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A Modified Ideal Rank Index Number Formula

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ABSTRACT

This paper proposes an index number method called Asanya-Jibasen-Mbaga index, the method satisfied Laspeyre's and Paasche's bounding test which is unbiased compared to existing methods. The proposed index methods used expenditure/quantity and rank as weights and it satisfied all the tests of consistency of elementary index formulae. The proposed method was found to approximate the Fisher's Ideal Price index (PF). The proposed method is an improvement on Jibasen –Gazali-Asanya rank price index. Numerical analyses conducted at low and high levels of aggregation revealed that the Jibasen-Gazali-Asanya rank price Index is biased and also failed Laspeyre's and Paasche's bounding test. Thus, based on the results of the findings, Asanya-Jibasen-Mbaga index method is recommended as the appropriate functional form of the elementary index method among other existing elementary index methods.

1. Introduction

Index numbers are statistical measures that reflect the relative changes in the level of a certain phenomenon in the current period with respect to its values in a period in the past and this past period is called the base period. The development of index number theory has a long history in literature. Few of the relevant work include; Dutot Price index, P^D (Dutot, 1740), Carli Price index, P^C (Carli, 1804), Jevons Price index, P^J (Jevons, 1865), Harmonic Mean of Price Relative index, P^H, others includes; Laspeyres Price index, P^L (Laspeyres, 1871), Paasche's Price Index, P^P (Paasche, 1874), Fisher's Price Index, P^F (Fisher, 1922) and Marshall - Edgeworth Price Index, P^{ME}. Frequently used index numbers formulae are; Laspeyres Price index, P^L, Paasche's Price Index, P^P, Fisher's Price Index, P^F, Marshall-Edgeworth Price Index, P^{ME} and Walsh Price index, P^W (Asanya, 2016). Indices based on the Fisher formula are considered ideal target indices in terms of consistency with utility theory, while Laspeyres-based indices have an upward bias and Paasche-based indices have a downward bias (Diewert, 1976 and Diewert, 1978). In the same vein, none of the existing elementary indices (weighted or unweighted) consistently approximates Fisher's Ideal index and as such, the index numbers made available by these methods could be misleading. Yuong and Xie (2005) argued that the causes of bias in the consumer price index (CPI) reflect: item substitution, outlet substitution, new goods not being priced, new outlets not being priced and quality change. A unique challenge of index methods is that index numbers constructed by different methods hardly converge to a unique value. In an attempt to solve problems of variations in the value of index numbers made available by several methods, Dalen, (1992) initiated an axiomatic approach for the determination of the functional form for aggregating price quotations at the lowest level where quantity information is not available. Jibasen, Gazali and Asanya (2018) proposed an elementary index number formula that uses rank and expenditure as dual weights. Their approach was applied to rents on residential apartments, where rents were taken as price, rank of the category of apartments, and expenditure on the apartment are the dual weights. Though the Jibasen-Gazali-Asanya rank price index satisfies all the tests of consistency of elementary index formulae, yet it is biased depending on the nature of the data and it has also failed Laspeyre's and Paasche's bounding test. Hence, the need to develop an improvement of the Jibasen-Gazali-Asanya rank price index in such a way that the improved version should attend Laspeyre's and Paasche's bounding test and be consistently unbiased with respect to the target index known as "Fisher's ideal index". The proposed improvement is called Asanya-Jibasen-Mbaga, P^{AJM} Index Method.

2. Methods

2.1 Development of the Proposed Asanya-Jibasen-Mbaga Index Formula

To overcome the shortcoming of the Jibasen-Gazali-Asanya rank price index method, Asanya-Jibasen-Mbaga Index Method was then developed. The proposed modification is the geometric mean of the Jibasen-Gazali-

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Asanya rank price index and the index of its reversed rank. Both the Jibasen-Gazali-Asanya rank price index and Asanya-Jibasen-Mbaga price index formulae use double weights. The weights are the expected expenditure on the house (EEH) (or items) and ranks based on the category of the house (or items). Jibasen-Gazali-Asanya rank price index (Jibasen, Gazali and Asanya, 2018) is given as;

$$P^{JGA} = \frac{\sum_{i=1}^{m} w_i p_{it} R_{it}^{L}}{\sum_{i=1}^{m} w_i p_{i0} R_{i0}^{L}} * 100 \quad (1)$$

This formulation is now modified and named Asanya-Jibasen-Mbaga price index the formula is given by;

$$P^{AJM} = \sqrt{\frac{\sum_{i=1}^{m} W_i P_{it} R_{it}^{H}}{\sum_{i=1}^{m} W_i P_{io} R_{io}^{H}} * \frac{\sum_{i=1}^{m} W_i P_{it} R_{it}^{L}}{\sum_{i=1}^{m} W_i P_{i0} R_{i0}^{L}}} * 100 \quad (2)$$

Where w_i is weight computed as follows;

$$w_i = \frac{Q_{io} + Q_{it}}{\sum_{i=1}^{m} (Q_{io} + Q_{it})} \quad (3)$$

(if quantity is available)

$$w_i = \frac{E_{io} + E_{it}}{\sum_{i=1}^{m} (Q_{io} + Q_{it})} \quad (4)$$

(if expected expenditure is available)

Where;

p_{it} = Price of item $i(i = 1,2,\ldots,m)$ in period t.

p_{io} = Price of item i(i =1, 2,...,m) in the base period.

Q_{it} = Quantity of item $i(i = l,2,\ldots,m)$ purchased in period t.

Q_{io} = Quantity of item $i(i = l,2,\ldots,m)$ purchased in the base period.

m = Number of items under study.

R_{it}^{H} = Rank of item $i(i = 1,2,\ldots,m)$ in period t such that the item with the highest price takes the first rank while the lowest get rank m.

R_{it}^{L} = Rank of item $i(i = 1,2,\ldots,m)$ in period t such that the item with the lowest price takes the first rank while the highest get rank m.

R_{io}^{H} = Rank of item i $(i = 1,2,\ldots,m)$ in base period such that the item with the highest price takes the first rank while the lowest get rank m.

R_{io}^{L} = Rank of item i $(i = 1,2,\ldots,m)$ in base period such that the item with the lowest price takes the first rank while the highest get rank m.

w_i is the aggregate percentage proportion of the criteria of weighting for both periods under consideration for item $i(i = 1,2,\ldots,m)$.

E_{it} and E_{io} represent expected expenditure on the house (EEH) in period $t$and the base period respectively.

$P_{it}$and P_{io} represent price in period t and the base period respectively

The proposed formula can be used to compute effectively the index numbers both at high levels of aggregation and at a low level of aggregation. These index numbers/indicators reflect the relative changes in the level of a certain phenomenon in any given period with respect to its values in some fixed period. The phenomena (variables) under consideration may be; rents paid, prices of goods and services, volume (quantity) of commodities or cost, of living of persons belonging to a particular income group.

2.2 Assumptions of the Proposed Asanya-Jibasen-Mbaga Index Method are:

The assumptions of the proposed method are:

i. That the quality of the items in the basket of goods and services priced remains constant over time. As this is not necessarily the case, changes in quality must be adjusted in to maintain a constant quality basket, that either quantity or expenditure information is available.
ii. The underlying phenomenon of interest is continuous and also non-negative.
iii. And also that the items can be ranked based on price, quality, or any other variable or attributes.

2.3 Procedures for Computing the Proposed Asanya-Jibasen-Mbaga Price Index, P^{AJM}:

i. For items $i(i = 1,2,\ldots,m)$ in period t and base period, assign rank $R_{it}^{H}$and R_{io}^{H} respectively, to each of the ith price such that the highest price takes the first rank and the lowest get rank m,
ii. For items $i(i = 1,2,\ldots,m)$ in period t and base period, assign rank $R_{it}^{L}$and R_{io}^{H} respectively, to each of the i^{th} price such that the lowest price takes the first rank and the highest get rank m.
iii. In each of the (i) and (ii) cases above, if ties occur, the items that have equal prices are assigned average of the ranks otherwise would have taken if ties did not occur.

iv. For items $i(i = 1,2,\dots,m)$ in base period and in period t, a common weight W_i (i.e the aggregate per cent proportion of the criteria for weighting in both periods under consideration) is assigned to reflect the importance of each i^{th} item.

v. For each of items $i(i = l,2,\dots,m)$, obtain $W_iP_{it}R_{it}^H$, $W_iP_{it}R_{it}^L$, $W_iP_{io}R_{io}^H$ and $W_iP_{io}R_{io}^L$

vi. Find the sum of the values obtained in step (v) over all the items to get

$$\sum_{i=1}^{m} W_iP_{it}R_{it}^H, \quad \sum_{i=1}^{m} W_iP_{it}R_{it}^L$$

$$\sum_{i=1}^{m} W_iP_{io}R_{io}^H and \sum_{i=1}^{m} W_iP_{io}R_{io}^L,$$

vii. Substitute the values obtained in step (vi) in equation (3) and compute for the required index number.

2.4 Test of Consistency of the Proposed Asanya-Jibasen-Mbaga Index Formula

The proposed Asanya-Jibasen-Mbaga price index, P^{AJM} was subjected to all the axiomatic tests of adequacy, and the mathematical proofs are presented below;

a) Invariance to Changes in Units/Commensurability Test: The proposed Asanya-Jibasen-Mbaga Price index, P^{AJM} satisfies invariance to changes in units/commensurability test if for $\lambda > 0$,

$$P^{AJM}(\lambda P_{io}, \lambda P_{it}) = P^{AJM}(P_{io}, P_{it}) \quad (5)$$

Proof:

$P^{AJM}(\lambda P_{io}, \lambda P_{it})$

$$= \sqrt{\frac{\sum_{i=1}^{m} w_i\lambda P_{it}R_{it}^H}{\sum_{i=1}^{m} w_i\lambda P_{io}R_{io}^H} * \frac{\sum_{i=1}^{m} w_i\lambda P_{it}R_{it}^L}{\sum_{i=1}^{m} w_i\lambda P_{io}R_{io}^L}} * 100 \quad (6)$$

$$= \sqrt{\frac{\lambda^2 \sum_{i=1}^{m} w_iP_{it}R_{it}^H}{\lambda^2 \sum_{i=1}^{m} w_iP_{io}R_{io}^H} * \frac{\sum_{i=1}^{m} w_iP_{it}R_{it}^L}{\sum_{i=1}^{m} w_iP_{io}R_{io}^L}} * 100 \quad (7)$$

$$= \sqrt{\frac{\sum_{i=1}^{m} w_iP_{it}R_{it}^H}{\sum_{i=1}^{m} w_iP_{io}R_{io}^H} * \frac{\sum_{i=1}^{m} w_iP_{it}R_{it}^L}{\sum_{i=1}^{m} w_iP_{io}R_{io}^L}} * 100 \quad (8)$$

$= P^{AJM}(P_{io}, P_{it})$ {proved}

Time Reversal Test: The proposed Asanya-Jibasen-Mbaga Price index, P^{AJM} satisfies time reversal test {without the factor l00};

$$P^{AJM}(P_{io}, P_{it}) = 1/P^{AJM}(P_{it}, P_{io}) \quad (9)$$

Proof:

Considering the RHS,

$$1/P^{AJM}(P_{it},P_{io}) = \frac{1}{\sqrt{\frac{\sum_{i=1}^{m} W_iP_{io}R_{i1}^H}{\sum_{i=1}^{m} W_iP_{it}R_{it}^H} * \frac{\sum_{i=1}^{m} W_iP_{io}R_{io}^L}{\sum_{i=1}^{m} W_iP_{it}R_{it}^L}}} \quad (10)$$

$$= 1 \div \frac{\sqrt{\sum_{i=1}^{m} W_iP_{io}R_{io}^H}}{\sqrt{\sum_{i=1}^{m} W_iP_{it}R_{it}^H}} * \frac{\sqrt{\sum_{i=1}^{m} W_iP_{io}R_{io}^L}}{\sqrt{\sum_{i=1}^{m} W_iP_{it}R_{it}^L}} \quad (11)$$

$$= 1 * \sqrt{\frac{\sum_{i=1}^{m} W_iP_{it}R_{it}^H}{\sum_{i=1}^{m} W_iP_{io}R_{io}^H} * \frac{\sum_{i=1}^{m} W_iP_{it}R_{it}^L}{\sum_{i=1}^{m} W_iP_{io}R_{io}^L}} \quad (12)$$

$$= \sqrt{\frac{\sum_{i=1}^{m} W_iP_{i1}R_{i1}^H}{\sum_{i=1}^{m} W_iP_{io}R_{io}^H} * \frac{\sum_{i=1}^{m} W_iP_{it}R_{io}^L}{\sum_{i=1}^{m} W_iP_{io}R_{io}^L}} \quad (13)$$

$= P^{AJM}(P_{io}, P_{it})$ LHS {Proved}

b) Circularity Test: Considering P_{i0}, P_{i1} and P_{i2} as prices of commodities at three successive periods, the proposed Asanya-Jibasen-Mbaga Price index, P^{AJM} satisfies circularity test (Without the factor l00);

$$P^{AJM}(P_{io}, P_{i1})\, P^{AJM}(P_{i1}, P_{i2}) = P^{AJM}(P_{i0}, P_{i2}) \tag{14}$$

Proof:

$$P^{AJM}(P_{io},P_{i1})P^{AJM}(P_{i1},P_{i2})=\sqrt{\frac{\sum_{i=1}^{m} W_i P_{i1} R_{i1}^{H}}{\sum_{i=1}^{m} W_i P_{io} R_{io}^{H}} * \frac{\sum_{i=1}^{m} W_i P_{i1} R_{i1}^{L}}{\sum_{i=1}^{m} W_i P_{io} R_{io}^{L}}} * \sqrt{\frac{\sum_{i=1}^{m} W_i P_{i2} R_{i2}^{H}}{\sum_{i=1}^{m} W_i P_{i1} R_{i1}^{H}} * \frac{\sum_{i=1}^{m} P_{i2} R_{i2}^{L}}{\sum_{i=1}^{m} W_i P_{i1} R_{i1}^{L}}} \tag{15}$$

$$= \sqrt{\frac{\sum_{i=1}^{m} W_i P_{i2} R_{i2}^{H}}{\sum_{i=1}^{m} W_i P_{io} R_{io}^{H}} * \frac{\sum_{i=1}^{m} W_i P_{i2} R_{i2}^{L}}{\sum_{i=1}^{m} W_i P_{io} R_{io}^{L}}} \tag{16}$$

$= P^{AJM}(P_{i0}, P_{i2})$ {Proved}

c) Multiperiod Identity Test: Considering P_{i0}, P_{i1}, P_{i2} as prices of commodities at three successive periods, the proposed Asanya-Jibasen-Mbaga Price index, P^{AJM} satisfies multiperiod identity test (without the factor 100);

$$P^{AJM}(P_{i0}, P_{i1})\, P^{AJM}(P_{i1}, P_{i2}) * P^{AJM}(P_{i2}, P_{i0}) = 1 \tag{17}$$

Proof

$P^{AJM}(P_{i0}, P_{i1})\, P^{AJM}(P_{i1}, P_{i2})\, P^{AJM}(P_{i2}, P_{i0})$

$$= \sqrt{\frac{\sum_{i=1}^{m} W_i P_{i1} R_{i1}^{H}}{\sum_{i=1}^{m} W_i P_{io} R_{io}^{H}} * \frac{\sum_{i=1}^{m} W_i P_{i1} R_{i1}^{L}}{\sum_{i=1}^{m} W_i P_{io} R_{io}^{L}}} * \sqrt{\frac{\sum_{i=1}^{m} W_i P_{i2} R_{i2}^{H}}{\sum_{i=1}^{m} W_i P_{i1} R_{i1}^{H}}\, x\, \frac{\sum_{i=1}^{m} W_i P_{i2} R_{i2}^{L}}{\sum_{i=1}^{m} W_i P_{i1} R_{i1}^{L}}} * \sqrt{\frac{\sum_{i=1}^{m} W_i P_{i0} R_{i0}^{H}}{\sum_{i=1}^{m} W_i P_{i2} R_{i2}^{H}} * \frac{\sum_{i=1}^{m} W_i P_{i0} R_{i0}^{L}}{\sum_{i=1}^{m} W_i P_{i2} R_{i2}^{L}}} \tag{18}$$

$= \sqrt{1} = 1$ {Proved}

d) Prices Bouncing Test: If price at P_{io} are rearranged as $P_{io}^{\sim}$ for any possible permutation of P_{io} and then returned to P_{i0}, the proposed Asanya-Jibasen-Mbaga Price index, P^{AJM} satisfies price bouncing test;

$$P^{AJM}(P_{io}, P_{io}^{\sim})\, P^{AJM}(P_{io}^{\sim}, P_{io}) = 1 \tag{19}$$

Proof

$P^{AJM}(P_{io}, P_{io}^{\sim})\, P^{A\text{-}J}(P_{io}^{\sim}, P_{io})$

$$= \sqrt{\frac{\sum_{i=1}^{m} w_i P_{io}^{\sim} R_{io}^{H}}{\sum_{i=1}^{m} w_i P_{io} R_{io}^{H}} * \frac{\sum_{i=1}^{m} w_i P_{io}^{\sim} R_{io}^{L}}{\sum_{i=1}^{m} w_i P_{io} R_{io}^{L}} * \frac{\sum_{i=1}^{m} w_i P_{io} R_{io}^{H}}{\sum_{i=1}^{m} w_i P_{io}^{\sim} R_{io}^{H}} * \frac{\sum_{i=1}^{m} w_i P_{io} R_{io}^{L}}{\sum_{i=1}^{m} w_i P_{io}^{\sim} R_{io}^{L}}} \tag{20}$$

$= \sqrt{1} = 1$ {proved}

The proposed method also satisfies the following test: Positivity Test, Continuity Test, Identity/Constant Prices Test, Proportionality in Current Prices Test, Inverse Proportionality in Base Prices Test, Monotonicity in Current Prices Test, Monotonicity in Base Prices Test, Mean Value Test, Commodity Reversal Test/Symmetric treatment of outlets. Thus, the proposed Asanya-Jibasen-Mbaga index formula satisfies all the tests of consistency for elementary index formulae.

3. Results

The analysis was carried out using empirical assessment of the index number formulae at high and low levels of aggregations. The data used for the high level of aggregation was obtained from [15] while the data used at low level of aggregation were the prices for house rent in Jimeta metropolis obtained from a housing agent Faruna and Co. Nigeria Limited (Jibasen Gazali and Asanya, 2018). The data used at a low level of aggregation includes mean house rent (**MHR**) and expected expenditure on the house (**EEH**) for a random sample of 30 houses of the same category. Two weights were employed as seen in equation (3);

(i) the weight $\mathbf{w}_i$ attached to each category of houses represents the aggregate percentage proportion of **EEH** for both periods under consideration and

(ii) the rank (R_i) of the rent (MHR) of the house while **MHR** for each category of houses represents the price (**P**).

The following existing index methods were employed in the comparative data analysis with the proposed index methods;

a) Weighted Dutot's Price Index: This price index formula is given by:

$$P^{WD} = \frac{\sum_{i=1}^{m} W_i P_{it}}{\sum_{i=1}^{m} W_i P_{io}} * 100 \tag{21}$$

b) Weighted Carli's Price Index: This is given by:

$$P^{WC} = \frac{\sum_{i=1}^{m}\left[w_i\left(\frac{P_{it}}{P_{iO}} * 100\right)\right]}{\sum_{i=1}^{m} w_i} = \frac{\sum_{i=1}^{m} w_i P}{\sum_{i=1}^{m} w_i} \quad (22)$$

Where $P = \frac{P_{it}}{P_{iO}} * 100$ (23)

Weighted Jevons Price index, P^{WJ}: This is given as,

$$P^{WJ} = \text{Antilog}\left[\frac{\sum_{i=1}^{m} W_i \log P}{\sum_{i=1}^{m} W_i}\right] \quad (24)$$

c) Laspeyres Price index, P^L: This formula is given by:

$$P^L = \frac{\sum_{i=1}^{m} P_{it}Q_{io}}{\sum_{i=1}^{m} P_{io}Q_{io}} * 100 \quad (25)$$

d) Paasche's Price Index, P^P: The formula is given by:

$$P^P = \frac{\sum_{i=1}^{m} P_{it}Q_{it}}{\sum_{i=1}^{m} P_{io}Q_{it}} * 100 \quad (26)$$

e) Fisher's Price Index, P^F: The formula is given by:

$$P^F = \sqrt{(P^L * P^P)} \quad (27)$$

Other statistical tools employed to study the performances of the index number methods are:

(i) Mean Absolute Deviation (MAD): The Mean Absolute Deviation (MAD) is given by:

$$\text{MAD} = \frac{\sum_{i=}^{m}(\bar{Y} - Y_i)}{m} \quad (28)$$

where: Y_i is the i[th] index number computed per index method.

$\bar{Y}$ is the mean of the computed indices per index method.

m is the number of items under consideration

(ii) Coefficient of Variation (CV): The coefficient of variation is given by: $\text{CV} = \frac{\sigma}{Y} * 100$

Where: σ is the standard deviation of the computed indices per index method.

(iii) Paasche's and Laspeyre's Bounding Test: The price index $P(P_{i0}, P_{it})$ which lies between the Laspeyre's and Paasche's indices, P^P and P^L, defined by equation (25) and (26) respectively is said to satisfy Paasche's and Laspeyre's bounding test. Both Bowley, (1901) and Fisher, (1922) endorsed this property for a price index formula.

3.1 Empirical Assessment of Index Number Formulae at High Level of Aggregation.

In this section, the price index for the commodities in Table 1 and Table 2 were computed using the Jibasen-Gazali-Asanya Rank Index method, the proposed Asanya-Jibasen-Mbaga index method, Weighted Dutot Index method, Weighted Carli Index method, Weighted Jevons Index method, Paasche's Index method, Laspeyre's Index method and Fisher's ideal Index method.

3.2.1 Data Presentation and Analysis at the High Level of Aggregation.

Table l: When Item per unit Quantity Information is Available

	1970		1980	
Commodities	Price	Quantity	Price	Quantity
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

Source: Gupta, (2011)

From Table 1, the price index number for the year 1980 with 1970 as the base year has the following results:

i. Jibasen-Gazali-Asanya rank price index, P^{JGA} = 120.91%
ii. Asanya-Jibasen-Mbaga Price index, P^{AJM} = 123.48%
iii. Weighted Dutot's Price index, P^{WD} = 123.32%
vi. Weighted Carli's Price index, P^{WC} = 123.56%
vii. Weighted Jevon's Price index, P^{WJ} = 114.37%
viii. Paasche's Price index, P^P = 121.77%
ix. Laspeyre's Price index, P^L = 124.699%

x. Fisher's Price Index, P^F = 123.23%

Table 2: When item per unit expenditure information is available

		1986		1996	
Commodities	Units	Price	Value	Price	Value
A	Kg	10	1500	11	1760
B	Kg	12	1080	13	1300
C	Metre	15	900	16	960
D	Packets	9	450	12	480

Source: Gupta, (2011)

From Table 2 the price index number for the year 1996 with 1986 as the base year are given as follows:

i. Jibasen-Gazali-Asanya rank price index, P^{JGA} = 99.69%
ii. Asanya-Jibasen-Mbaga Price index, P^{AJM} = 111.26%
iii. Weighted Dutot`s Price index, P^{WD} = 110.55%
iv. Weighted Carli`s Price index, P^{WC} = 111.3 7%
v. Weighted Jevon's Price index, P^{WJ} = 111.07%
vi. Paasche's Price index, P^P = 110.84%
vii. Laspeyre's Price index, P^L = 111.45%
viii. Fisher's Price Index, P^F = 111.14%

3.2.2 Empirical Assessment of Index Number Formulae at Low Level of Aggregation.

In this section, the House Rent Price Index (HRPI) of Jimeta Metropolis was computed using Jibasen-Gazali-Asanya rank price index method, the proposed Asanya-Jibasen-Mbaga index method, Weighted Dutot Index method, P^{WD}, Weighted Carli Index method, P^{WC}, and Weighted Jevons index method, P^{WJ}. The coefficient of variation (CV) and mean absolute deviation (MAD) for the distribution of price indices were also obtained with respect to each index method.

3.2.3 Data Presentation and Analysis at the Low Level of Aggregation

Table 3: Mean House Rents (in "000 Naira) and Expected Expenditure on the House (in "000 Naira) for a random sample of 30 Houses

	2000		2005		2010		2015	
Karewa	MHR	EEH	MHR	EEH	MHR	EEH	MHR	EEH
SR	15	450	30	900	40	1,200.00	80	2,400
RP	30	900	60	1,800	80	2,400	100	3,000
OBS	50	1,500	80	2,400	120	3,600	200	6,000
2BF	80	2,400	130	3,900	150	4,500	350	10,500.
3BF	100	3,000	180	5,400	220	6,600	500	15,000.
A/B								
SR	15	450	30	900	40	1,200	80	2,400
RP	30	900	60	1,800	80	2,400	100	3,000
OBS	50	1,500	80	2,400	120	3,600	200	6,000
2BF	80	2,400	130	3,900	150	4,500	350	10,500
3BF	100	3,000	180	5,400	220	6,600	500	15,000
Bekaji								
SR	30	900	30	900	50	1,500	100	3,000
RP	60	1,800	60	1,800	70	2,100	150	4,500
OBS	70	2,100	80	2,400	100	3,000	175	5,250
2BF	100	3,000	130	3,900	150	4,500	275	8,250
3BF	120	3,600	180	5,400	200	6,000	375	11,250

PZ								
SR	10	300	20	600	30	900	80	2,400
RP	20	600	40	1,200	70	2,100	120	3,600
OBS	40	1,200	50	1,500	100	3,000	150	4,500
2BF	60	1,800	110	3,300	100	3,900	225	6,750
3BF	100	3,000	150	4,500	200	6,000	300	9,000
Jambutu								
SR	10	300	15	450	25	750	50	1,500
RP	20	600	30	900	50	1,500	70	2,100
OBS	80	2,400	45	1,350	70	2,100	100	3,000
2BF	100	3,000	100	3,000	130	3,900	175	5,250
3BF	120	3,600	140	4,200	170	5,100	250	7,500
Shagari I								
SR	30	900	60	1,800	80	2,400	100	3,000
RP	50	1,500	80	2,400	90	2,700	110	3,300
OBS	70	2,100	100	3,000	105	3,150	125	3,750
2BF	100	3,000	150	4,500	180	5,400	200	6,000
3BF	150	4,500	200	6,000	235	7,050	275	8,250

Source: Faruna and Co. Nigeria Limited, Yola.

Where, SR – Single room
RP –Room and parlour
OBS– One bedroom self-contain,
2BF– Two-bedroom
3BF– Three-bedroom

Table 4: HRPI of Jimeta Metropolis for 2015 using 2000 as the base year.

Area/Methods	P^{JGA}	P^{AJM}	P^{WD}	P^{WC}	P^{WJ}
Karewa	471.91	458.21	464.33	453.22	449.57
Army Barrack	451.17	428.22	438.95	422.03	414.00
Bekaji	292.91	285.46	288.11	285.15	283.6
Behind PZ	330.13	357.76	345.94	410.95	390.48
Jambutu	190.99	188.06	189.32	214.56	199.80
Shagari phase 1	189.29	197.47	194.34	207.65	203.75
CV	34.78	32.82	33.63	30.22	30.86
MAD	96.67	95.53	96.24	96.47	94.48

The proposed Asanya-Jibasen-Mbaga price index performs comparatively well with the other four methods.

Table 5. HRPI of Jimeta Metropolis for 2015 using 2005 as the base year.

Area/Methods	P^{JGA}	P^{AJM}	P^{WD}	P^{WC}	P^{WJ}
Karewa	271.29	264.45	267.51	259.72	257.34
Army Barrack	258.38	246.04	251.79	239.35	414
Bekaji	211.89	218.21	215.69	226.95	283.6
Behind PZ	210.1	223.7	217.98	246.17	390.48
Jambutu	180.87	186.71	183.99	199.99	199.8
Shagari phase 1	135.68	136.75	136.37	137.78	203.75
CV	21.56	19.56	20.38	18.54	18.27
MAD	35.82	33.94	34.96	32.96	32.13

With the shift in the base year, the proposed Asanya-Jibasen-Mbaga price index still performs comparatively well with the other four methods.

Table 6. HRPI of Jimeta Metropolis for 2015 using 2010 as the base year.

Area/Methods	P^{JGA}	P^{AJM}	P^{WD}	P^{WC}	P^{WJ}
Karewa	220.63	211.27	215.33	206.6	202.86
Army Barrack	210.14	196.6	202.72	191.04	185.91
Bekaji	186.26	187.05	187.05	189.05	188.71
Behind PZ	156.61	160.95	159.24	167.86	165.65
Jambutu	143.33	143.35	143.34	143.35	145.08
Shagari phase 1	115.85	116.73	116.42	117.49	117.41
CV	21.52	19.20	20.17	17.84	17.32
MAD	33.54	29.06	31.02	25.93	24.89

The proposed Asanya-Jibasen-Mbaga price index performs comparatively well with the other four methods, even with the shift in base year.

Table 7. HRPI of Jimeta Metropolis for 2010 using 2005 as the Base Year.

Area/Methods	P^{JGA}	P^{AJM}	P^{WD}	P^{WC}	P^{WJ}
Karewa	123.18	125.48	124.54	127.7	127.15
Army Barrack	123.18	125.48	124.54	124.7	127.15
Bekaji	113.74	116.24	115.22	119.55	118.82
Behind PZ	134.04	138.65	136.6	146.43	143.75
Jambutu	126.17	130.09	128.26	128.26	135.02
Shagari phase 1	117.12	117.12	117.12	117.12	116.98
CV	5.27	6.09	5.69	7.64	7.16
MAD	4.98	5.91	5.47	8.09	7.49

4.0 Discussion

The results of this work as evident in section 2.4 and the data analyses in Tables 1, 2, 4, and 6 revealed that the Jibasen-Gazali-Asanya rank price index, P^{JGA} failed Paasche's and Laspeyres' bounding test and the Weighted Dutot's Price Index, P^{WD} satisfied Paasche's and Laspeyres' bounding test in the analysis of data in Table 1 but failed Paasche's and Laspeyres' bounding tests in the analysis of data in Table 2. Further, the results show that the Weighted Carli's Price Index, P^{WC} satisfied Paasche's and Laspeyres' bounding test in both analyses, while, Weighted Jevon's Price index, P^{WJ} failed Paasche's and Laspeyres' bounding test in the analysis of data in Table 1 but satisfied Paasche's and Laspeyres' bounding test in the analysis of data in Table 2.

The Weighted Carli's Price Index, P^{WC} and Weighted Jevon's Price Index, P^{WJ} obtained for each Area in Jimeta Metropolis agreed with (Reinsdorf and Moulton, 1994). The arithmetic mean of price ratios are greater than or equal to the geometric mean of price ratios. Considering the CV and MAD for index methods in Table 4, Table 5, Table 6, and Table 7, the Carli index has the highest deviation compared to other indices, the weighted Jevons index has the lowest deviation while Asanya-Jibasen-Mbaga Price index is always bounded by these extreme values. These Tables further show that the Carli index overestimates prices compared to other indices, the weighted Jevons index underestimates, while Asanya-Jibasen-Mbaga Price index is relatively unbiased.

Further, the results revealed that the proposed Asanya-Jibasen-Mbaga index, P^{AJM} method satisfies all the test of consistency of elementary index formulae as shown in section 3 and also satisfies Paasche's and Laspeyres' bounding test. Practically, the proposed Asanya-Jibasen-Mbaga Price index consistently approaches Fisher's ideal price index, as seen in all the analyses. Hence, Asanya-Jibasen-Mbaga index method in comparison with the other four (4) methods considered is said to be relatively not biased (either upwardly or downwardly).

We also observed that at a high level of aggregation, the Asanya-.libasen-Mbaga index approximates Fisher's Index and it is consistently bounded by Laspeyre`s and Paasche's indices. Considering the axiomatic approach of elementary price indices and the result of the analyses conducted, Asanya-Jibasen-Mbaga index method emerged as the appropriate (best) elementary index method at a low level of aggregation and at a high level of aggregation.

Furthermore, we discovered that the Asanya-Jibasen-Mbaga index method is capable of solving index number problems both at a low level of aggregation and at a high level of aggregation. Also, the method overcomes the problem of varying weights by creating a constant (common) weight for both periods under consideration. Asanya-Jibasen-Mbaga's method is an efficient and dual-purpose elementary index formula and can be used both for categorical and continuous variables.

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