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Paper Title

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Abstract

The abstract must state the background of the study by briefly introducing the general problem area and its significance. It must clearly indicate the motivation for the work and, where appropriate, the gap in existing knowledge that the study addresses. The abstract should summarise the main aim of the paper and briefly describe the approach or methods used. It must present the key results obtained in a concise form, without detailed derivations or excessive technicality. Finally, it should state the main conclusion and the broader implications or relevance of the findings. The abstract should be self-contained, clear, and understandable without reference to the full paper, and it should not contain citations, figures, or undefined abbreviations.

1 Introduction

The **introduction** should begin by introducing the general topic of the study and explaining its scientific or practical relevance. It should provide sufficient background in-

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formation to orient the reader and establish the context of the research problem. The introduction must include a concise review of relevant literature, highlighting key contributions in the field and showing how the topic has been previously studied. It should then clearly identify the research gap, limitation, or unresolved problem that motivates the present work. After this, the introduction must state the main objectives of the study and outline what the paper aims to achieve. Finally, it should briefly highlight the structure of the article by indicating how the remaining sections are organised. The introduction should be clear, focused, and logically guide the reader from general context to the specific contribution of the paper.

Refer to the example below:

Crime remains one of the most persistent challenges affecting societies worldwide, with significant implications for economic stability, social welfare, and public safety (Agnew, 2012; Kakar, 2025; Okpuvwie et al., 2021). It is widely recognised that criminal activity does not occur in isolation but emerges from complex interactions between offenders, potential victims, and law enforcement systems (Fafchamps and Moser, 2003). Understanding these interactions is essential for designing effective policies aimed at crime prevention and control.

In recent times, mathematical modelling has increasingly been used as a tool to analyse crime dynamics (McMillon et al., 2014; Nadiyadra et al., 2024; Stojičić et al., 2025). Traditional approaches have largely relied on statistical and econometric models to study crime trends and the effects of deterrence policies. However, these approaches often fail to capture the dynamic and interactive nature of criminal behaviour. As a result, there has been growing interest in the application of models from population dynamics, particularly those inspired by predator–prey systems, to describe the interaction between criminal and non-criminal populations.

Predator–prey models, originally developed by Lotka and Volterra (Lotka, 1925; Volterra, 1926), have been widely used to describe interactions between species in ecological systems. These models have since been extended to various fields, including economics, epidemiology, and social dynamics (Brauer and Castillo-Chavez, 2001). In the context of crime, criminals may be viewed as predators who target vulnerable individuals or resources, while the general population represents the prey. This analogy has been explored in several studies, where crime dynamics are modelled through interactions between criminal agents and potential victims, often incorporating the role of law enforcement as a controlling mechanism.

For instance, Sooknanan et al. (2012) examined a predator–prey system in which criminals act as predators and are subject to removal through policing strategies, modelled as harvesting mechanisms. Their work incorporated important features such as prey migration, group defence, and predator switching, capturing phenomena such as spatial crime displacement. Similarly, Abbas et al. (2017) developed a dynamical system describing interactions between criminal and non-criminal populations, highlighting the role of law enforcement in reducing criminal activity and identifying threshold conditions under which criminal populations decline or persist.

Despite these advances, most existing models are deterministic in nature and often assume homogeneous environments. Such assumptions limit their applicability to real-world settings, where crime dynamics are influenced by spatial heterogeneity and random external factors. In reality, urban environments consist of multiple interconnected regions with varying socioeconomic conditions, policing intensity, and population density. Criminal activity often shifts between these regions in response to enforcement strategies, a phenomenon widely referred to as spatial displacement (Eck, 1993; Morgan, 2014).

Motivated by these considerations, this study develops a stochastic predator–prey model for criminal dynamics in a multi-patch environment. The remainder of the paper is organised as follows. In Section 3, we present the mathematical model formulation. Section 4 is devoted to analytical study, including positivity, boundedness, and stochastic stability analysis using Lyapunov methods. Section 5 presents numerical simulations. Conclusions and policy implications are discussed in Section 8.

2 Methods/Methodology

The **methodology** (or **methods**) section should clearly and rigorously describe the mathematical or theoretical framework used in the study. For analytical or modelling papers, it should begin by stating all underlying assumptions explicitly and precisely, ensuring that the conditions under which the model or theory is developed are unambiguous. It should present all governing equations, definitions, and mathematical formulations in a clear and logically structured manner. Where applicable, it must explain the construction of models, derivation of expressions, and any analytical techniques employed, such as stability analysis, perturbation methods, or stochastic analysis. If computational or numerical work is included, the section should describe algorithms, simulation procedures, and implementation details sufficiently to ensure reproducibility. Overall, the methodology should be written in a formal, precise, and self-contained manner, allowing readers to understand and, if necessary, replicate the results.

The following sections are some examples of methodological approaches (Sections 3, 4 and 5):

3 Mathematical Model Formulation

In this study, we propose a stochastic multi-region predator–prey model to describe the dynamics of criminal activity across spatially structured regions. We consider a system consisting of three interconnected regions. Let $V_i(t)$ denote the density of potential victims or economic opportunities in region i, and $C_i(t)$ denote the density of active criminal agents in region i, for $i = 1, 2, 3$. The total population at time t is

$$\Lambda(t) = \sum_{i=1}^{3} \big(V_i(t) + C_i(t)\big). \tag{1}$$

Based on the modelling assumptions (logistic opportunity growth, bilinear criminal–victim interaction, enforcement removal, and inter-region migration with stochastic perturbations), the system is governed by

$$\begin{cases} dV_1 = \left[r_1V_1\left(1-\frac{V_1}{\kappa_1}\right) - a_1V_1C_1 - \nu_1V_1 + \alpha_4V_2 + \alpha_3V_3 - (\alpha_1+\alpha_6)V_1\right]dt + \sigma_1V_1\,dW_1(t), \\ dC_1 = \left[\gamma_1V_1C_1 - (\mu_1+\nu_1)C_1\right]dt + \sigma_1C_1\,dW_1(t), \\ dV_2 = \left[r_2V_2\left(1-\frac{V_2}{\kappa_2}\right) - a_2V_2C_2 - \nu_2V_2 + \alpha_1V_1 + \alpha_5V_3 - (\alpha_2+\alpha_4)V_2\right]dt + \sigma_2V_2\,dW_2(t), \\ dC_2 = \left[\gamma_2V_2C_2 - (\mu_2+\nu_2)C_2\right]dt + \sigma_2C_2\,dW_2(t), \\ dV_3 = \left[r_3V_3\left(1-\frac{V_3}{\kappa_3}\right) - a_3V_3C_3 - \nu_3V_3 + \alpha_2V_2 + \alpha_6V_1 - (\alpha_3+\alpha_5)V_3\right]dt + \sigma_3V_3\,dW_3(t), \\ dC_3 = \left[\gamma_3V_3C_3 - (\mu_3+\nu_3)C_3\right]dt + \sigma_3C_3\,dW_3(t). \end{cases} \tag{2}$$

Initial conditions: $V_i(0) \geq 0$, $C_i(0) \geq 0$ for $i = 1, 2, 3$.

4 Analysis of the Model

4.1 Existence and Uniqueness

Theorem 4.1. *There exists a unique solution to the deterministic version of system (2) for any initial condition in $\mathbb{R}^6$.*

Proof. *The vector field $f(X)$ consists of polynomial components in $X = (V_1, C_1, V_2, C_2, V_3, C_3)^\top$; hence f is locally Lipschitz. By the Picard–Lindelöf theorem (Coddington and Levinson, 1955; Teschl, 2012), a unique solution exists on a maximal interval $[0, t_{\max})$.*

Theorem 4.2 (Existence and Uniqueness for the SDE system)**.** *Suppose the drift F and diffusion Σ of system (2) are locally Lipschitz. Then there exists a unique continuous $\mathbb{R}^6_+$-valued adapted process solving (2) up to the explosion time τ_e. If F and Σ are globally Lipschitz, then $\tau_e = \infty$ almost surely.*

Proof. *For a detailed proof see Hsu (2002).*

4.2 Positivity and Boundedness

Theorem 4.3 (Positivity)**.** *Under the initial conditions $V_i(0) \geq 0$, $C_i(0) \geq 0$, all solutions of system (2) remain non-negative for all $t \geq 0$.*

Proof. *The logistic growth term ensures $V_i \geq 0$ when $V_i = 0$. Interaction and enforcement sinks vanish at zero. Migration inflows are non-negative. Similarly, $dC_i/dt|_{C_i=0} = 0$. Hence, by the standard comparison argument, all state variables remain non-negative.*

Proposition 4.4 (Boundedness)**.** *The total population $\Lambda(t)$ defined in (1) remains bounded for all $t \geq 0$.*

Proof. *One can show constants $\varepsilon > 0$, $\delta > 0$ such that $d\Lambda/dt \leq \varepsilon - \delta\Lambda(t)$, giving $\Lambda(t) \leq \max\big(\Lambda(0), \varepsilon/\delta\big)$ by the Comparison Principle (Hale, 2009). Boundedness of individual populations follows. The LaSalle Invariance Principle (Wiggins, 2023) confirms global attractiveness of the bounded region.*

4.3 Stability Analysis

We introduce the Lyapunov function

$$\mathcal{L}(V, C) = \frac{1}{2}\sum_{i=1}^{3}\left(V_i^2 + C_i^2\right).$$

Applying Itô's Lemma (Hassler, 2016) and collecting terms, the expected rate of change satisfies

$$\mathbb{E}\left[\frac{d\mathcal{L}}{dt}\right] = \sum_{i=1}^{3}\left[V_i Y_i + C_i \psi_i + \tfrac{1}{2}\left(Z_i^2 + \Omega_i^2\right)\right],$$

where Y_i, ψ_i are the deterministic drift terms and $Z_i = \sigma_i V_i$, $\Omega_i = \sigma_i C_i$. Mean-square stability (Nouri et al., 2020; Tocino and Senosiain, 2012) is achieved when

$$\mathcal{R}_{0i} := \frac{r_i + \gamma_i V_i}{\mu_i + \nu_i + \frac{1}{2}\sigma_i^2} < 1, \quad i = 1, 2, 3.$$

Remark 4.5. $\mathcal{R}_{0i} < 1$ *ensures that enforcement, natural decay, and stochastic damping jointly dominate growth and criminal reinforcement. When* $\mathcal{R}_{0i} > 1$ *the system becomes unstable and criminal activity may grow unboundedly.*

5 Numerical Simulations

Numerical simulations were conducted in Python using the Euler–Maruyama scheme. Parameter values are summarised in Table 1.

Table 1. Parameter values used in simulations.

Parameter	Description	Value	Source
ν_1, ν_2, ν_3	Enforcement intensities	0.02	Estimated
$\kappa_1, \kappa_2, \kappa_3$	Carrying capacities	10, 8, 6	Estimated
a_1, a_2, a_3	Crime interaction rates	0.2, 0.3, 0.4	Estimated
r_1, r_2, r_3	Growth rates of opportunities	0.5, 0.4, 0.3	Estimated
μ_1, μ_2, μ_3	Natural removal rates	0.1	Estimated
$\gamma_1, \gamma_2, \gamma_3$	Criminal reinforcement rates	0.01	Estimated
σ_i	Noise intensities, $i = 1, 2, 3$	Variable	Estimated
α_1–α_6	Migration rates	0.03	Estimated

When $\mathcal{R}_{0i} < 1$ (Figure 1) the Lyapunov function remains bounded, confirming stochastic stability. When $\mathcal{R}_{0i} > 1$ (Figure 2) the function grows without bound, indicating instability. The results confirm that higher enforcement intensity and moderate noise promote stability (Nouri et al., 2020; Tocino and Senosiain, 2012).

6 Results

The results section should present the main findings of the study in a clear, logical, and objective manner. It should report analytical results, such as theorems, propositions, stability conditions, or threshold values, in a structured form, with minimal interpreta-

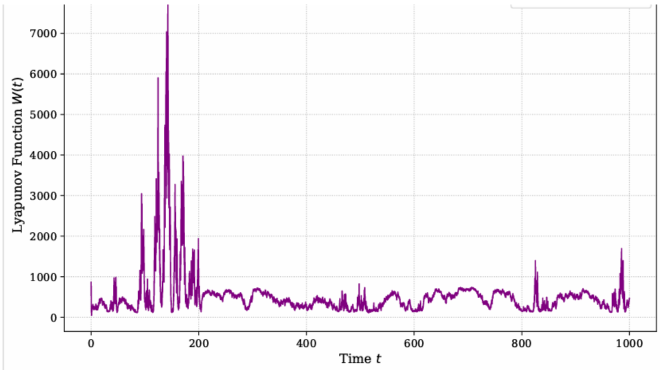

Figure 1. Lyapunov function $\mathcal{L}(t)$ over time when $\mathcal{R}_{0i} < 1$ (stochastically stable regime).

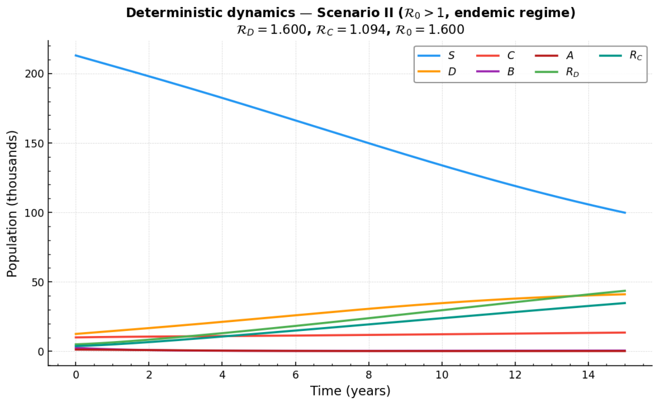

Figure 2. Lyapunov function $\mathcal{L}(t)$ over time when $\mathcal{R}_{0i} > 1$ (unstable regime).

tion. Where applicable, it should include numerical or simulation outcomes that support the theoretical analysis, using tables, figures, or descriptive summaries as needed. The section should clearly highlight the behaviour of the system under different conditions and emphasise key patterns or relationships observed in the results. All results should be presented without extensive discussion or justification, as interpretation is typically reserved for the discussion section. The aim is to communicate what was obtained from the analysis or computations in a precise and reproducible way.

Table 2 shows a simple presentation of data, while Table 3 illustrates a more refined format using `booktabs`.

Table 2. Basic table (simple presentation)

Region	Crime rate	Population
A	12	1000
B	18	1500
C	9	800

Table 3. Professional table (booktabs style)

Region	Crime rate	Population
A	12	1000
B	18	1500
C	9	800

Table 4 presents the observed growth of trees over time, showing an increasing trend in length as the number of days increases.

Table 4. Growth of trees over time (days vs length)

Days	Tree length (cm)
1	2.1
3	4.5
5	7.8
7	11.2
10	15.6

Table 5 shows the reported number of people suffering from Ebola across different regions as of 1995.

Table 5. Table of people suffering from Ebola as of 1995

Country/Region	Number of Cases	Remarks
DR Congo	315	Major outbreak region
Uganda	120	Localized cases
Sudan	220	Early epidemic phase
Gabon	95	Sporadic outbreaks
Congo	150	Rural transmission zones

7 Discussion

The discussion section should interpret and explain the significance of the results obtained in the study. It should relate the findings back to the research objectives and clearly explain how they address the problem posed in the introduction. The section should compare the results with existing literature, highlighting agreements, differences, or improvements over previous work where appropriate. It should provide a critical analysis of the implications of the findings, including their theoretical, practical, or applied significance. Any unexpected results should be explained or reasonably interpreted within the context of the model or methodology used. The discussion should also consider the limitations of the study and their possible effects on the results. Overall, this section should move beyond presentation of results to provide meaningful interpretation and insight into their relevance and contribution to the field.

8 Conclusion

The conclusion section should provide a concise summary of the main findings of the study, restating the key results without introducing new information. It should briefly reflect on how the objectives of the work were achieved and highlight the main contributions of the paper in a clear and direct manner. The conclusion should emphasise the significance of the results in relation to the problem addressed, including any theoretical, practical, or applied implications. It may also briefly mention limitations of the study and suggest possible directions for future research. Overall, the conclusion should give the reader a clear and final understanding of the importance and impact of the work.

A Note on References

IJDM allows the use of APA-style referencing. Authors may either use `BibTeX` or the `thebibliography` environment for manual entry of references. All references cited in the text must appear in the reference list, and all entries in the reference list must be cited in the text.

When using the IJDM citation system (based on `natbib`), authors are strongly required to use:

- `\citet{key}` for textual citations, e.g. Brauer Brauer and Castillo-Chavez (2001) developed mathematical models in population biology.
- `\citep{key}` for parenthetical citations, e.g. (Brauer and Castillo-Chavez, 2001) introduced key epidemiological modelling concepts.

Authors must use `\citet` and `\citep` consistently throughout the manuscript.

For works with multiple authors, the following rules apply:

- **Two authors:** Both names must always be written.
  - Example (textual): Brauer and Castillo-Chavez (2001)
  - Example (parenthetical): (Brauer and Castillo-Chavez, 2001)
- **Three or more authors:** Use *et al.* after the first author in in-text citations.
  - Example (textual): Sooknanan Sooknanan et al. (2012)
  - Example (parenthetical): (Sooknanan et al., 2012)

The correct APA in-text rendering will automatically appear as:

- (Sooknanan et al., 2012)
- Sooknanan et al. (2012)

Important:

- Do not manually type "et al." in citations.
- It is automatically handled by the `natbibapa` style.
- Always use `\citet` or `\citep` instead of plain `\cite`.

All references must be included in the `thebibliography` environment in APA format as shown below:

```
\begin{thebibliography}{999}

 \bibitem[Brauer and Castillo-Chavez(2001)]{Brauer2001}
 Brauer, F.; Castillo-Chavez, C. \textit{Mathematical Models in Population Biology a
 Springer: New York, USA, 2001.

 \bibitem[Lotka(1925)]{Lotka1925}
 Lotka, A.J. \textit{Elements of Physical Biology}; Williams and Wilkins: Baltimore,
```

\end{thebibliography}

The reference list must follow strict APA formatting rules, and authors are responsible for ensuring consistency, completeness, and correctness of all entries.

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