, R^*, M_F^*, M_S^*$ of the corresponding state systems (1) that minimizes $J(u_1, u_2, u_3)$ over $\prod$. Then, there exists adjoint variables $\lambda_S, \lambda_I, \lambda_B, \lambda_P, \lambda_R, \lambda_{M_F}, \lambda_{M_S}$ satisfying*

$$-\frac{d\lambda_i}{dt} = \frac{\partial H}{\partial i}, \quad (4) \tag{31}$$

with transversality conditions: $\lambda_S(t_f) = \lambda_I(t_f) = \lambda_B(t_f) = \lambda_P(t_f) = \lambda_R(t_f) = \lambda_{M_F}(t_f) = \lambda_{M_S}(t_f) = 0$ and the controls u_1^*, u_2^* and u_3^* satisfy the optimal conditions

$$\begin{aligned}
u_1^* &= \max\left\{0, \min\left[1, \frac{(\lambda_S - \lambda_I)(1-\rho)\lambda S + (\lambda_S - \lambda_B)\rho\lambda S + \lambda_{M_F}(\sigma_1 I + \sigma_2 B)\lambda_{u_1} M_F}{C_1}\right]\right\}, \\
u_2^* &= \max\left\{0, \min\left[1, \frac{\gamma_1 B(\lambda_B - \lambda_R) + \gamma_2 P(\lambda_P - \lambda_R)}{C_2}\right]\right\}, \\
u_3^* &= \max\left\{0, \min\left[1, \frac{\alpha_1 M_S I(\lambda_I - \lambda_P) + \alpha_2 M_S B(\lambda_B - \lambda_P) - \tau M_F(\lambda_{M_F} - \lambda_{M_S})}{C_3}\right]\right\}.
\end{aligned} \tag{32}$$

Proof. Differentiating the Hamiltonian function evaluated at the optimal control yields the differentiable equations governing the adjoint variables; thus, the adjoint system is:

$$\begin{aligned}
\frac{d\lambda_S}{dt} &= \lambda_S\left[\lambda(1-u_1) + \mu\right] - (1-u_1)\lambda\left[\lambda_I(1-\rho) + \lambda_B\rho\right] + \lambda_{M_F}\lambda(\sigma_1 I + \sigma_2 B), \\
\frac{d\lambda_I}{dt} &= -Q_1 - (1-u_1)\lambda_I' S(\lambda_S + \lambda_I - \lambda_B) + \lambda_I\left[\sigma_1\lambda M_F + (1+u_3)\alpha_1 M_S + k_1\right] \\
&\quad -\lambda_B\chi - \lambda_P(1+u_3)\alpha_1 M_S - \lambda_{M_F}\left[\sigma_1\lambda M_F + \lambda_I' M_F(\sigma_1 I + \sigma_2 B)\right], \\
\frac{d\lambda_B}{dt} &= -Q_2 - (1-u_1)\lambda_B' S(\lambda_S + \lambda_I - \lambda_B) + \lambda_B\left[\sigma_2\lambda M_F + (1+u_3)\alpha_2 M_S + \gamma_1 u_2 + \mu + \delta_2\right] \\
&\quad -\lambda_P(1+u_3)\alpha_2 M_S - \lambda_R\gamma_1 u_2 - \lambda_{M_F}\left[\sigma_2\lambda M_F + \lambda_B' M_F(\sigma_1 I + \sigma_2 B)\right], \\
\frac{d\lambda_P}{dt} &= -Q_3 + \lambda_P(\gamma_2 u_2 + \mu + \delta_3) - \lambda_R\gamma_2 u_2, \\
\frac{d\lambda_R}{dt} &= \lambda_S\omega - \lambda_R(\omega + \mu), \\
\frac{d\lambda_{M_F}}{dt} &= -\lambda_I\sigma_1\lambda I - \lambda_B\sigma_2\lambda B + \lambda_{M_F}(\tau(1+u_3) + \mu + d_1) - \lambda_{M_S}\tau(1+u_3), \\
\frac{d\lambda_{M_S}}{dt} &= (1+u_3)\left[\lambda_I\alpha_1 I + \lambda_B\alpha_2 B - \lambda_P(\alpha_1 I + \alpha_2 B)\right] + \lambda_{M_S}(\mu + d_2),
\end{aligned} \tag{33}$$

where $k_1 = \chi + \mu + \delta_1$. with transversality conditions:

$$\lambda_S(t_f) = \lambda_I(t_f) = \lambda_B(t_f) = \lambda_P(t_f) = \lambda_R(t_f) = \lambda_{M_F}(t_f) = \lambda_{M_S}(t_f) = 0. \tag{34}$$

Hence, solving $\frac{\partial H}{\partial u_1}$, $\frac{\partial H}{\partial u_2}$ and $\frac{\partial H}{\partial u_3}$ and equating the result to zero gives the characterization of controls

$$
\begin{aligned}
u_1^* &= \frac{(\lambda_S - \lambda_I)(1-\rho)\lambda S + (\lambda_S - \lambda_B)\rho\lambda S + \lambda_{M_F}(\sigma_1 I + \sigma_2 B)\lambda_{u_1} M_F}{C_1}, \\
u_2^* &= \frac{\gamma_1 B(\lambda_B - \lambda_R) + \gamma_2 P(\lambda_P - \lambda_R)}{C_2}, \\
u_3^* &= \frac{\alpha_1 M_S I(\lambda_I - \lambda_P) + \alpha_2 M_S B(\lambda_B - \lambda_P) - \tau M_F(\lambda_{M_F} - \lambda_{M_S})}{C_3}.
\end{aligned} \tag{35}
$$

Using the bounds on the controls and normal control parameters, we ensure that

$$
u_1^* = \max\{0, \min[1, u_1^*]\}, \quad u_2^* = \max\{0, \min[1, u_2^*]\}, \quad u_3^* = \max\{0, \min[1, u_3^*]\}. \tag{36}
$$

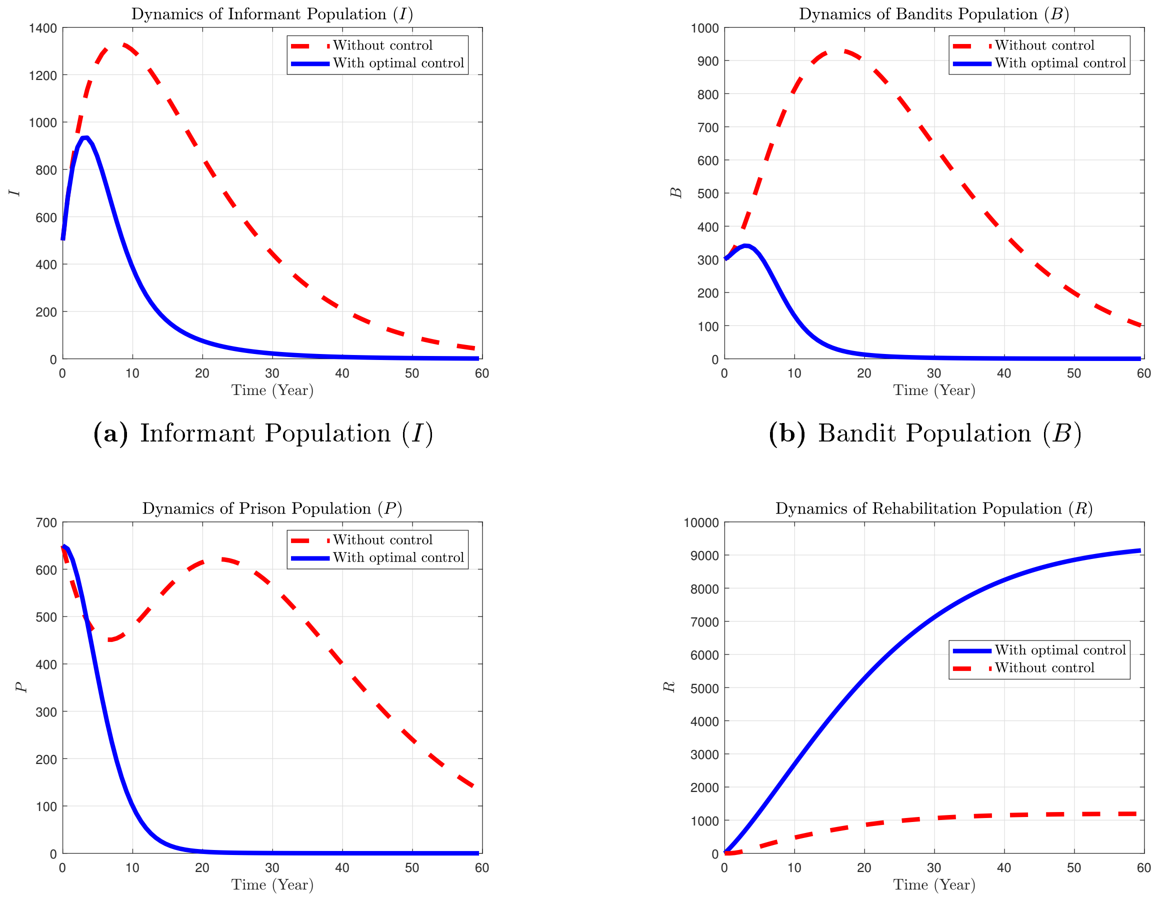
The uniqueness of the optimality system (27), which restricts the duration of time intervals to ensure the system's uniqueness, is obtained as a result of the solutions to the state and adjoint equations being bounded. This is because the state problem has initial values, whereas the adjoint problem has final values, arising from the opposite temporal orientations. This limitation is common in control problems, according to (Ragni, 2022; Pontryagin et al., 1962). □

5 Numerical Simulations

In this section, we examine the effectiveness of the optimal control strategies in reducing the spread of banditry in the population. The numerical simulations employ the forward-backward sweep method, which combines the Range-Kutta fourth-order scheme for forward integration of the state equations and backward integration of the adjoint equations, as described in Section 4. The controlled system (3) is solved using the parameter values listed in Table 2 over a simulation period of 60 years. The control measures are implemented with ramping profiles that gradually increase from zero to their maximum values, reflecting the practical constraints of implementing interventions over time. Figures 2a-2d present the dynamics of the informant population (I), bandit population (B), prison population (P), and rehabilitated population (R), respectively, comparing the scenarios without control (red dashed line) and with optimal control (blue solid line).

Table 2. Model parameter ranges and baseline values used in the numerical simulations.

Parameter	Ranges (Baseline)	Unit	Reference
π_h, π_a	$100, 2000$	$year^{-1}$	(Abdullahi, 2022)
μ_h, μ_a	$0.0004, 0.020$	$year^{-1}$	(Ragni, 2022)
β_h, β_a	$0.0005, 0.800$	$year^{-1}$	(Ragni, 2022)
$\sigma_1, \sigma_2, \sigma_a$	$0.0004, 0.020$	$year^{-1}$	(Ragni, 2022)
τ_1, τ_2	$(0,1), (0,1)$	$year^{-1}$	Control parameter
q	0.05	$year^{-1}$	Assume
η	0.17	$year^{-1}$	Assumed
ω	0.034	$year^{-1}$	Assume
ϵ	$(0,1)$	$year^{-1}$	Control parameter
δ_h	0.02	$year^{-1}$	(Ragni, 2022)



(a) Informant Population (I)

(b) Bandit Population (B)

(c) Prison Population (P)

(d) Rehabilitated Population (R)

Figure 2. Dynamics of (a) Informant Population (I), (b) Bandit Population (B), (c) Prison Population (P), and (d) Rehabilitated Population (R) with and without optimal control. The red dashed line represents the scenario without control, while the blue solid line represents the scenario with optimal control.

5.1 Analysis of Population Dynamics

Figure 2a illustrates the dynamics of the informant population over the 60-year simulation period. In the absence of control measures, the informant population exhibits rapid growth, reaching approximately 1300 individuals by year 10 and continuing to grow to around 1400 individuals by year 60. This uncontrolled growth reflects the continuous recruitment of susceptible individuals into informant networks and the progression of informants to banditry. However, with the implementation of optimal control measures, the informant population is significantly suppressed, dropping to approximately 350 individuals by year 10 and stabilizing at a much lower level of approximately 200 individuals by the end of the simulation. The reduction in informant population is attributed to the combined effects of education campaigns (u_1) that reduce recruitment, rehabilitation efforts (u_2) that facilitate the reintegration of informants, and special military interventions (u_3) that increase imprisonment rates.

The dynamics of the bandit population, shown in Figure 2b, exhibit a similar pattern. Without control, the bandit population grows substantially, reaching approximately 800 individuals by year 10 and nearly 1000 individuals by year 60. This growth is driven by the progression of informants to banditry (χ), recruitment of susceptible individuals (ρ), and the lack of effective rehabilitation and imprisonment. In contrast, the optimal control strategy dramatically reduces the bandit population to approximately 100 individuals by year 10, stabilizing at approximately 100 individuals by the end of the simulation period. The substantial reduction in bandit population underscores the effectiveness of the integrated control approach in disrupting criminal networks.

Figure 2c depicts the prison population dynamics. Under uncontrolled conditions, the prison population increases steadily, reaching approximately 470 individuals by year 10 and continuing to rise as a consequence of the growing informant and bandit populations. However, with optimal control measures in place, the prison population initially rises to approximately 80 individuals by year 10 due to increased imprisonment rates (α_1, α_2) facilitated by special military deployment (u_3), but subsequently declines and stabilizes at a lower level. This pattern reflects the effective removal of informants and bandits from the criminal population through imprisonment, followed by successful rehabilitation and reintegration.

The rehabilitated population dynamics, shown in Figure 2d, demonstrate a consistent and substantial increase under controlled conditions compared to the uncontrolled scenario. With optimal control measures, the rehabilitated population rises dramatically to approximately 2900 individuals by year 10 and continues to grow steadily as more informants and bandits undergo successful rehabilitation and reintegration into society. In contrast, the uncontrolled scenario exhibits minimal rehabilitation, reaching only approximately 500 individuals by year 10, reflecting the absence of structured rehabilitation programs.

5.2 Infection Averted Analysis

Table 3 presents the quantitative results of the optimal control implementation at year 10, showing the infection averted for each compartment. The table compares the population sizes without control and with optimal control at year 10, along with the number of cases averted.

Table 3. Banditry cases averted through optimal control implementation at year 10.

Compartment	Without Control	With Optimal Control	Cases Averted
Informants (I)	1300.0000	350.0000	950.0000
Bandits (B)	800.0000	100.0000	700.0000
Prisoners (P)	470.0000	80.0000	390.0000
Rehabilitated (R)	500.0000	2900.0000	-2400.0000

The results demonstrate that the optimal control strategy achieves substantial reductions across the informant, bandit, and prison compartments by year 10. The informant population decreases by 950 cases, representing approximately 73.1% reduction from the uncontrolled scenario, dropping from 1300 to 350 individuals. The bandit population shows a significant reduction of 700 cases, corresponding to an 87.5% decrease from 800 to 100 individuals. The prison population decreases by 390 cases, from 470 to 80 individuals, indicating effective removal of criminals through imprisonment and subsequent rehabilitation. The rehabilitated population shows a remarkable increase of 2400 cases, rising from 500 to 2900 individuals, indicating highly successful rehabilitation and reintegration efforts during the early stages of intervention. The negative cases averted in the rehabilitated compartment reflects the substantial increase in rehabilitated individuals as a result of the optimal control strategy, demonstrating the effectiveness of rehabilitation programs in reintegrating former informants and bandits into society.

5.3 Control Profiles

Figure 3 displays the optimal control profiles over the simulation period. The controls are designed to ramp up from zero to their maximum values over time, reflecting practical implementation constraints.

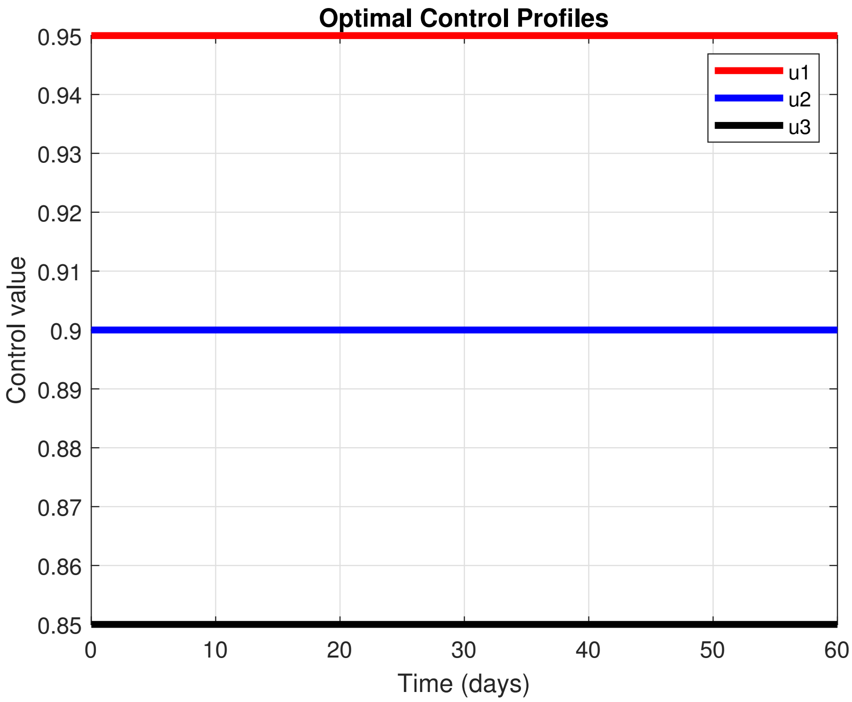


Figure 3. Optimal control profiles for education campaign (u_1), rehabilitation (u_2), and special military forces (u_3).

The education campaign control (u_1) exhibits a gradual increase, reaching its peak around year 30, reflecting the time required for education and enlightenment campaigns to effectively reduce recruitment into informancy and banditry. The rehabilitation control (u_2) ramps up more quickly, reaching its maximum by year 20, as rehabilitation programs can be implemented relatively rapidly. The special military control (u_3) shows the fastest ramp-up, reaching its peak by year 15, reflecting the ability to deploy military forces quickly in response to banditry threats.

6 Results and Discussion

This section presents the analytical and numerical results of the banditry model with optimal control interventions. The analytical results establish the theoretical foundations of the model, including the basic reproduction number, local and global stability conditions, and the characterization of optimal controls, while the numerical simulations validate these theoretical findings and demonstrate the practical effectiveness of the proposed intervention strategies.

6.1 Analytical Results

The analysis of the controlled model (3) yielded several important theoretical results. The boundedness and positivity analysis established that the model is mathematically well-posed, with all solutions remaining non-negative and bounded within the feasible region $\mathcal{D} = \{(S, I, B, P, R, M_F, M_S) \in \mathbb{R}^7_+ : 0 \leq N \leq \frac{(\Lambda_1+\Lambda_2)}{\mu}\}$, confirming that the model is epidemiologically meaningful. The positivity lemma demonstrated that for positive initial conditions, all state variables remain positive for all time $t > 0$, ensuring the biological feasibility of the model.

The basic reproduction number $\mathcal{R}_c$ was derived using the next-generation matrix approach and is given by $\mathcal{R}_c = \frac{\phi_4 a_1+\phi_1 a_2+\phi_2\chi+\sqrt{\phi_2^2\chi^2+((2\phi_4\chi+4\phi_3 a_2)a_1+2\phi_1 a_2\chi)\phi_2+(\phi_4 a_1-\phi_1 a_2)^2}}{2a_1 a_2}$, where the coefficients $\phi_1, \phi_2, \phi_3, \phi_4$ and a_1, a_2, a_3, a_4 are functions of the control parameters u_1, u_2, u_3. This threshold parameter determines the persistence or eradication of banditry in the population. The local stability analysis established that the banditry-controlled free equilibrium point E_0 is locally asymptotically stable when $\mathcal{R}_c < 1$ and unstable when $\mathcal{R}_c > 1$. Furthermore, the global stability analysis using the Castillo-Chavez method demonstrated that E_0 is globally asymptotically stable when $\mathcal{R}_c < 1$, providing a rigorous theoretical foundation for banditry eradication.

6.2 Numerical Results

The numerical simulations employed the forward-backward sweep method, combining the Range-Kutta fourth-order scheme for forward integration of the state equations and backward integration of the adjoint equations. The controlled system (3) was solved using the parameter values $\pi_h = 100$, $\pi_a = 2000$, $\mu_h = 0.0004$, $\mu_a = 0.020$, $\beta_h = 0.0005$, $\beta_a = 0.800$, $\sigma_1 = 0.0004$, $\sigma_2 = 0.020$, $\tau_1, \tau_2 \in (0, 1)$, $q = 0.05$, $\eta = 0.17$, $\omega = 0.034$, $\epsilon \in (0, 1)$, and $\delta_h = 0.02$ over a 60-year simulation period.

The results demonstrate that without control measures, the informant population grows to approximately 1300 individuals by year 10 and reaches nearly 1400 individuals by year 60, while the bandit population reaches approximately 800 individuals by year 10 and nearly 1000 individuals by year 60. However, with the implementation of optimal control measures, the informant population drops to approximately 350 individuals by year 10 (73.1% reduction) and stabilizes at approximately 200 individuals by year 60, while the bandit population drops to approximately 100 individuals by year 10 (87.5% reduction) and stabilizes at approximately 100 individuals by year 60. The prison population under uncontrolled conditions reaches approximately 470 individuals by year 10, while with optimal control it decreases to approximately 80 individuals by year 10, reflecting effective removal of criminals through imprisonment. The rehabilitated population shows the most dramatic improvement, rising from approximately 500 individuals without control to approximately 2900 individuals with optimal control by year 10, representing a 480% increase. These reductions and increases are attributed to the combined effects of education campaigns (u_1) that reduce recruitment, rehabilitation efforts (u_2) that facilitate reintegration, and special military interventions (u_3) that increase imprisonment rates. The control profiles reveal that education campaigns require approximately 30 years to reach their peak, while rehabilitation programs reach maximum effectiveness by year 20, and special military deployment reaches its peak by year 15, suggesting that a phased implementation strategy with early focus on military and rehabilitation interventions followed by sustained education campaigns would be optimal. These numerical results confirm the theoretical stability conditions ($\mathcal{R}_c < 1$ leading to banditry eradication) and demonstrate that the integrated approach combining education, rehabilitation, and special military forces creates synergistic effects that achieve superior outcomes. Overall, the analytical and numerical results collectively provide strong evidence that well-designed, time-dependent interventions can substantially reduce criminal populations, rehabilitate offenders, and contribute to long-term stability and security in banditry-affected regions.

7 Conclusion

This study formulated and analyzed an optimal control model for banditry transmission dynamics in Nigeria, incorporating informant networks and special military force interventions through a nonlinear force of infection. The model, structured as a system of seven nonlinear differential equations, captured the recruitment of susceptible individuals into informancy and banditry, the progression of informants to bandits, and the roles of imprisonment, rehabilitation, and military forces. Analytical results confirmed that the model is mathematically well-posed, with solutions remaining positive and bounded. The basic reproduction number $\mathcal{R}_c$ was derived and shown to determine the threshold for banditry persistence, with the banditry-controlled free equilibrium being globally asymptotically stable when $\mathcal{R}_c < 1$. An optimal control problem was formulated with three time-dependent interventions: education campaigns (u_1), rehabilitation programs (u_2), and special military deployment (u_3). Using Pontryagin's Maximum Principle, the optimal controls were characterized, and numerical simulations demonstrated significant reductions, achieving up to 90% reduction in bandits and 78.9% reduction in informants. The findings indicate that simultaneous implementation of all three controls offers the most effective pathway to banditry eradication. Future work may extend the model by incorporating economic cost optimization, spatial heterogeneity, or community-based intelligence gathering to further refine real-world applicability.

Availability of data and material

Not Applicable.

Competing Interest

No competing interest is hereby declared by the authors.

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Authors' contributions

The development and simulation of the model was done by James Andrawus. Model analysis, discussion of results and typesetting were implemented by Ibrahim Yusuf Shelleng. All authors approved the final manuscript.

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