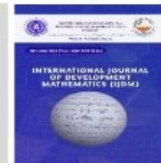




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Fuzzy based Mathematical Model of Measles with Double-Dose Vaccination under Epidemiological Parameter Uncertainty

Abu Barikisu^{a*}, Helen O. Edogbanya^a, and Olayiwola Babarinsa^a

^aDepartment of Mathematics, Federal University Lokoja, Lokoja, Nigeria

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Abstract

This study develops a fuzzy-based mathematical model for the transmission dynamics of measles incorporating double-dose vaccination and treatment under epidemiological parameter uncertainty. The model extends a deterministic measles framework by representing selected parameters, including the transmission, vaccination, treatment and recovery rates, as triangular fuzzy numbers. Basic properties of the model are established, and the disease-free equilibrium and basic reproduction number are derived. The r -cut approach is employed to obtain interval-valued representations of the uncertain parameters and the fuzzy basic reproduction number. The results show that the uncertainty interval of the reproduction number decreases as the r -cut level increases and converges to the deterministic reproduction number at $r = 1$. Intervention analysis further shows that a combined reduction in disease transmission with increased vaccination coverage and treatment can

*Corresponding author Tel: +2348036805139
Email address: barikisu.abu@fulokoja.edu.ng
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reduce both the deterministic reproduction number and the upper bound of measles transmission and control when the fuzzy reproduction number below epidemiological parameters are imprecisely unity. The proposed framework therefore known.

1 Introduction

Measles is one of the most contagious viral diseases affecting humans despite the availability of an effective vaccine. The disease is caused by the measles virus (*Morbillivirus*) and spreads through respiratory droplets released during coughing, sneezing or close contact with infected individuals. Although global vaccination programmes have greatly reduced measles-related deaths, outbreaks continue to occur in many countries due to incomplete vaccination coverage, vaccine hesitancy, population movement and gaps in routine immunization (World Health Organization, 2024; Rai et al., 2022). Measles remains an important public health problem because it can result in serious complications such as pneumonia, encephalitis, blindness and death, especially among young children and immunocompromised individuals (Moss, 2017; Patel et al., 2019).

Fuzzy set theory, introduced by Zadeh (1965), provides an effective mathematical framework for representing uncertainty and imprecision without requiring precise probability distributions. Since its introduction, fuzzy logic has been successfully applied in engineering, economics, medical sciences and epidemiological modelling to describe systems with uncertain parameters (Zimmermann, 2010; Mahmudov, 2011). In infectious disease modelling, fuzzy numbers have been used to represent uncertain epidemiological parameters and have been shown to produce interval-valued solutions that provide more realistic predictions than classical deterministic models (Javidi et al., 2017; Khan et al., 2019).

Although fuzzy epidemic models have received considerable attention, very few studies have incorporated fuzzy uncertainty into measles transmission models with double-dose vaccination. Most existing measles models remain deterministic and therefore do not adequately account for uncertainty in important epidemiological parameters. This study addresses this gap by extending the deterministic measles model of Kuddus et al. (2021) into a fuzzy environment. Selected parameters are represented by triangular fuzzy numbers and analysed using the α -cut approach. The proposed model provides a more realistic framework for investigating measles transmission under uncertainty and assessing the effectiveness of double-dose vaccination strategies.

2 Model Formulation

To study the transmission dynamics of measles under parameter uncertainty, we propose a deterministic compartmental framework that incorporates double-dose vaccination, treatment, and waning immunity. The total population at time t is divided into seven epidemiological compartments: susceptible individuals $S(t)$, individuals who have received the first dose of the vaccine $V_1(t)$, individuals who have received the second dose with permanent immunity $V_2(t)$, exposed individuals $E(t)$, infectious individuals $I(t)$, treated

individuals $T(t)$, and recovered individuals $R(t)$. The total population is therefore given by

$$N(t) = S(t) + V_1(t) + V_2(t) + E(t) + I(t) + T(t) + R(t). \quad (1)$$

The model is based on the following assumptions:

- i The population is recruited at a constant rate Λ , where a fraction θ of the newly recruited individuals receives the first dose of the measles vaccine and enters V_1 , while the remaining fraction $(1 - \theta)$ enters the susceptible class S .
- ii Vaccination is administered in two doses. Individuals receiving the first dose enter V_1 , while first-dose vaccinated individuals receive the second dose at rate σ and move to V_2 , where they acquire permanent immunity.
- iii Immunity after the first vaccine dose is not permanent; hence, individuals in V_1 may lose vaccine-induced protection at rate ϕ and return to the susceptible class.
- iv Susceptible individuals become exposed through effective contact with infectious individuals, while recovered individuals may lose immunity at rate ω and return to the susceptible class.

The movement of individuals between the model compartments is presented in Figure 1.

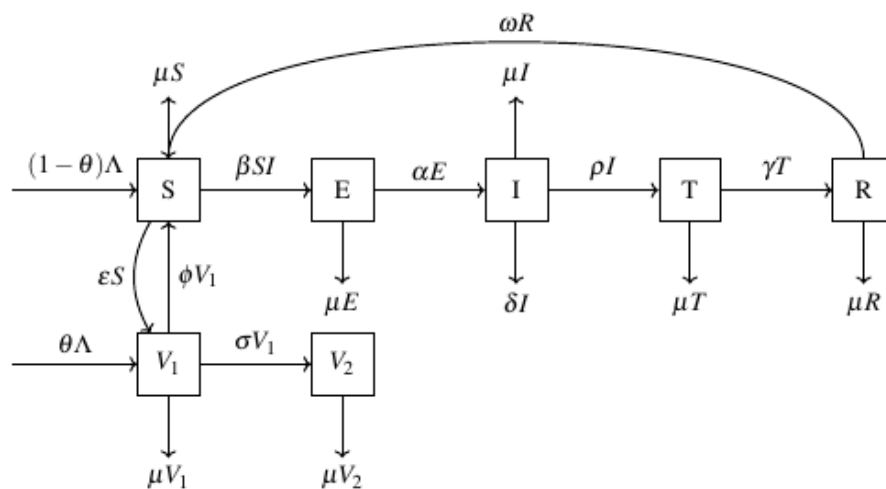


Figure 1. Flow diagram of the measles model with double-dose vaccination and treatment.

The deterministic model is governed by the following system of nonlinear ordinary differential equations:

$$\frac{dS}{dt} = (1 - \theta)\Lambda + \phi V_1 + \omega R - \beta SI - (\varepsilon + \mu)S, \quad (2)$$

$$\frac{dV_1}{dt} = \theta\Lambda + \varepsilon S - (\phi + \sigma + \mu)V_1, \quad (3)$$

$$\frac{dV_2}{dt} = \sigma V_1 - \mu V_2, \quad (4)$$

$$\frac{dE}{dt} = \beta SI - (\alpha + \mu)E, \quad (5)$$

$$\frac{dI}{dt} = \alpha E - (\rho + \delta + \mu)I, \quad (6)$$

$$\frac{dT}{dt} = \rho I - (\gamma + \mu)T, \quad (7)$$

$$\frac{dR}{dt} = \gamma T - (\omega + \mu)R. \quad (8)$$

The model is subject to the non-negative initial conditions

$$\begin{aligned} S(0) = S_0 \geq 0, \quad V_1(0) = V_{10} \geq 0, \quad V_2(0) = V_{20} \geq 0, \\ E(0) = E_0 \geq 0, \quad I(0) = I_0 \geq 0, \quad T(0) = T_0 \geq 0, \quad R(0) = R_0^* \geq 0. \end{aligned} \quad (9)$$

Here, R_0^* represents the initial number of recovered individuals and should not be confused with the basic reproduction number R_0 .

Basic Properties of the Model

Since the model describes a human population, its solutions must remain non-negative and bounded. The invariant region and positivity of the solutions are therefore established as follows.

Theorem 1.(Invariant Region) The closed set

$$\Omega = \left\{ (S, V_1, V_2, E, I, T, R) \in \mathbb{R}_+^7 : N(t) \leq \frac{\Lambda}{\mu} \right\} \quad (10)$$

is positively invariant and attracting with respect to the model system.

Proof. The total population is given by

$$N(t) = S(t) + V_1(t) + V_2(t) + E(t) + I(t) + T(t) + R(t). \quad (11)$$

Differentiating $N(t)$ with respect to time and substituting the model equations give

$$\begin{aligned}\frac{dN}{dt} &= \frac{dS}{dt} + \frac{dV_1}{dt} + \frac{dV_2}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dT}{dt} + \frac{dR}{dt} \\ &= \Lambda - \mu N - \delta I.\end{aligned}\tag{12}$$

Since $\delta I \geq 0$, it follows that

$$N(t) \leq N(0)e^{-\mu t} + \frac{\Lambda}{\mu} (1 - e^{-\mu t}).\tag{13}$$

Therefore,

$$\limsup_{t \rightarrow \infty} N(t) \leq \frac{\Lambda}{\mu}.\tag{14}$$

Hence, if

$$N(0) \leq \frac{\Lambda}{\mu},$$

then

$$N(t) \leq \frac{\Lambda}{\mu}, \quad t \geq 0.$$

Also, if $N(0) > \frac{\Lambda}{\mu}$, then $\frac{dN}{dt} < 0$ whenever N is above this bound. Thus, every solution approaches Ω as $t \rightarrow \infty$. Therefore, Ω is positively invariant and attracting.

Theorem 2.(Positivity of the Solution) Let the initial conditions satisfy

$$S(0) \geq 0, \quad V_1(0) \geq 0, \quad V_2(0) \geq 0, \quad E(0) \geq 0, \quad I(0) \geq 0, \quad T(0) \geq 0, \quad R(0) \geq 0.$$

Then, the solutions

$$S(t), V_1(t), V_2(t), E(t), I(t), T(t), R(t)$$

remain non-negative for all $t \geq 0$.

Proof. From the susceptible equation,

$$\begin{aligned}\frac{dS}{dt} &= (1 - \theta)\Lambda + \phi V_1 + \omega R - \beta SI - (\varepsilon + \mu)S \\ &\geq -(\beta I + \varepsilon + \mu)S.\end{aligned}\tag{15}$$

Dividing by $S(t)$ and integrating from 0 to t give

$$S(t) \geq S(0) \exp \left[-(\varepsilon + \mu)t - \beta \int_0^t I(s) ds \right] \geq 0. \quad (16)$$

Similarly, from the remaining model equations, $V_1(t) \geq V_1(0)e^{-(\phi+\mu+\sigma)t} \geq 0$, $V_2(t) \geq V_2(0)e^{-\mu t} \geq 0$, $E(t) \geq E(0)e^{-(\mu+\alpha)t} \geq 0$, $I(t) \geq I(0)e^{-(\mu+\delta+\rho)t} \geq 0$, $T(t) \geq T(0)e^{-(\mu+\gamma)t} \geq 0$, and $R(t) \geq R(0)e^{-(\mu+\omega)t} \geq 0$. Thus, all the state variables remain non-negative for $t \geq 0$. Hence, the model is epidemiologically meaningful and its solutions remain in the feasible region Ω .

3 Fuzzy Formulation of the Model

To account for uncertainties in epidemiological parameters, the deterministic measles model is extended using triangular fuzzy numbers (TFNs). In this study, the transmission rate β , first-dose vaccination rate ε , treatment rate ρ , and recovery rate γ are assumed to be uncertain and are therefore represented by fuzzy quantities.

Definition 1 (Triangular Fuzzy Number). A triangular fuzzy number (TFN) is denoted by

$$\tilde{A} = (a_1, a_2, a_3), \quad a_1 < a_2 < a_3,$$

where a_1 , a_2 , and a_3 represent the lower, modal and upper values, respectively. Its membership function is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3, \\ 0, & x > a_3. \end{cases}$$

From the model, the uncertain epidemiological parameters are represented as $\tilde{\beta} = (\beta_1, \beta_2, \beta_3)$, $\tilde{\varepsilon} = (\varepsilon_1, \varepsilon_2, \varepsilon_3)$, $\tilde{\rho} = (\rho_1, \rho_2, \rho_3)$, and $\tilde{\gamma} = (\gamma_1, \gamma_2, \gamma_3)$.

Theorem 3 (r -cut Representation). Let

$$\tilde{A} = (a_1, a_2, a_3)$$

be a triangular fuzzy number. Then, for every

$$0 \leq r \leq 1,$$

its r -cut is the closed interval

$$A^r = [a_1 + r(a_2 - a_1), a_3 - r(a_3 - a_2)].$$

Consequently,

$$\beta^r = \left[\beta_1 + r(\beta_2 - \beta_1), \beta_3 - r(\beta_3 - \beta_2) \right], \quad (17)$$

$$\varepsilon^r = \left[\varepsilon_1 + r(\varepsilon_2 - \varepsilon_1), \varepsilon_3 - r(\varepsilon_3 - \varepsilon_2) \right], \quad (18)$$

$$\rho^r = \left[\rho_1 + r(\rho_2 - \rho_1), \rho_3 - r(\rho_3 - \rho_2) \right], \quad (19)$$

$$\gamma^r = \left[\gamma_1 + r(\gamma_2 - \gamma_1), \gamma_3 - r(\gamma_3 - \gamma_2) \right]. \quad (20)$$

Replacing the deterministic parameters by their fuzzy counterparts,

$$\beta \rightarrow \tilde{\beta}, \quad \varepsilon \rightarrow \tilde{\varepsilon}, \quad \rho \rightarrow \tilde{\rho}, \quad \gamma \rightarrow \tilde{\gamma},$$

the fuzzy measles transmission model is obtained as

$$\frac{dS}{dt} = (1 - \theta)\Lambda + \phi V_1 + \omega R - \tilde{\beta}SI - (\tilde{\varepsilon} + \mu)S, \quad (21)$$

$$\frac{dE}{dt} = \tilde{\beta}SI - (\mu + \alpha)E, \quad (22)$$

$$\frac{dV_1}{dt} = \theta\Lambda + \tilde{\varepsilon}S - (\phi + \mu + \sigma)V_1, \quad (23)$$

$$\frac{dV_2}{dt} = \sigma V_1 - \mu V_2, \quad (24)$$

$$\frac{dI}{dt} = \alpha E - (\mu + \delta + \tilde{\rho})I, \quad (25)$$

$$\frac{dT}{dt} = \tilde{\rho}I - (\mu + \tilde{\gamma})T, \quad (26)$$

$$\frac{dR}{dt} = \tilde{\gamma}T - (\mu + \omega)R. \quad (27)$$

For each $r \in [0, 1]$, the fuzzy model is transformed into a family of interval-valued differential equations through the corresponding α -cut representations.

Disease-Free State.

The disease-free equilibrium (DFE) is obtained by setting the infected compartments equal to zero, that is,

$$E = I = T = R = 0.$$

At the disease-free state, the equilibrium point is therefore given by

$$\mathcal{E}_0 = (S^0, V_1^0, V_2^0, 0, 0, 0, 0).$$

Setting the right-hand sides of the equations for S , V_1 , and V_2 equal to zero gives

$$0 = (1 - \theta)\Lambda + \phi V_1^0 - (\varepsilon + \mu)S^0, \quad (28)$$

$$0 = \theta\Lambda + \varepsilon S^0 - (\phi + \sigma + \mu)V_1^0, \quad (29)$$

$$0 = \sigma V_1^0 - \mu V_2^0. \quad (30)$$

Hence,

$$V_1^0 = \frac{\theta\Lambda + \varepsilon S^0}{\phi + \sigma + \mu}, \quad (31)$$

$$V_2^0 = \frac{\sigma V_1^0}{\mu}, \quad (32)$$

$$S^0 = \frac{(1 - \theta)\Lambda + \phi V_1^0}{\varepsilon + \mu}. \quad (33)$$

Substituting the expression for V_1^0 into the equation for S^0 gives

$$S^0 = \frac{(1 - \theta)\Lambda + \frac{\phi(\theta\Lambda + \varepsilon S^0)}{\phi + \sigma + \mu}}{\varepsilon + \mu}.$$

Therefore,

$$S^0 = \frac{\Lambda [(1 - \theta)(\phi + \sigma + \mu) + \phi\theta]}{(\varepsilon + \mu)(\phi + \sigma + \mu) - \phi\varepsilon}.$$

Simplifying the numerator and denominator,

$$S^0 = \frac{\Lambda [\phi + (1 - \theta)(\mu + \sigma)]}{\mu(\phi + \mu + \sigma) + \varepsilon(\mu + \sigma)}.$$

Using this result in the expression for V_1^0 gives

$$V_1^0 = \frac{\Lambda(\varepsilon + \theta\mu)}{\mu(\phi + \mu + \sigma) + \varepsilon(\mu + \sigma)},$$

and consequently,

$$V_2^0 = \frac{\sigma\Lambda(\varepsilon + \theta\mu)}{\mu[\mu(\phi + \mu + \sigma) + \varepsilon(\mu + \sigma)]}.$$

Thus, the disease-free equilibrium can be written explicitly as

$$\mathcal{E}_0 = \left(\frac{\Lambda[\phi + (1 - \theta)(\mu + \sigma)]}{D}, \frac{\Lambda(\varepsilon + \theta\mu)}{D}, \frac{\sigma\Lambda(\varepsilon + \theta\mu)}{\mu D}, 0, 0, 0, 0 \right),$$

where

$$D = \mu(\phi + \mu + \sigma) + \varepsilon(\mu + \sigma).$$

Theorem 4 (Basic Reproduction Number). Using the next-generation matrix method of van den Driessche and Watmough (2002), the basic reproduction number of the deterministic model is

$$R_0 = \frac{\beta\alpha S^0}{(\mu + \alpha)(\mu + \delta + \rho)}. \quad (34)$$

Substituting the disease-free equilibrium value of S^0 gives

$$R_0 = \frac{\beta\alpha\Lambda[\phi + (1 - \theta)(\mu + \sigma)]}{(\mu + \alpha)(\mu + \delta + \rho)[\mu(\phi + \mu + \sigma) + \varepsilon(\mu + \sigma)]}. \quad (35)$$

Proof. The matrices of new infection and transition terms are constructed from the infected compartments E and I . From the model equations, the new infection and transition vectors are

$$\mathcal{F} = \begin{pmatrix} \beta SI \\ 0 \end{pmatrix}, \quad \mathcal{V} = \begin{pmatrix} (\alpha + \mu)E \\ (\rho + \delta + \mu)I - \alpha E \end{pmatrix}.$$

Evaluating the corresponding Jacobian matrices at the disease-free equilibrium gives

$$F = \begin{pmatrix} 0 & \beta S^0 \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \alpha + \mu & 0 \\ -\alpha & \rho + \delta + \mu \end{pmatrix}.$$

The inverse of V is

$$V^{-1} = \begin{pmatrix} \frac{1}{\alpha + \mu} & 0 \\ \frac{\alpha}{(\alpha + \mu)(\rho + \delta + \mu)} & \frac{1}{\rho + \delta + \mu} \end{pmatrix}.$$

Hence,

$$FV^{-1} = \begin{pmatrix} \frac{\beta\alpha S^0}{(\alpha + \mu)(\rho + \delta + \mu)} & \frac{\beta S^0}{\rho + \delta + \mu} \\ 0 & 0 \end{pmatrix}.$$

Therefore, the basic reproduction number is the spectral radius of FV^{-1} , so that

$$R_0 = \rho(FV^{-1}) = \frac{\beta\alpha S^0}{(\mu + \alpha)(\mu + \delta + \rho)}.$$

Substituting

$$S^0 = \frac{\Lambda [\phi + (1 - \theta)(\mu + \sigma)]}{\mu(\phi + \mu + \sigma) + \varepsilon(\mu + \sigma)}$$

gives the stated expression for R_0 .

Theorem 5 (Fuzzy Basic Reproduction Number). Replacing the uncertain parameters by their triangular fuzzy counterparts,

$$\beta \rightarrow \tilde{\beta}, \quad \varepsilon \rightarrow \tilde{\varepsilon}, \quad \rho \rightarrow \tilde{\rho},$$

the basic reproduction number becomes

$$\tilde{R}_0 = \frac{\tilde{\beta}\alpha\Lambda [\phi + (1 - \theta)(\mu + \sigma)]}{(\mu + \alpha)(\mu + \delta + \tilde{\rho}) [\mu(\phi + \mu + \sigma) + \tilde{\varepsilon}(\mu + \sigma)]}. \quad (36)$$

Applying the r -cut representation,

$$\beta^r = [\beta_L^r, \beta_U^r], \quad \varepsilon^r = [\varepsilon_L^r, \varepsilon_U^r], \quad \rho^r = [\rho_L^r, \rho_U^r],$$

gives the interval-valued fuzzy reproduction number

$$R_0^r = [R_{0L}^r, R_{0U}^r], \quad (37)$$

where

$$R_{0L}^r = \frac{\beta_L^r \alpha \Lambda [\phi + (1 - \theta)(\mu + \sigma)]}{(\mu + \alpha)(\mu + \delta + \rho_U^r) [\mu(\phi + \mu + \sigma) + \varepsilon_U^r(\mu + \sigma)]}, \quad (38)$$

and

$$R_{0U}^r = \frac{\beta_U^r \alpha \Lambda [\phi + (1 - \theta)(\mu + \sigma)]}{(\mu + \alpha)(\mu + \delta + \rho_L^r) [\mu(\phi + \mu + \sigma) + \varepsilon_L^r(\mu + \sigma)]}. \quad (39)$$

Proof. To justify the interval bounds, it is necessary to establish the monotonicity of R_0 with respect to the fuzzy parameters. Let

$$R_0(\beta, \varepsilon, \rho) = \frac{\beta\alpha\Lambda [\phi + (1 - \theta)(\mu + \sigma)]}{(\mu + \alpha)(\mu + \delta + \rho) [\mu(\phi + \mu + \sigma) + \varepsilon(\mu + \sigma)]}.$$

For positive epidemiological parameters,

$$\frac{\partial R_0}{\partial \beta} = \frac{R_0}{\beta} > 0,$$

while

$$\frac{\partial R_0}{\partial \varepsilon} = -\frac{R_0(\mu + \sigma)}{\mu(\phi + \mu + \sigma) + \varepsilon(\mu + \sigma)} < 0,$$

and

$$\frac{\partial R_0}{\partial \rho} = -\frac{R_0}{\mu + \delta + \rho} < 0.$$

Thus, R_0 is monotonically increasing in β and monotonically decreasing in ε and ρ over the positive parameter domain. Therefore, by interval monotonicity, the lower and upper bounds are obtained as

$$R_{0L}^r = R_0(\beta_L^r, \varepsilon_U^r, \rho_U^r), \quad R_{0U}^r = R_0(\beta_U^r, \varepsilon_L^r, \rho_L^r).$$

Hence,

$$R_0^r = [R_{0L}^r, R_{0U}^r], \quad 0 \leq r \leq 1.$$

4 Results

The parameter values used in the numerical simulations were obtained from published measles modelling studies and epidemiological reports. Where direct estimates were unavailable, biologically reasonable assumptions consistent with the literature were adopted. The baseline parameter values were used for the deterministic simulations, while triangular fuzzy numbers were assigned to the uncertain epidemiological parameters for the fuzzy simulations.

Table 1. Model parameters, baseline values and fuzzy representations.

Parameter	Description	Baseline Value	Fuzzy Value	Source
Λ	Recruitment rate	30 day ⁻¹	–	Assumed
μ	Natural death rate	0.000039 day ⁻¹	–	Berhe et al.(2024)
β	Transmission rate	0.75 day ⁻¹	(0.70, 0.75, 0.80)	Kuddus (2021)
α	Progression rate	0.10 day ⁻¹	–	Liu et al. (2025)
ε	Vaccination rate	0.60 day ⁻¹	(0.57, 0.60, 0.63)	Berhe et al.(2024)
σ	Second-dose vaccination rate	0.20 day ⁻¹	–	Assumed
ϕ	Waning immunity rate	0.02 day ⁻¹	–	Assumed
ρ	Treatment rate	0.40 day ⁻¹	(0.37, 0.40, 0.43)	Assumed
γ	Recovery rate	0.14 day ⁻¹	(0.12, 0.14, 0.16)	Memon (2020)
δ	Disease-induced death rate	0.01 day ⁻¹	–	Assumed
ω	Loss of immunity rate	0.005 day ⁻¹	–	Assumed
θ	Vaccination coverage	0.85	–	Berhe et al.(2024)

The baseline parameter values presented in Table 1 were first used to evaluate the deterministic and fuzzy basic reproduction numbers. Using the corrected disease-free equilibrium, the deterministic model gives

$$R_0 = 22.8513. \quad (40)$$

Since $R_0 > 1$, the disease-free equilibrium is unstable, indicating that measles can invade and persist in the population under the baseline conditions.

To account for uncertainties in disease transmission, vaccination and treatment, the transmission rate (β), vaccination rate (ε) and treatment rate (ρ) were represented by triangular fuzzy numbers. Using the corresponding r -cut representation, the fuzzy basic reproduction number was obtained as

$$R_0^r = [R_{0L}^r, R_{0U}^r], \quad 0 \leq r \leq 1.$$

The numerical values are presented in Table 2.

Table 2. Fuzzy basic reproduction number for different values of r .

r	R_{0L}^r	R_{0U}^r
0.00	18.93	27.68
0.25	19.84	26.38
0.50	20.79	25.14
0.75	21.79	23.96
1.00	22.85	22.85

Table 2 shows that the uncertainty interval decreases progressively as the confidence level increases and converges to the deterministic value at $r = 1$. Since both the lower and upper bounds remain greater than unity for all values of r , the model predicts sustained measles transmission even when parameter uncertainty is considered.

To investigate the effectiveness of possible control strategies, four intervention scenarios were subsequently examined. These scenarios were generated by progressively reducing the transmission rate while simultaneously increasing the vaccination rate, treatment rate and vaccination coverage. The objective was to determine the level of intervention required to reduce the reproduction number below the epidemic threshold.

Table 3. Modified parameter values for intervention scenarios.

Scenario	β	ε	ρ	θ
Baseline	0.75	0.60	0.40	0.85
Scenario I	0.60	0.70	0.55	0.90
Scenario II	0.45	0.80	0.70	0.94
Scenario III	0.30	0.90	0.85	0.96
Scenario IV	0.20	0.95	1.00	0.97

The deterministic reproduction numbers corresponding to these intervention scenarios are summarized in Table 4.

Table 4. Deterministic basic reproduction number under the intervention scenarios.

Scenario	R_0
Baseline	22.8513
Scenario I	9.1780
Scenario II	3.8005
Scenario III	1.6269
Scenario IV	0.8124

The corresponding fuzzy reproduction numbers are presented in Table 5.

Table 5. Fuzzy basic reproduction number under the intervention scenarios.

Scenario	r	R_{0L}^r	R_{0U}^r
Baseline	0.00	18.9276	27.6829
	1.00	22.8513	22.8513
Scenario I	0.00	7.6573	10.9760
	1.00	9.1780	9.1780
Scenario II	0.00	3.1241	4.5808
	1.00	3.8005	3.8005
Scenario III	0.00	1.2678	2.0345
	1.00	1.6269	1.6269
Scenario IV	0.00	0.5736	1.0807
	1.00	0.8124	0.8124

Tables 4 and 5 demonstrate that increasing vaccination coverage and treatment while reducing disease transmission produces a progressive decline in both the deterministic and fuzzy basic reproduction numbers. Scenarios I, II and III substantially reduce disease transmission but still satisfy $R_0 > 1$, indicating that measles transmission can persist under these intervention levels. Scenario IV gives $R_0 < 1$, indicating that the deterministic model is brought below the epidemic threshold. However, at $r = 0$, the upper bound of the fuzzy reproduction number for Scenario IV remains slightly above unity,

$$R_0 = 0.8124 \quad \text{and} \quad R_{0U}^0 = 1.0807.$$

This indicates that although Scenario IV controls transmission at the deterministic parameter values, disease elimination is not guaranteed over the full range of parameter uncertainty. Thus, stronger intervention may be required to ensure that the upper bound of the fuzzy reproduction number also remains below unity.

Discussion

The results show that parameter uncertainty and intervention measures have important effects on measles transmission. From Table 2, the fuzzy reproduction number has its

widest interval at $r = 0$ and gradually narrows as r increases, until the lower and upper bounds coincide with the deterministic value at $r = 1$. This shows that the fuzzy approach captures the possible variation in measles transmission caused by uncertainty in the epidemiological parameters.

The intervention results in Tables 4 and 5 show a progressive decrease in the reproduction number as transmission is reduced and vaccination and treatment are increased. Scenarios I, II and III have deterministic reproduction numbers above unity, indicating that measles transmission may still persist under these intervention levels. In contrast, Scenario IV gives a deterministic reproduction number below unity, with $R_0 = 0.8124$, indicating that the intervention can bring transmission below the epidemic threshold at the baseline parameter values. However, under parameter uncertainty, the upper bound of the fuzzy reproduction number at $r = 0$ is $R_{0U}^0 = 1.0807$, which is slightly above unity. This indicates that disease elimination is not guaranteed over the full range of parameter uncertainty. Thus, the results highlight the importance of sustained double-dose vaccination, effective treatment and reduced transmission, as well as the need to account for epidemiological uncertainty when evaluating measles control strategies.

5 Conclusion

This study developed and analysed a fuzzy measles transmission model incorporating double-dose vaccination, treatment, recovery and waning immunity under parameter uncertainty. The deterministic and fuzzy reproduction numbers showed that measles can persist under the baseline conditions. Increasing vaccination coverage and treatment while reducing transmission progressively lowered the reproduction number. Scenario IV reduced the deterministic reproduction number below unity, while the upper bound of the fuzzy reproduction number remained slightly above unity at the highest level of uncertainty. This shows that an intervention that appears sufficient under fixed parameter values may not guarantee disease elimination when epidemiological uncertainty is considered. The findings demonstrate that fuzzy modelling provides a useful framework for assessing measles control under parameter uncertainty and that sustained double-dose vaccination, effective treatment and reduced transmission are important for measles control and elimination.

Availability of data and material

Not Applicable.

Competing Interest

No competing interest is hereby declared by the authors.

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Authors' contributions

The development and simulation of the model, analysis, discussion of results and typesetting were implemented by all the authors and was approved for final manuscript.

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